

# Unit 01: Linear, Polynomial, and Rational Functions (Weeks1-6)

Content Area: **Template**

Course(s):

Time Period: **Full Year**

Length:

Status: **Published**

## Standards Alignment

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### New Jersey Student Learning Standards

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MATH.9-12.F.BF.A.1	Write a function that describes a relationship between two quantities.
LA.K-12.NJSLSA.R2	Determine central ideas or themes of a text and analyze their development; summarize the key supporting details and ideas.
MATH.9-12.F.BF.A.1.b	Combine standard function types using arithmetic operations.
LA.K-12.NJSLSA.R3	Analyze how and why individuals, events, and ideas develop and interact over the course of a text.
MATH.9-12.F.BF.A.1.c	Compose functions.
LA.K-12.NJSLSA.R4	Interpret words and phrases as they are used in a text, including determining technical, connotative, and figurative meanings, and analyze how specific word choices shape meaning or tone.
LA.K-12.NJSLSA.R5	Analyze the structure of texts, including how specific sentences, paragraphs, and larger portions of the text (e.g., a section, chapter, scene, or stanza) relate to each other and the whole.
MATH.9-12.F.BF.B	Build new functions from existing functions
MATH.9-12.F.BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$ , $k f(x)$ , $f(kx)$ , and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MATH.9-12.F.BF.B.4	Find inverse functions.
MATH.9-12.F.BF.B.4.a	Solve an equation of the form $f(x) = c$ for a simple function $f$ that has an inverse and write an expression for the inverse.
LA.K-12.NJSLSA.R10	Read and comprehend complex literary and informational texts independently and proficiently with scaffolding as needed.
MATH.9-12.F.BF.B.4.b	Verify by composition that one function is the inverse of another.
MATH.9-12.F.BF.B.4.c	Read values of an inverse function from a graph or a table, given that the function has an inverse.
MATH.9-12.F.BF.B.4.d	Produce an invertible function from a non-invertible function by restricting the domain.
LA.RST.11-12.2	Determine the central ideas, themes, or conclusions of a text; summarize complex concepts, processes, or information presented in a text by paraphrasing them in simpler but still accurate terms.
LA.K-12.NJSLSA.W	Writing

MATH.9-12.F.IF	Interpreting Functions
LA.RST.11-12.3	Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.
MATH.9-12.F.IF.A	Understand the concept of a function and use function notation
LA.K-12.NJSLSA.W1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
LA.RST.11-12.4	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 11-12 texts and topics.
LA.K-12.NJSLSA.W2	Write informative/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.
MATH.9-12.F.IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$ . The graph of $f$ is the graph of the equation $y = f(x)$ .
MATH.9-12.F.IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
LA.RST.11-12.5	Analyze how the text structures information or ideas into categories or hierarchies, demonstrating understanding of the information or ideas.
MATH.9-12.F.IF.A.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
LA.K-12.NJSLSA.W4	Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.
MATH.9-12.F.IF.B	Interpret functions that arise in applications in terms of the context
MATH.9-12.F.IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MATH.9-12.F.IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
LA.RST.11-12.10	By the end of grade 12, read and comprehend science/technical texts in the grades 11-CCR text complexity band independently and proficiently.
MATH.9-12.F.IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
LA.K-12.NJSLSA.W9	Draw evidence from literary or informational texts to support analysis, reflection, and research.
MATH.9-12.F.IF.C	Analyze functions using different representations
MATH.9-12.F.IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
LA.WHST.11-12.1	Write arguments focused on discipline-specific content.
MATH.9-12.F.IF.C.7.a	Graph linear and quadratic functions and show intercepts, maxima, and minima.
MATH.9-12.F.IF.C.7.b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
MATH.9-12.F.IF.C.7.c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
LA.WHST.11-12.1.D	Establish and maintain a style and tone appropriate to the audience and purpose (e.g.,

	formal and objective for academic writing) while attending to the norms and conventions of the discipline in which they are writing.
MATH.9-12.F.IF.C.7.d	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
LA.WHST.11-12.1.E	Provide a concluding paragraph or section that supports the argument presented.
LA.WHST.11-12.2	Write informative/explanatory texts, including the narration of historical events, scientific procedures/experiments, or technical processes.
MATH.9-12.F.IF.C.8.a	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
MATH.9-12.F.IF.C.8.b	Use the properties of exponents to interpret expressions for exponential functions.
LA.WHST.11-12.2.D	Use precise language, domain-specific vocabulary and techniques such as metaphor, simile, and analogy to manage the complexity of the topic; convey a knowledgeable stance in a style that responds to the discipline and context as well as to the expertise of likely readers.
LA.WHST.11-12.2.E	Provide a concluding paragraph or section that supports the argument presented.
MATH.9-12.F.IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
LA.WHST.11-12.4	Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.
LA.WHST.11-12.9	Draw evidence from informational texts to support analysis, reflection, and research.

## **Integration of Career Readiness, Life Literacies and Key Skills**

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CRP.K-12.CRP1	Act as a responsible and contributing citizen and employee.
CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP3	Attend to personal health and financial well-being.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP5	Consider the environmental, social and economic impacts of decisions.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP9	Model integrity, ethical leadership and effective management.
CRP.K-12.CRP10	Plan education and career paths aligned to personal goals.
CRP.K-12.CRP11	Use technology to enhance productivity.
CRP.K-12.CRP12	Work productively in teams while using cultural global competence.

## **Technology / Integration of Computer Science and Design Thinking**

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## **Interdisciplinary Connections: NJSLs for ELA, Social Studies, Science and/or Math**

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## Section

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### Capacities of the Literate Individual

#### Students Who are College and Career Ready in Reading, Writing, Speaking, Listening, & Language

They demonstrate independence.

They build strong content knowledge.

They comprehend as well as critique.

They value evidence.

MATH.K-12.1	Make sense of problems and persevere in solving them
MA.F-IF.A	Understand the concept of a function and use function notation
MA.F-IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$ . The graph of $f$ is the graph of the equation $y = f(x)$ .
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MATH.K-12.4	Model with mathematics
MA.F-IF.B	Interpret functions that arise in applications in terms of the context
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MATH.K-12.6	Attend to precision
MA.F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
MA.F-IF.C	Analyze functions using different representations
MA.N-CN	The Complex Number System
MA.N-CN.A	Perform arithmetic operations with complex numbers.
MA.F-IF.C.7a	Graph linear and quadratic functions and show intercepts, maxima, and minima.
MA.F-IF.C.7b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
MA.A-APR	Arithmetic with Polynomials and Rational Expressions
MA.F-IF.C.7c	Graph polynomial functions, identifying zeros when suitable factorizations are available,

	and showing end behavior.
MA.N-CN.A.3	Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
MA.F-IF.C.7d	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
MA.A-APR.B	Understand the relationship between zeros and factors of polynomials
MA.A-APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
MA.F-IF.C.8a	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
MA.N-CN.C.7	Solve quadratic equations with real coefficients that have complex solutions.
MA.N-CN.C.9	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
MA.A-APR.D	Rewrite rational expressions
MA.F-BF.A	Build a function that models a relationship between two quantities
MA.A-APR.D.6	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$ , where $a(x)$ , $b(x)$ , $q(x)$ , and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.
MA.A-APR.D.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
MA.A-CED.A	Create equations that describe numbers or relationships
MA.A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems.
MA.F-BF.A.1c	Compose functions.
MA.F-BF.B	Build new functions from existing functions
MA.F-BF.B.4	Find inverse functions.
MA.F-BF.B.4b	Verify by composition that one function is the inverse of another.
MA.F-BF.B.4c	Read values of an inverse function from a graph or a table, given that the function has an inverse.
MA.F-BF.B.4d	Produce an invertible function from a non-invertible function by restricting the domain.
MA.A-REI.B	Solve equations and inequalities in one variable
MA.A-REI.B.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
MA.A-REI.B.4	Solve quadratic equations in one variable.
MA.A-REI.D	Represent and solve equations and inequalities graphically
MA.A-REI.D.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

## **Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media LiteracyNew Section**

see Crosswalks

## 21st Century Life and Careers

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### Stage I: Desired Results

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### Transfer/Overview/Rationale

#### Transfer / Overview / Rationale

Unit Rationale

The purpose of this unit...

### Meaning

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### Essential Questions

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Essential Questions

- How do you identify intercepts and symmetry in order to sketch graphs of equations?
- What are the important defining characteristics and representations of a function?
- How is the graph of a function used to determine the key elements of a function?
- How do you write and draw graphs for simple transformations of functions?
- What is the inverse of a function?
- How are quadratic, polynomial, and rational functions used to model real-world phenomena?
- Why are zeros of a polynomial function important?

### Enduring Understanding/Indicators of Understanding

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Enduring Understanding/Indicators of Understanding

- Functions can be represented numerically, graphically, algebraically, and verbally.
- The properties of functions are used to model and analyze real world applications.
- Graphs of functions can help you visualize relationships between variables in real life.
- Knowledge of the graphs of common functions and the effects of various transformations make it possible to sketch a wide variety of functions.
- Real world data does not follow a set equation, but different regression formulas can be used to model and analyze data.

## **Acquisition (Student Learning Objectives)**

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### **Knowledge**

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#### Knowledge

Students will know...

- Cartesian Plane
- Distance
- Midpoint
- Intercepts
- Symmetry
- Linear equation
- Parallel and perpendicular
- Function Notation
- Domain and Range
- Zeros
- Increasing / decreasing functions
- Parent functions
- Vertical / horizontal shifts
- Reflections
- Non-ridged transformations
- Composite functions
- Inverses
- Mathematical models
- Quadratic function
- Polynomial function
- Zeros of a polynomial
- Synthetic division
- Complex numbers
- The Fundamental Theorem of Algebra
- Rational Functions
- Polynomial inequalities

### **Skills**

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## Skills

Student will be skilled at ...

- Plot points
- Use distance and midpoint formula
- Find intercepts
- Use symmetry to sketch graphs
- Find slope
- Write linear equations in two variables
- Graph, identify, use parallel and perpendicular lines
- Use function notation and evaluate functions
- Find domain and range
- Find zeros of a function
- Determine intervals of increase / decrease / points of max / min
- Identify even and odd functions
- Identify and graph the eight important parent functions
- Create composite functions and evaluate them
- Find the inverse of a function and verify that they are inverses
- Use mathematical models in all above skills
- Analyze and write quadratic functions in standard form
- Find rational and complex zeros of polynomial functions
- Use long and synthetic division to divide polynomials
- Add, subtract, multiply, and divide complex numbers
- Analyze and graph rational functions

## Stage 3: Learning Plan

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## Resource and Mentor Texts

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Resources and Mentor Texts

Larson, Ron and Robert Hostetler. *Precalculus*. Boston: Houghton Mifflin Company, 2007.

- Teacher created materials.
- Resources supplied by publisher.
- Resource from the Internet.

## **Formative Assessment Strategies**

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### Formative Assessment Strategies

-Announced quizzes

-Short unannounced quizzes

-Homework quizzes

-Exit tickets

-Teacher observations

## **Learning Activities/Unit of Study**

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### Learning Activities/Unit of Study

#### **Chapter 1**

##### **1.1-1.3: Select topics as needed**

- notes/examples

##### **1.4: Functions**

- notes/examples
- practice in groups

##### **1.5: Analyzing Graphs of Functions**

- notes/examples
- group activity: exploring even and odd functions (in notes)

## **1.6: Parent Functions**

- notes/examples

## **1.7: Transformations of Functions**

- notes/examples
- group exploration: investigating transformations (in notes)
- group exploration: investigating the order of transformations (in note)

## **1.8: Composite Functions**

- notes/examples
- practice in group

## **1.9: Inverse Functions**

- notes/examples
- group exploration: investigating graphs of inverse functions (in notes)

## **1.10: Mathematical Modeling and Variation**

- notes/examples

## **Chapter 2**

### **2.1: Quadratic Functions and Models**

- notes/examples
- group exploration: investigating graphs of parabolas (in notes)
- group activity: Applications of Quadratic Equations (attached)

### **2.2: Polynomial Functions of Higher Degree**

- notes/examples
- group exploration: investigating the leading coefficient test

### 2.3: Polynomial and Synthetic Division

- notes/examples

### 2.4: Complex Numbers

- notes/examples

### 2.5: Zeros of Polynomial Functions

- notes/examples

### 2.6: Rational Functions

- notes/examples
- group exploration: analyzing graphs of rational functions (in notes)

[Honors Trig Summer Packet 2012.doc](#)

[trig chap 1 notes.pdf](#)

[trig ch 2 notes.pdf](#)

[quadratic word problems.pdf](#)

[trig 9 18 13.doc](#)

[trig 9 26 13.doc](#)

[trig 10 04 13.doc](#)

[trigo 10 16 13.doc](#)

[trig 10 24 13.doc](#)

[trig 11 1 13.doc](#)

## **Modifications and/or Accommodations**

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### **Suggested Modifications (ELL, Sp. Ed, Gifted, At-risk of Failure)**

#### **English Language Learners**

Native language support: The teacher provides auditory or written content to students in their native language.

Adjusted Speech: The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.

**Visuals:** The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subjects at hand.

**Front-Loading Vocabulary:** The teacher front loads vocabulary. This means providing students with a list of important vocabulary words they will need to know for a book, lesson, etc. prior to the lesson being taught. Including pictures to go with the vocabulary words is also very beneficial for the students.

## Special Education Students

**Chunking:** The teacher presents information in a way that makes it easy for students to understand and remember. Chunking is based on the presumption that our working memory is easily overloaded by excessive detail. The best way to deliver information is to organize it into meaningful units. Because students with special needs get overloaded easily, chunking is an effective strategy to use with them.

**Checking for Understanding:** It is important to constantly check for understanding, especially for students who have accommodations. Teachers want to make sure students understand the concepts being covered in a way that makes sense to them.

**Extra time:** The teacher provides students with special needs extra time to complete work or answer questions. It is important to give students enough time to process their thoughts.

**Oral Reading:** The teacher will read work orally to students. Class work such as tests and literature circles may need to be read aloud to the student.

**Timers:** The teacher will use timers as an instructional tool. The use of timers is beneficial for students who have trouble completing tasks. Timers can be helpful so the student is aware of how much time they have to complete an assignment.

## Students with 504 Plans

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**Checking for Understanding:** It is important to constantly check for understanding, especially for students who have accommodations. Teachers want to make sure students understand the concepts being covered in a way that makes sense to them.

**Extra time:** The teacher provides students with special needs extra time to complete work or answer questions. It is important to give students enough time to process their thoughts.

## Gifted & Talented Strategies

Extensions/Enrichments: Teachers will provide gifted and talented students with extension/enrichment projects. Students will be challenged to further their understanding, to apply acquired knowledge, and/or to produce something in reference to acquired knowledge.

Modify/Change Activities: Teachers will monitor and modify activities to accommodate those students who need to be challenged further. Additional reading, problem-solving, writing, or project work is necessary for those students who are ready to move on at a rate more accelerated than their peers. In this way, G & T students are provided the same opportunity for support as special needs students.

## Students at Risk of School Failure

Directions or Instructions: Make sure directions and/or instructions are given in limited numbers. Give directions/instructions verbally and in simple written format. Ask students to repeat the instructions or directions to ensure understanding occurs. Check back with the student to ensure he/she hasn't forgotten.

Peer Support: Peers can help build confidence in other students by assisting in peer learning. Many teachers use the 'ask 3 before me' approach. This is fine, however, a student at risk may have to have a specific student or two to ask. Set this up for the student so he/she knows who to ask for clarification before going to you.

Alternate or Modified Assignments: Always ask yourself, "How can I modify this assignment to ensure the students at risk are able to complete it?" Sometimes you'll simplify the task, reduce the length of the assignment or allow for a different mode of delivery. For instance, many students may hand something in, the at-risk student may jot notes and give you the information verbally. Or, it just may be that you will need to assign an alternate assignment.

Increase One to One Time: When other students are working, always touch base with your students at risk and find out if they're on track or needing some additional support. A few minutes here and there will go a long way to intervene as the need presents itself.

Contracts: It helps to have a working contract between you and your students at risk. This helps prioritize the tasks that need to be done and ensure completion happens. Each day write down what needs to be completed, as the tasks are done, provide a checkmark or happy face. The goal of using contracts is to eventually have the student come to you for completion sign-offs.

Hands On: As much as possible, think in concrete terms and provide hands-on tasks. This means a child doing math may require a calculator or counters. The child may need to tape record comprehension activities instead of writing them. A child may have to listen to a story being read instead of reading it him/herself.

Tests/Assessments: Tests can be done orally if need be. Break tests down in smaller increments by having a portion of the test in the morning, another portion after lunch and the final part the next day.

Seating: Seat students near a helping peer or with quick access to the teacher. Those with hearing or sight issues need to be close to the instruction which often means near the front.

