Unit 03: Integrals (5 Weeks)

Content Area: **Template**
Course(s):
Time Period: **Full Year**
Length: **FY**
Status: **Published**

Standards Alignment

New Jersey Student Learning Standards

Capacities of the Literate Individual
Students Who are College and Career Ready in Reading, Writing, Speaking, Listening, & Language

They demonstrate independence.

They build strong content knowledge.

They respond to the varying demands of audience, task, purpose, and discipline.

They comprehend as well as critique.

They value evidence.

They use technology and digital media strategically and capably.

MA.F-IF	Interpreting Functions
LA.K-12.NJSLSA.R	Reading
MA.F-IF.B	Interpret functions that arise in applications in terms of the context
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.F-IF.C	Analyze functions using different representations
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
LA.K-12.NJSLSA.R10	Read and comprehend complex literary and informational texts independently and proficiently with scaffolding as needed.
LA.K-12.NJSLSA.W	Writing
LA.K-12.NJSLSA.W2	Write informative/explanatory texts to examine and convey complex ideas and

	information clearly and accurately through the effective selection, organization, and analysis of content.
LA.RST.11-12.10	By the end of grade 12, read and comprehend science/technical texts in the grades 11-CCR text complexity band independently and proficiently.
LA.WHST.11-12.2	Write informative/explanatory texts, including the narration of historical events, scientific procedures/experiments, or technical processes.
MA.G-GMD	Geometric Measurement and Dimension
MA.G-GMD.B	Visualize relationships between two-dimensional and three-dimensional objects
MA.G-GMD.B.4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
MA.G-MG	Modeling with Geometry
MA.G-MG.A	Apply geometric concepts in modeling situations
MA.G-MG.A.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
MA.G-MG.A.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

Integration of Career Readiness, Life Literacies and Key Skills

CRP.K-12.CRP1	Act as a responsible and contributing citizen and employee.
CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP3	Attend to personal health and financial well-being.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP5	Consider the environmental, social and economic impacts of decisions.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP9	Model integrity, ethical leadership and effective management.
CRP.K-12.CRP10	Plan education and career paths aligned to personal goals.
CRP.K-12.CRP11	Use technology to enhance productivity.
CRP.K-12.CRP12	Work productively in teams while using cultural global competence.

Technology / Integration of Computer Science and Design Thinking

TECH.8.2.12.E	Computational Thinking: Programming: Computational thinking builds and enhances problem solving, allowing students to move beyond using knowledge to creating knowledge.
TECH.8.2.12.E.1	Demonstrate an understanding of the problem-solving capacity of computers in our world.

Interdisciplinary Connections: NJSLS for ELA, Social Studies, Science and/or Math Section

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MATH.K-12.1	Make sense of problems and persevere in solving them
LA.K-12.NJSLSA.R	Reading
	Key Ideas and Details
MATH.K-12.2	Reason abstractly and quantitatively
LA.K-12.NJSLSA.R1	Read closely to determine what the text says explicitly and to make logical inferences and relevant connections from it; cite specific textual evidence when writing or speaking to support conclusions drawn from the text.
MATH.K-12.3	Construct viable arguments and critique the reasoning of others
MATH.K-12.4	Model with mathematics
MATH.K-12.5	Use appropriate tools strategically
MATH.K-12.6	Attend to precision
MATH.K-12.7	Look for and make use of structure
MATH.K-12.8	Look for and express regularity in repeated reasoning
LA.RI.11-12.1	Accurately cite strong and thorough textual evidence, (e.g., via discussion, written response, etc.), to support analysis of what the text says explicitly as well as inferentially, including determining where the text leaves matters uncertain.

Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media LiteracyNew Section

see Crosswalks

21st Century Life and Careers

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.

Stage I: Desired Results

Transfer/Overview/Rationale

Transfer / Overview / Rationale

Unit Rationale

The purpose of this unit…

Integrals are used in a variety of practical and theoretical applications. Definite integrals can be approximated using different methods and can be computed using geometry. Integrals are used in area, volume, and motion applications, and as an accumulation function.

Meaning

Essential Questions

Essential Questions

- How do you find the average value of a function?
- What is the Fundamental Theorem of Calculus?
- When can you use geometry and symmetry help to solve an integral?
- In what situations would you use rectangles or trapezoids to approximate integrals, rather than finding the exact value?
- How do you know if you need to use washers or discs to find a volume?

Enduring Understanding/Indicators of Understanding

Enduring Understanding/Indicators of Understanding

- Antidifferentiation if the inverse process of differentiation.
- The definite integral of a function is the limit of a Riemann sum over that interval and can be calculated using a variety of strategies.
- The Fundamental Theorem of Calculus, which has two distinct formulations, connects differentiation and integration.
- The definite integral of a function over an interval is a mathematical tool with many interpretations and applications involving accumulation.
- Antidifferentiation is an underlying concept involved in solving separable differential equations.
- Calculus can be used to compute area of complex regions of the plane and volumes of complex 3-d figures.

Acquisition (Student Learning Objectives)

Knowledge

Students will know…

- Justification
- Reasoning
- Modeling
- Intepretation
- Application
- Solutions to differential equations are functions or families of functions
- Slope fields provide visual clues to the behavior of solutions to first order differential equations
- Antiderivatives
- Differentiation rules provide the foundation for finding antiderivatives
- A Riemann sum, which requires a partition of an interval, is the sum of products, each of which is the value of a

function at a point in a subinterval multiplied by the length of that subinterval of the partition
- Definite integrals can be approximated for functions that are represented graphically, verbally, numerically, and algebraically
- Definite integrals can be approximated using a left, right, or midpoint Riemann sum, or a trapezoidal sum.
- In some cases, a definite integral can be evaluated by using geometry and the connection between the definite integral and area
- Properties of an integral
- Fundamental Theorem of Calculus
- Techniques for finding antiderivatives include algebraic manipulation and substitution
- A function defined as an integral represents an accumulation of a rate of change
- The definite integral of the rate of change of a quantity over an integral gives the net change of that quantity over that integral
- For a particle in rectilinear motion over an interval of time, the definite integral of velocity represents the particle's displacement over the interval of time, and the definite integral of speed represents the particle's total distance traveled over the interval of time
- Antidifferentiation can be used to find specific solutions to differential equations with given initial conditions
- Some differential equations can be solved by separation of variables
- Solutions to differential equations may be subject to domain restrictions
- The model for exponential growth and decay that arises from the statement "The rate of change of a quantity is proportional to the size of the quantity is dy/dt = ky
- Areas of certain regions in the plan can be calculated with definite integrals.
- The definite integral can be used to express information about accumulation and net change in many applied contexts.
- Volumes of solids with known cross sections, including discs and washers, can be calculated with definite integrals.
- For differential equations, Euler's method provides a procedure for approximating a solution or a point on a solution curve.
- An improper integral is an integral that has one or both limits infinite or has an integrand that is unbounded in the interval of integration.
- Improper integrals can be determined using limits of definite integrals.
- Techniques for finding antiderivatives include integration by parts and nonrepeating linear partial fractions.
- The length of a planar curve defined by a function can be calculated using a definite integral.
- The model for logistic growth that arises from the statement "The rate of change of a quantity is jointly proportional to the size of the quantity and the difference between the quantity and the carrying capacity" is dy/dt = ky(a-y).

Skills

Student will be skilled at …

- Reason with definitions and theorems
- Connect concepts
- Implement algebraic/computational processes
- Connect multiple representations (graphical, numerical, analytical, verbal)
- Build notational fluency
- Communicate methods, justifications, and conclusions accurately and with precise language
- Recognize antiderivatives of basic functions.
- Interpret the definite integral as the limit of a Riemann sum
- Approximate a definite integral
- Calculate a definite integral using areas and properties of definite integrals
- Analyze functions defined by an integral
- Calculate antiderivatives
- Evaluate definite integrals

- Interpret the meaning of a definite integral within a problem
- Apply definite integrals to problems involving the average value of a function, motion, area, volume, and length of a curve
- Analyze differential equations to obtain general and specific solutions
- Interpret, create, and solve differential equations from problems in context
- Use the definite integral to solve problems in various contexts
- Evaluate an improper integral or show that an improper integral diverges

Stage 3: Learning Plan

Resource and Mentor Texts

Finney, Ross L., Franklin D. Demana, Bert K. Waits, and Daniel Kennedy. *Calculus-Graphical, Numerical, Algebraic*. Menlow Park: Scott Forseman Addison Wesley, 2012.

Cade, Sharon, Rhea Caldwell, and Jeff Lucia. Fast Track to a 5: *Preparing for the AP Calculus Am and Calculus BC Examinations*. Evanston: McDougal Littell, 2006.

Lederman, David. *Multiple-Choice & Free-response Questions in Preparation for the AP Calculus (AB) Examination*. New York: D&S Marketing Systems, Inc., 2004.

Lifshitz, Maxine. *AP Calculus AB/BC: Preparing for the Advanced Placement Examinations*. New York: Amsco School Publications, Inc., 2004.

Barron's AP Calculus prep book

http://jamesrahn.com/pages/calculus%20ap/calculus_labs.htm

http://apcentral.collegeboard.com/apc/public/courses/teachers_corner/2178.html

https://www.khanacademy.org/

Formative Assessment Strategies

- Announced quizzes
- Short unannounced quizzes
- Take home AP problem packets
- Teacher observations
- Homework (quick check, collect, quiz)
- Exit tickets
- Teacher observations
- Student work on the board
- Kahoot
- Teacher guided questions
- Students submit picture of quiz/problem solution to google classroom, then as a class we review the answer so they can correct their work and don't have to wait until next class to review.
- Use google forms, todays meet, and/or google hangouts for do nows/exit tickets, spot checks, and back channeling
- Chapter tests

calc 6.1-6.2 quiz.doc
calc 6.1-6.3 quiz.doc
calc 6.1 quiz.doc
calc 6.1-6.2 quiz2.doc
calc 6.3 quiz.doc
calc 6.4 quiz.doc
calc 7.3 7.3 quiz.doc
calc 7.3 quiz 1.doc

calc bc chapter 6 test

calc bc chapter 7 test

Learning Activities/Unit of Study

- Review homework
- Warm Up activity
- Guided notes
- Group work (matching, war, practice problems, exploration, work on problems at the board)
- Calculator explorations
- Foerster calculus explorations
- Stations - (Small group instruction, skills practice - scavenger hunts, online games, board work)
- Kahoot to reinforce skills
- Review and practice skills using a variety of materials - (text, workbook, chromebook, games, activities,

discussion)
- Foldables--creates an organized study guide per chapter
- Jeopardy style review games
- Students “as teachers” where they present a method or formula they discovered through investigation

calc ch 6.pdf
calc ch 7.pdf
integrals foerster.pdf

Modifications and/or Accommodations

Suggested Modifications (ELL, Sp. Ed, GIfted, At-risk of Failure)

English Language Learners

Native language support: The teacher provides auditory or written content to students in their native language.

Adjusted Speech: The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.

Visuals: The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subjects at hand.

Front-Loading Vocabulary: The teacher front loads vocabulary. This means providing students with a list of important vocabulary words they will need to know for a book, lesson, etc. prior to the lesson being taught. Including pictures to go with the vocabulary words is also very beneficial for the students.

Special Education Students

Chunking: The teacher presents information in a way that makes it easy for students to understand and remember. Chunking is based on the presumption that our working memory is easily overloaded by excessive detail. The best way to deliver information is to organize it into meaningful units. Because students with special needs get overloaded easily, chunking is an effective strategy to use with them.

Checking for Understanding: It is important to constantly check for understanding, especially for students who have accommodations. Teachers want to make sure students understand the concepts being covered in a way that makes sense to them.

Extra time: The teacher provides students with special needs extra time to complete work or answer questions. It is important to give students enough time to process their thoughts.

Oral Reading: The teacher will read work orally to students. Class work such as tests and literature

circles may need to be read aloud to the student.

Timers: The teacher will use timers as an instructional tool. The use of timers is beneficial for students who have trouble completing tasks. Timers can be helpful so the student is aware of how much time they have to complete an assignment.

Students with 504 Plans

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Gifted & Talented Strategies

Extensions/Enrichments: Teachers will provide gifted and talented students with extension/enrichment projects. Students will be challenged to further their understanding, to apply acquired knowledge, and/or to produce something in reference to acquired knowledge.

Modify/Change Activities: Teachers will monitor and modify activities to accommodate those students who need to be challenged further. Additional reading, problem-solving, writing, or project work is necessary for those students who are ready to move on at a rate more accelerated than their peers. In this way, G & T students are provided the same opportunity for support as special needs students.

Students at Risk of School Failure

Directions or Instructions: Make sure directions and/or instructions are given in limited numbers. Give directions/instructions verbally and in simple written format. Ask students to repeat the instructions or directions to ensure understanding occurs. Check back with the student to ensure he/she hasn't forgotten.

Peer Support: Peers can help build confidence in other students by assisting in peer learning. Many teachers use the 'ask 3 before me' approach. This is fine, however, a student at risk may have to have a specific student or two to ask. Set this up for the student so he/she knows who to ask for clarification before going to you.

Alternate or Modified Assignments: Always ask yourself, "How can I modify this assignment to ensure the students at risk are able to complete it?" Sometimes you'll simplify the task, reduce the length of the assignment or allow for a different mode of delivery. For instance, many students may hand something in, the at-risk student may jot notes and give you the information verbally. Or, it just may be that you will need to assign an alternate assignment.

Increase One to One Time: When other students are working, always touch base with your students at risk and find out if they're on track or needing some additional support. A few minutes here and there will go a long way to intervene as the need presents itself.

Contracts: It helps to have a working contract between you and your students at risk. This helps prioritize the tasks that need to be done and ensure completion happens. Each day write down what needs to be completed, as the tasks are done, provide a checkmark or happy face. The goal of using contracts is to eventually have the student come to you for completion sign-offs.

Hands On: As much as possible, think in concrete terms and provide hands-on tasks. This means a child doing math may require a calculator or counters. The child may need to tape record comprehension activities instead of writing them. A child may have to listen to a story being read instead of reading it him/herself.

Tests/Assessments: Tests can be done orally if need be. Break tests down in smaller increments by having a portion of the test in the morning, another portion after lunch and the final part the next day.

Seating: Seat students near a helping peer or with quick access to the teacher. Those with hearing or sight issues need to be close to the instruction which often means near the front.