

# Unit 04: Sequences, L'Hopital's Rule, Improper Integrals (9 weeks)

Content Area: **Template**  
Course(s):  
Time Period: **Full Year**  
Length: **FY**  
Status: **Published**

## Standards Alignment

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### New Jersey Student Learning Standards

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MA.F-IF	Interpreting Functions
MA.F-IF.A	Understand the concept of a function and use function notation
MA.F-IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$ . The graph of $f$ is the graph of the equation $y = f(x)$ .
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.F-IF.A.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
MA.F-IF.B	Interpret functions that arise in applications in terms of the context
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MA.F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
MA.F-IF.C	Analyze functions using different representations
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.F-IF.C.7a	Graph linear and quadratic functions and show intercepts, maxima, and minima.
MA.F-IF.C.7b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
MA.F-IF.C.7c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
MA.F-IF.C.7d	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
MA.F-IF.C.7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
MA.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

MA.F-IF.C.8a	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
MA.F-IF.C.8b	Use the properties of exponents to interpret expressions for exponential functions.
MA.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

## **Integration of Career Readiness, Life Literacies and Key Skills**

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CRP.K-12.CRP1	Act as a responsible and contributing citizen and employee.
CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP3	Attend to personal health and financial well-being.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP5	Consider the environmental, social and economic impacts of decisions.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP9	Model integrity, ethical leadership and effective management.
CRP.K-12.CRP10	Plan education and career paths aligned to personal goals.
CRP.K-12.CRP11	Use technology to enhance productivity.
CRP.K-12.CRP12	Work productively in teams while using cultural global competence.

## **Technology / Integration of Computer Science and Design Thinking**

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TECH.8.2.12.E	Computational Thinking: Programming: Computational thinking builds and enhances problem solving, allowing students to move beyond using knowledge to creating knowledge.
TECH.8.2.12.E.1	Demonstrate an understanding of the problem-solving capacity of computers in our world.

## **Interdisciplinary Connections: NJSLs for ELA, Social Studies, Science and/or Math Section**

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### **Capacities of the Literate Individual Students Who are College and Career Ready in Reading, Writing, Speaking, Listening, & Language**

They demonstrate independence.

They build strong content knowledge.  
They respond to the varying demands of audience, task, purpose, and discipline.

They comprehend as well as critique.  
They value evidence

They use technology and digital media strategically and capably.

MATH.K-12.1	Make sense of problems and persevere in solving them
LA.K-12.NJSLSA.R	Reading Key Ideas and Details
MATH.K-12.2	Reason abstractly and quantitatively
LA.K-12.NJSLSA.R1	Read closely to determine what the text says explicitly and to make logical inferences and relevant connections from it; cite specific textual evidence when writing or speaking to support conclusions drawn from the text.
MATH.K-12.3	Construct viable arguments and critique the reasoning of others
MATH.K-12.4	Model with mathematics
MATH.K-12.5	Use appropriate tools strategically
MATH.K-12.6	Attend to precision
MATH.K-12.7	Look for and make use of structure
MATH.K-12.8	Look for and express regularity in repeated reasoning
LA.RI.11-12.1	Accurately cite strong and thorough textual evidence, (e.g., via discussion, written response, etc.), to support analysis of what the text says explicitly as well as inferentially, including determining where the text leaves matters uncertain.

## **Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media Literacy**

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see Crosswalks

## **21st Century Life and Careers**

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CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP12	Work productively in teams while using cultural global competence.

## **Stage I: Desired Results**

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## Transfer/Overview/Rationale

### Transfer / Overview / Rationale

#### Unit Rationale

The purpose of this unit...

**Methods for evaluating definite integrals are extended to integrals where one or both limits of integration are infinite and to integrals whose integrands become unbounded on the interval of integration.**

## Meaning

## Essential Questions

Essential Questions

- How can the definite integral be used to find the net or total change of a quantity whose rate of change is known?
- What is the connection between the Fundamental Theorem of Calculus and applications of the integral?
- How do you use the definite integral to find area, volume, and arc length?

## Enduring Understanding/Indicators of Understanding

Enduring Understanding/Indicators of Understanding

- The definite integral of a function over an interval is the limit of the Riemann sum over that interval and can be calculated using a variety of strategies.
- The concept of a limit can be used to understand the behavior of functions.

- The sum of an infinite number of real numbers may converge.

## **Acquisition (Student Learning Objectives)**

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### **Knowledge**

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Students will know...

- An improper integral is an integral that has one or both limits infinite or has an integrand that is unbounded in the interval of integration
- Improper integrals can be determined using limits of definite integrals
- Limits of the indeterminate form may be evaluated using L'Hopital's rule
- A sequence may converge

### **Skills**

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Student will be skilled at ...

- Evaluate an improper integral or show that an improper integral diverges
- Determine the limit of a function using L'Hopital's rule
- Deduce and interpret behavior of functions using limits
- Express limits symbolically using correct notation
- Interpret limits symbolically
- Estimate limits of functions

## Stage 3: Learning Plan

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### Resource and Mentor Texts

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Finney, Ross L., Franklin D. Demana, Bert K. Waits, and Daniel Kennedy. *Calculus-Graphical, Numerical, Algebraic*. Menlo Park: Scott Foresman Addison Wesley, 2012.

Cade, Sharon, Rhea Caldwell, and Jeff Lucia. *Fast Track to a 5: Preparing for the AP Calculus AB and Calculus BC Examinations*. Evanston: McDougal Littell, 2006.

Lederman, David. *Multiple-Choice & Free-response Questions in Preparation for the AP Calculus (AB) Examination*. New York: D&S Marketing Systems, Inc., 2004.

Lifshitz, Maxine. *AP Calculus AB/BC: Preparing for the Advanced Placement Examinations*. New York: Amsco School Publications, Inc., 2004.

Barron's AP Calculus prep book

[http://jamesrahn.com/pages/calculus%20ap/calculus\\_labs.htm](http://jamesrahn.com/pages/calculus%20ap/calculus_labs.htm)

[http://apcentral.collegeboard.com/apc/public/courses/teachers\\_corner/2178.html](http://apcentral.collegeboard.com/apc/public/courses/teachers_corner/2178.html)

<https://www.khanacademy.org/>

<http://webspaceship.edu/msrenault/GeoGebraCalculus/GeoGebraCalculusApplets.html>

## **Formative Assessment Strategies**

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- Announced quizzes
- Short unannounced quizzes
- Take home AP problem packets
- Teacher observations
- Homework (quick check, collect, quiz)
- Exit tickets
- Teacher observations
- Student work on the board
- Kahoot
- Teacher guided questions
- Students submit picture of quiz/problem solution to google classroom, then as a class we review the answer so they can correct their work and don't have to wait until next class to review.
- Use google forms, todays meet, and/or google hangouts for do nows/exit tickets, spot checks, and back channeling
- Chapter tests

## **Learning Activities/Unit of Study**

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- Review homework
- Warm Up activity
- Guided notes
- Group work (matching, war, practice problems, exploration, work on problems at the board)
- Calculator explorations
- Foerster calculus explorations
- Stations - (Small group instruction, skills practice - scavenger hunts, online games, board work)
- Kahoot to reinforce skills
- Review and practice skills using a variety of materials - (text, workbook, chromebook, games, activities, discussion)
- Foldables--creates an organized study guide per chapter
- Jeopardy style review games
- Students "as teachers" where they present a method or formula they discovered through investigation

## **Modifications and/or Accommodations**

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### **Suggested Modifications (ELL, Sp. Ed, Gifted, At-risk of Failure)**

#### **English Language Learners**

Native language support: The teacher provides auditory or written content to students in their native

language.

**Adjusted Speech:** The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.

**Visuals:** The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subjects at hand.

**Front-Loading Vocabulary:** The teacher front loads vocabulary. This means providing students with a list of important vocabulary words they will need to know for a book, lesson, etc. prior to the lesson being taught. Including pictures to go with the vocabulary words is also very beneficial for the students.

## Special Education Students

**Chunking:** The teacher presents information in a way that makes it easy for students to understand and remember. Chunking is based on the presumption that our working memory is easily overloaded by excessive detail. The best way to deliver information is to organize it into meaningful units. Because students with special needs get overloaded easily, chunking is an effective strategy to use with them.

**Checking for Understanding:** It is important to constantly check for understanding, especially for students who have accommodations. Teachers want to make sure students understand the concepts being covered in a way that makes sense to them.

**Extra time:** The teacher provides students with special needs extra time to complete work or answer questions. It is important to give students enough time to process their thoughts.

**Oral Reading:** The teacher will read work orally to students. Class work such as tests and literature circles may need to be read aloud to the student.

**Timers:** The teacher will use timers as an instructional tool. The use of timers is beneficial for students who have trouble completing tasks. Timers can be helpful so the student is aware of how much time they have to complete an assignment.

## Students with 504 Plans

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## Gifted & Talented Strategies

Extensions/Enrichments: Teachers will provide gifted and talented students with extension/enrichment projects. Students will be challenged to further their understanding, to apply acquired knowledge, and/or to produce something in reference to acquired knowledge.

Modify/Change Activities: Teachers will monitor and modify activities to accommodate those students who need to be challenged further. Additional reading, problem-solving, writing, or project work is necessary for those students who are ready to move on at a rate more accelerated than their peers. In this way, G & T students are provided the same opportunity for support as special needs students.

## Students at Risk of School Failure

Directions or Instructions: Make sure directions and/or instructions are given in limited numbers. Give directions/instructions verbally and in simple written format. Ask students to repeat the instructions or directions to ensure understanding occurs. Check back with the student to ensure he/she hasn't forgotten.

Peer Support: Peers can help build confidence in other students by assisting in peer learning. Many teachers use the 'ask 3 before me' approach. This is fine, however, a student at risk may have to have a specific student or two to ask. Set this up for the student so he/she knows who to ask for clarification before going to you.

Alternate or Modified Assignments: Always ask yourself, "How can I modify this assignment to ensure the students at risk are able to complete it?" Sometimes you'll simplify the task, reduce the length of the assignment or allow for a different mode of delivery. For instance, many students may hand something in, the at-risk student may jot notes and give you the information verbally. Or, it just may be that you will need to assign an alternate assignment.

Increase One to One Time: When other students are working, always touch base with your students at risk and find out if they're on track or needing some additional support. A few minutes here and there will go a long way to intervene as the need presents itself.

Contracts: It helps to have a working contract between you and your students at risk. This helps prioritize the tasks that need to be done and ensure completion happens. Each day write down what needs to be completed, as the tasks are done, provide a checkmark or happy face. The goal of using contracts is to eventually have the student come to you for completion sign-offs.

Hands On: As much as possible, think in concrete terms and provide hands-on tasks. This means a child doing math may require a calculator or counters. The child may need to tape record comprehension activities instead of writing them. A child may have to listen to a story being read instead of reading it him/herself.

Tests/Assessments: Tests can be done orally if need be. Break tests down in smaller increments by having a portion of the test in the morning, another portion after lunch and the final part the next day.

Seating: Seat students near a helping peer or with quick access to the teacher. Those with hearing or sight issues need to be close to the instruction which often means near the front.