

Unit 04: Vectors, Matrices, and Probability (Weeks 19-27)

Content Area: **Template**

Course(s):

Time Period: **Full Year**

Length: **FY**

Status: **Published**

Standards Alignment

New Jersey Student Learning Standards

LA.K-12.NJSLSA.R2	Determine central ideas or themes of a text and analyze their development; summarize the key supporting details and ideas.
LA.K-12.NJSLSA.R3	Analyze how and why individuals, events, and ideas develop and interact over the course of a text.
LA.K-12.NJSLSA.R4	Interpret words and phrases as they are used in a text, including determining technical, connotative, and figurative meanings, and analyze how specific word choices shape meaning or tone.
LA.K-12.NJSLSA.R5	Analyze the structure of texts, including how specific sentences, paragraphs, and larger portions of the text (e.g., a section, chapter, scene, or stanza) relate to each other and the whole.
LA.K-12.NJSLSA.R10	Read and comprehend complex literary and informational texts independently and proficiently with scaffolding as needed.
MATH.9-12.S.IC	Making Inferences and Justifying Conclusions
LA.K-12.NJSLSA.W	Writing
LA.RST.11-12.2	Determine the central ideas, themes, or conclusions of a text; summarize complex concepts, processes, or information presented in a text by paraphrasing them in simpler but still accurate terms.
LA.RST.11-12.3	Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.
LA.K-12.NJSLSA.W1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
LA.K-12.NJSLSA.W2	Write informative/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.
LA.RST.11-12.4	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 11-12 texts and topics.
LA.RST.11-12.5	Analyze how the text structures information or ideas into categories or hierarchies, demonstrating understanding of the information or ideas.
LA.K-12.NJSLSA.W4	Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.
MATH.9-12.S.CP	Conditional Probability and the Rules of Probability

LA.RST.11-12.10	By the end of grade 12, read and comprehend science/technical texts in the grades 11-CCR text complexity band independently and proficiently.
LA.K-12.NJSLSA.W9	Draw evidence from literary or informational texts to support analysis, reflection, and research.
MATH.9-12.A.REI.C	Solve systems of equations
LA.WHST.11-12.1	Write arguments focused on discipline-specific content.
MATH.9-12.A.REI.C.8	Represent a system of linear equations as a single matrix equation in a vector variable.
LA.WHST.11-12.1.D	Establish and maintain a style and tone appropriate to the audience and purpose (e.g., formal and objective for academic writing) while attending to the norms and conventions of the discipline in which they are writing.
LA.WHST.11-12.1.E	Provide a concluding paragraph or section that supports the argument presented.
MATH.9-12.A.REI.C.9	Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater).
LA.WHST.11-12.2	Write informative/explanatory texts, including the narration of historical events, scientific procedures/experiments, or technical processes.
MATH.9-12.S.MD	Using Probability to Make Decisions
LA.WHST.11-12.2.D	Use precise language, domain-specific vocabulary and techniques such as metaphor, simile, and analogy to manage the complexity of the topic; convey a knowledgeable stance in a style that responds to the discipline and context as well as to the expertise of likely readers.
LA.WHST.11-12.2.E	Provide a concluding paragraph or section that supports the argument presented.
LA.WHST.11-12.4	Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.
LA.WHST.11-12.9	Draw evidence from informational texts to support analysis, reflection, and research.

Integration of Career Readiness, Life Literacies and Key Skills

CRP.K-12.CRP1	Act as a responsible and contributing citizen and employee.
CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP3	Attend to personal health and financial well-being.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP5	Consider the environmental, social and economic impacts of decisions.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP9	Model integrity, ethical leadership and effective management.
CRP.K-12.CRP10	Plan education and career paths aligned to personal goals.
CRP.K-12.CRP11	Use technology to enhance productivity.
CRP.K-12.CRP12	Work productively in teams while using cultural global competence.

Technology / Integration of Computer Science and Design Thinking

Interdisciplinary Connections: NJSL for ELA, Social Studies, Science and/or Math Section

Capacities of the Literate Individual

Students Who are College and Career Ready in Reading, Writing, Speaking, Listening, & Language

They demonstrate independence.

They build strong content knowledge.

They comprehend as well as critique.

They value evidence.

MATH.K-12.1	Make sense of problems and persevere in solving them
MATH.K-12.2	Reason abstractly and quantitatively
MATH.K-12.4	Model with mathematics
MA.N-VM	Vector and Matrix Quantities
MA.N-VM.A	Represent and model with vector quantities.
MA.N-VM.A.1	Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., \boldsymbol{v} , $ \boldsymbol{v} $, $ \boldsymbol{v} $, v).
MA.N-VM.A.2	Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
MA.N-VM.A.3	Solve problems involving velocity and other quantities that can be represented by vectors.
MA.N-VM.B	Perform operations on vectors.
MA.N-VM.B.4	Add and subtract vectors.
MA.N-VM.B.4a	Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
MA.N-VM.B.4b	Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
MA.N-VM.B.4c	Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of \boldsymbol{w} , with the same magnitude as \boldsymbol{w} and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
MA.N-VM.B.5	Multiply a vector by a scalar.

MA.N-VM.C	Perform operations on matrices and use matrices in applications.
MA.N-VM.C.6	Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
MA.N-VM.C.7	Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
MA.N-VM.C.8	Add, subtract, and multiply matrices of appropriate dimensions.
MA.N-VM.C.9	Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
MA.N-VM.C.10	Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
MA.N-VM.C.11	Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
MA.N-VM.C.12	Work with 2×2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media Literacy

see Crosswalks

21st Century Life and Careers

Stage I: Desired Results

Transfer/Overview/Rationale

Transfer / Overview / Rationale

Unit Rationale

The purpose of this unit...

Meaning

Essential Questions

Essential Questions

- How do you represent and perform operations with vector quantities?
- How do you write a vector as a sum of two vector components?
- How do you use matrices to solve systems of equations?
- How do you perform operations on matrices?
- How do you find and use the inverse of a square matrix?
- How do you find the determinant of a square matrix?
- How do you use matrices to solve systems of equations, find areas of triangles, and write coded messages?
- How do you find the probability that a series of events will occur?

Enduring Understanding/Indicators of Understanding

Enduring Understanding/Indicators of Understanding

- Vectors and matrices can be used to model and solve real life problems.
- Matrices can be used to solve linear equations in two or more variables.
- Determinants are used in multiple branches of mathematics.

Acquisition (Student Learning Objectives)

Knowledge

Knowledge

Students will know...

- vectors
- initial point
- terminal point
- magnitude
- component form
- standard position
- unit vector
- scalar multiplication
- vector addition
- linear combination
- direction angle

- dot products
- orthogonal
- matrix
- augmented matrix
- row-echelon form
- matrix multiplication
- inverse matrices
- determinants
- Cramer's rule
- probability

Skills

Skills

Student will be skilled at ...

- representing vectors as directed line segments
- writing the component form of a vector
- performing basic vector operations
- writing vectors as linear combinations of unit vectors
- finding the direction angles of vectors
- finding the dot product of two vectors
- finding the angle between two vectors
- writing a vector as the sum of two vector components
- writing matrices and identifying their orders
- performing operations on matrices
- using matrices to solve systems of linear equations
- verifying that two matrices are inverses of each other
- finding inverses
- using inverses to solve systems of linear equations
- finding determinants
- using Cramer's rule
- using determinants to find areas of triangles, test for collinear points and find an equations of a line passing through two points
- finding probabilities of events

Stage 3: Learning Plan

Resource and Mentor Texts

Resources and Mentor Texts

Larson, Ron and Robert Hostetler. *Precalculus*. Boston: Houghton Mifflin Company, 2007.

- Teacher created materials.
- Resources supplied by publisher.
- Resource from the Internet.

Formative Assessment Strategies

Formative Assessment Strategies

-Announced quizzes

-Short unannounced quizzes

-Homework quizzes

-Exit tickets

-Teacher observations

Learning Activities/Unit of Study

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Chapter 6

6.3: Vectors in the Plane

- notes/examples

6.4: Vectors and Dot Products

- notes/examples

Chapter 8

8.1: Matrices and Systems of Equations

- notes/examples

8.2: Operations with Matrices

- notes/examples

8.3: The Inverse of a Square Matrix

- notes/examples

8.4: The Determinant of a Square Matrix

- notes/examples

8.5: Applications of Matrices and Determinants

- notes/examples

Chapter 9

9.7: Probability

- notes/examples

Modifications and/or Accommodations

Suggested Modifications (ELL, Sp. Ed, Gifted, At-risk of Failure)

English Language Learners

Native language support: The teacher provides auditory or written content to students in their native language.

Adjusted Speech: The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.

Visuals: The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subjects at hand.

Front-Loading Vocabulary: The teacher front loads vocabulary. This means providing students with a list of important vocabulary words they will need to know for a book, lesson, etc. prior to the lesson being taught. Including pictures to go with the vocabulary words is also very beneficial for the students.

Special Education Students

Chunking: The teacher presents information in a way that makes it easy for students to understand and remember. Chunking is based on the presumption that our working memory is easily overloaded by excessive detail. The best way to deliver information is to organize it into meaningful units. Because students with special needs get overloaded easily, chunking is an effective strategy to use with them.

Checking for Understanding: It is important to constantly check for understanding, especially for students who have accommodations. Teachers want to make sure students understand the concepts being covered in a way that makes sense to them.

Extra time: The teacher provides students with special needs extra time to complete work or answer questions. It is important to give students enough time to process their thoughts.

Oral Reading: The teacher will read work orally to students. Class work such as tests and literature circles may need to be read aloud to the student.

Timers: The teacher will use timers as an instructional tool. The use of timers is beneficial for students who have trouble completing tasks. Timers can be helpful so the student is aware of how much time they have to complete an assignment.

Students with 504 Plans

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Gifted & Talented Strategies

Extensions/Enrichments: Teachers will provide gifted and talented students with extension/enrichment projects. Students will be challenged to further their understanding, to apply acquired knowledge, and/or to produce something in reference to acquired knowledge.

Modify/Change Activities: Teachers will monitor and modify activities to accommodate those students who need to be challenged further. Additional reading, problem-solving, writing, or project work is necessary for those students who are ready to move on at a rate more accelerated than their peers. In this way, G & T students are provided the same opportunity for support as special needs students.

Students at Risk of School Failure

Directions or Instructions: Make sure directions and/or instructions are given in limited numbers. Give directions/instructions verbally and in simple written format. Ask students to repeat the instructions or directions to ensure understanding occurs. Check back with the student to ensure he/she hasn't forgotten.

Peer Support: Peers can help build confidence in other students by assisting in peer learning. Many teachers use the 'ask 3 before me' approach. This is fine, however, a student at risk may have to have a specific student or two to ask. Set this up for the student so he/she knows who to ask for clarification before going to you.

Alternate or Modified Assignments: Always ask yourself, "How can I modify this assignment to ensure the students at risk are able to complete it?" Sometimes you'll simplify the task, reduce the length of the assignment or allow for a different mode of delivery. For instance, many students may hand something in, the at-risk student may jot notes and give you the information verbally. Or, it just may be that you will need to assign an alternate assignment.

Increase One to One Time: When other students are working, always touch base with your students at risk and find out if they're on track or needing some additional support. A few minutes here and there will go a long way to intervene as the need presents itself.

Contracts: It helps to have a working contract between you and your students at risk. This helps

prioritize the tasks that need to be done and ensure completion happens. Each day write down what needs to be completed, as the tasks are done, provide a checkmark or happy face. The goal of using contracts is to eventually have the student come to you for completion sign-offs.

Hands On: As much as possible, think in concrete terms and provide hands-on tasks. This means a child doing math may require a calculator or counters. The child may need to tape record comprehension activities instead of writing them. A child may have to listen to a story being read instead of reading it him/herself.

Tests/Assessments: Tests can be done orally if need be. Break tests down in smaller increments by having a portion of the test in the morning, another portion after lunch and the final part the next day.

Seating: Seat students near a helping peer or with quick access to the teacher. Those with hearing or sight issues need to be close to the instruction which often means near the front.