Unit 05: Probability

Content Area: **Template**
Course(s):
Time Period: **Full Year**
Length: **FY**
Status: **Published**

Standards Alignment

New Jersey Student Learning Standards

MA.S-ID	Interpreting Categorical and Quantitative Data
MA.S-ID.A	Summarize, represent, and interpret data on a single count or measurement variable
MA.S-ID.A.2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
MA.S-ID.A.3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
MA.S-ID.A.4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
MA.S-ID.B	Summarize, represent, and interpret data on two categorical and quantitative variables
MA.S-ID.B.6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
MA.S-ID.B.6b	Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology.
MA.A-APR	Arithmetic with Polynomials and Rational Expressions
MA.S-IC	Making Inferences and Justifying Conclusions
MA.S-IC.A	Understand and evaluate random processes underlying statistical experiments
MA.S-IC.A.1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
MA.A-APR.C	Use polynomial identities to solve problems
MA.S-IC.B	Make inferences and justify conclusions from sample surveys, experiments, and observational studies
MA.S-IC.B.3	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
MA.S-IC.B.4	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
MA.A-APR.C.5	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
MA.S-IC.B.5	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

MA.S-IC.B.6	Evaluate reports based on data.
MA.S-CP	Conditional Probability and the Rules of Probability
MA.S-CP.A	Understand independence and conditional probability and use them to interpret data
MA.S-CP.A.2	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
MA.S-CP.A.3	Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
MA.S-CP.A.5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
MA.S-CP.B	Use the rules of probability to compute probabilities of compound events in a uniform probability model
MA.S-CP.B.6	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
MA.S-CP.B.7	Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.
MA.S-CP.B.8	Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = [P(A)] \times [P(B \mid A)] = [P(B)] \times [P(A \mid B)]$, and interpret the answer in terms of the model.
MATH.9-12.S.MD	Using Probability to Make Decisions
MA.S-CP.B.9	Use permutations and combinations to compute probabilities of compound events and solve problems.
MATH.9-12.S.MD.B	Use probability to evaluate outcomes of decisions
MATH.9-12.S.MD.B.5	Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
MATH.9-12.S.MD.B.5.b	Evaluate and compare strategies on the basis of expected values.
MATH.9-12.S.MD.B.7	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Integration of Career Readiness, Life Literacies and Key Skills

CRP.K-12.CRP1	Act as a responsible and contributing citizen and employee.
CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP3	Attend to personal health and financial well-being.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP5	Consider the environmental, social and economic impacts of decisions.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP9	Model integrity, ethical leadership and effective management.
CRP.K-12.CRP10	Plan education and career paths aligned to personal goals.
CRP.K-12.CRP11	Use technology to enhance productivity.

CRP.K-12.CRP12	Work productively in teams while using cultural global competence.

Technology / Integration of Computer Science and Design Thinking

TECH.8.1.12	Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.
TECH.8.1.12.A	Technology Operations and Concepts: Students demonstrate a sound understanding of technology concepts, systems and operations.
TECH.8.1.12.A.3	Collaborate in online courses, learning communities, social networks or virtual worlds to discuss a resolution to a problem or issue.
TECH.8.1.12.C	Communication and Collaboration: Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
TECH.8.1.12.C.1	Develop an innovative solution to a real world problem or issue in collaboration with peers and experts, and present ideas for feedback through social media or in an online community.

Interdisciplinary Connections: NJSLS for ELA, Social Studies, Science and/or Math Section

Capacities of the Literate Individual
Students Who are College and Career Ready in Reading, Writing, Speaking, Listening, & Language

They demonstrate independence.

They build strong content knowledge.

They comprehend as well as critique.

They use technology and digital media strategically and capably.

LA.K-12.NJSLSA.R	Reading
MATH.K-12.1	Make sense of problems and persevere in solving them
	Key Ideas and Details
LA.K-12.NJSLSA.R1	Read closely to determine what the text says explicitly and to make logical inferences and relevant connections from it; cite specific textual evidence when writing or speaking to support conclusions drawn from the text.
MATH.K-12.2	Reason abstractly and quantitatively

LA.K-12.NJSLSA.R2	Determine central ideas or themes of a text and analyze their development; summarize the key supporting details and ideas.
MATH.K-12.3	Construct viable arguments and critique the reasoning of others
MATH.K-12.4	Model with mathematics
MATH.K-12.5	Use appropriate tools strategically
MATH.K-12.6	Attend to precision
MATH.K-12.7	Look for and make use of structure
MATH.K-12.8	Look for and express regularity in repeated reasoning
LA.RI.11-12.1	Accurately cite strong and thorough textual evidence, (e.g., via discussion, written response, etc.), to support analysis of what the text says explicitly as well as inferentially, including determining where the text leaves matters uncertain.
LA.RI.11-12.2	Determine two or more central ideas of a text, and analyze their development and how they interact to provide a complex analysis; provide an objective summary of the text.
LA.K-12.NJSLSA.W1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
LA.K-12.NJSLSA.W2	Write informative/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.
LA.K-12.NJSLSA.W6	Use technology, including the Internet, to produce and publish writing and to interact and collaborate with others.
	Research to Build and Present Knowledge
LA.K-12.NJSLSA.W7	Conduct short as well as more sustained research projects, utilizing an inquiry-based research process, based on focused questions, demonstrating understanding of the subject under investigation.
LA.W.11-12.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
LA.K-12.NJSLSA.SL	Speaking and Listening
	Comprehension and Collaboration
LA.K-12.NJSLSA.SL1	Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.
LA.K-12.NJSLSA.SL2	Integrate and evaluate information presented in diverse media and formats, including visually, quantitatively, and orally.
LA.W.11-12.2	Write informative/explanatory texts to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.
	Presentation of Knowledge and Ideas
LA.K-12.NJSLSA.SL4	Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.
LA.K-12.NJSLSA.SL5	Make strategic use of digital media and visual displays of data to express information and enhance understanding of presentations.
LA.K-12.NJSLSA.L	Language
	Conventions of Standard English
LA.K-12.NJSLSA.L1	Demonstrate command of the conventions of standard English grammar and usage when writing or speaking.

	Knowledge of Language
LA.K-12.NJSLSA.L3	Apply knowledge of language to understand how language functions in different contexts, to make effective choices for meaning or style, and to comprehend more fully when reading or listening.
LA.W.11-12.6	Use technology, including the Internet, to produce, share, and update individual or shared writing products in response to ongoing feedback, including new arguments or information.
LA.W.11-12.7	Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.
LA.SL.11-12.1	Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.
LA.SL.11-12.1.A	Come to discussions prepared, having read and researched material under study; explicitly draw on that preparation by referring to evidence from texts and other research on the topic or issue to stimulate a thoughtful, well reasoned exchange of ideas.
LA.SL.11-12.1.B	Collaborate with peers to promote civil, democratic discussions and decision-making, set clear goals and assessments (e.g., student developed rubrics), and establish individual roles as needed.
LA.SL.11-12.2	Integrate multiple sources of information presented in diverse media or formats (e.g., visually, quantitatively, qualitatively, orally) evaluating the credibility and accuracy of each source.
LA.SL.11-12.4	Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
LA.SL.11-12.5	Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.
LA.L.11-12	Language
	Conventions of Standard English
LA.L.11-12.1	Demonstrate command of the conventions of standard English grammar and usage when writing or speaking.
LA.L.11-12.3	Apply knowledge of language to understand how language functions in different contexts, to make effective choices for meaning or style, and to comprehend more fully when reading or listening.

Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media LiteracyNew Section

see Crosswalks

21st Century Life and Careers

Stage I: Desired Results

Transfer/Overview/Rationale

Transfer / Overview / Rationale

Unit Rationale

The purpose of this unit…

Probability and statistics is used on a daily basis as it is commonly seen on TV, commercials, News, Informercials, and at work all the time. Having a background allows us to better understand the world around us. Students will understand the way that data is collected, organized and displayed influences interpretation and the probability of an event's occurrence can be predicted with varying degrees of confidence. Moreover, students will learn how to critically evaluate studies and determine why they might be biased. When they are shown a study from a company about gas mileage for a car, they should know that the first thing they need to ask is who did the study and why it's important. This unit will teach students how to identify the different ways of collecting data, and understand that the careful design of a study will affect the conclusions that can be drawn.

Meaning

Essential Questions

Essential Questions

- **What is standard deviation and how can it allow us to 'compare apples to oranges'?**
- **Why do we have different descriptive statistics to describe the center of a set of data and why are some better than others in different situations?**
- **What are some things we need to pay attention to when designing a study or sample?**
- **What is a bell curve and how does it relate to a set of data?**

Enduring Understanding/Indicators of Understanding

Enduring Understanding/Indicators of Understanding

- **Mean is more than just what your grade is in Algebra class. It is used to describe the center of a set of data.**
- **Just because something happened before like a roulette wheel coming up black over and over doesn't mean it's more likely NOT to happen the next time.**
- **Pay attention to how a study is conducted because, often, certain methods are used to lead to bias.**

Acquisition (Student Learning Objectives)

Knowledge

Students will know…

- **understand the properties of the normal distribution and the vocabulary related to it**
- **recognize appropriate uses for summarizing, representing, and interpreting given data**
- **understand random processes**
- **recognize the purposes of and differences among sample surveys, experiments, and observational studies**
- **understand the properties of false positives or false negatives**
- **discern between theoretical probability and experimental probability**
- **grasp the fundamental differences between permutations and combinations and the appropriate situations in which they should be applied**
- **Understand how conditions can change the outcome of an event.**
- **Understand what a bell curve is and how it applies to data sets.**
- **What descriptive statistics should be used to analyze a set of data**
- **Understand what probability is and how it is used to make decisions**
- **When to apply a binomial model**
- **Make inferences and justify conclusions**
- **Identify properties of a normal distribution and apply those properties to determine probabilities associated with areas under the standard normal curve.**

Skills

Student will be skilled at …

- **calculate amounts of outcomes using permutations and combinations on a calculator**
- **determine when order matters and when it does not matter**
- **calculate probabilities: with conditions, of multiple events, and of one event or another**
- **create probability distributions**
- **calculate standard deviation on a calculator**
- **conduct a survey using appropriate sampling methods and unbiased questions**
- **apply a binomial model using a calculator when the situation is appropriate**
- **apply a normal distribution to make predictions and calculate probabilities for data sets**
- **use the mean and standard deviation to fit a normal distribution and to make estimates**
- **use technology to estimate areas under the normal curve**
- **be able to make inferences about population parameters based on a sample**
- **evaluate a specific model to verify consistencies with results from a particular process**
- **explain how randomization as related to different studies**
- **Use data from a sample survey to estimate a population mean or proportion**
- **Develop a margin of error through the use of simulation models for random sampling**
- **use data from randomized experiment to compare two treatments**
- **use simulations to decide if differences between parameters are significant**
- **evaluate reports based on data**

Stage 3: Learning Plan

Resource and Mentor Texts

- **Algebra 2: Common Core, Pearson 2012**
- **TI-83 calculator**
- **Internet Resources (Sample Problem Based Tasks in Learning Activities Section)**
- **Bellringers**
- **Whiteboards (individual and group)**
- **Desmos**
- **Parcc Website**

Formative Assessment Strategies

- **Prepared Quizzes**
- **Daily Warm - Up Problems**
- **Homework**
- **Exit Tickets**
- **Open Ended Short Answer Questions**
- **Sample Parcc Questions**

Learning Activities/Unit of Study

- **Do Now--problem of the day related to previous learned skills or bellringers problems**
- **Review/Check Homework - (group check, partner check, whiteboard check)- Lecture**
- **Lecture**
- **Work together to understand and practice the skill - partner work/larger group work to read lesson, and practice skills through "On Your Own" problems incorporated throughout each lesson**
- **Stations - (Small group instruction, skills practice - scavenger hunts, online games, board work)**
- **Board/White Board Work - (solve problems/practice skills at board, or at seat with individual white boards)**
- **Kahoot to reinforce skills**
- **Review and practice skills using a variety of materials - (text, workbook, chromebook, games, activities, discussion)**
- **Scavenger hunts--self-checking, out of seats activity**
- **Jeopardy style review games**
- **Relay races--each student does one part of a problem, hands it to the next student to check then completes the next part, etc.**
- **Partner/Group investigation where students must create a formula, method, or strategy to solve a problem.**

Sample Problem-Based Tasks:

- **"Iron Cross Activity" Students see how long they can hold up textbooks and determine the winning team using statistics.**
- **Class survey: students survey the class on 5 basic metrics such as height and head size and determine outliers using statistics and normal distributions.**
- **Dice/Card/Coin Experimental Probability Activity**
- **Woah- Wheel of Fortune http://www.101qs.com/1774**
- **Fred's Fun Factory https://www.illustrativemathematics.org/content-standards/tasks/1197**
- **Return to Fred's Fun Factory https://www.illustrativemathematics.org/content-standards/tasks/1198**
- **Yellow Starburst http://threeacts.mrmeyer.com/yellowstarbursts/**
- **Cards and Independence https://www.illustrativemathematics.org/content-standards/tasks/943**
- **How Do You Get To School https://www.illustrativemathematics.org/content-standards/tasks/1025**
- **Lucky Envelopes https://www.illustrativemathematics.org/content-standards/tasks/944**
- **Breakfast Before School https://www.illustrativemathematics.org/content-standards/tasks/1019**
- **Three Shots http://www.mathalicious.com/lessons/three-shots**

- **Modeling with Conditional Probability: Lucky Dip** http://map.mathshell.org/lessons.php?collection=8&unit=9425
- **The Titanic 1** https://www.illustrativemathematics.org/content-standards/tasks/949
- **The Titanic 2** https://www.illustrativemathematics.org/content-standards/tasks/950
- **The Titanic 3** https://www.illustrativemathematics.org/content-standards/tasks/951
- **Coffee At Mom's Diner** https://www.illustrativemathematics.org/content-standards/tasks/1024
- **School Advisory Panel** https://www.illustrativemathematics.org/content-standards/tasks/186
- **Why Randomize** https://www.illustrativemathematics.org/content-standards/tasks/191
- **Strict Parents** https://www.illustrativemathematics.org/content-standards/tasks/122
- **Musical Preferences** https://www.illustrativemathematics.org/content-standards/tasks/123
- **Block Scheduling** https://www.illustrativemathematics.org/content-standards/tasks/125
- **NBA Officiating** https://emergentmath.com/2011/04/19/the-dallas-mavericks-are-2-16-in-playoff-games-officiated-by-danny-crawford-is-this-statistically-significant/
- **Accuracy of Carbon Dating** https://www.illustrativemathematics.org/content-standards/tasks/782
- **High Blood Pressure** https://www.illustrativemathematics.org/content-standards/tasks/1100
- **Words and Music** https://www.illustrativemathematics.org/content-standards/tasks/1029
- **Bellringers**

Modifications and/or Accommodations

Suggested Modifications (ELL, Sp. Ed, GIfted, At-risk of Failure)

English Language Learners

Native language support: The teacher provides auditory or written content to students in their native language.

Adjusted Speech: The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.

Visuals: The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subjects at hand.

Front-Loading Vocabulary: The teacher front loads vocabulary. This means providing students with a list of important vocabulary words they will need to know for a book, lesson, etc. prior to the lesson being taught. Including pictures to go with the vocabulary words is also very beneficial for the students.

Special Education Students

Chunking: The teacher presents information in a way that makes it easy for students to understand and remember. Chunking is based on the presumption that our working memory is easily overloaded by excessive detail. The best way to deliver information is to organize it into meaningful units. Because students with special needs get overloaded easily, chunking is an effective strategy

to use with them.

Checking for Understanding: It is important to constantly check for understanding, especially for students who have accommodations. Teachers want to make sure students understand the concepts being covered in a way that makes sense to them.

Extra time: The teacher provides students with special needs extra time to complete work or answer questions. It is important to give students enough time to process their thoughts.

Oral Reading: The teacher will read work orally to students. Class work such as tests and literature circles may need to be read aloud to the student.

Timers: The teacher will use timers as an instructional tool. The use of timers is beneficial for students who have trouble completing tasks. Timers can be helpful so the student is aware of how much time they have to complete an assignment.

Students with 504 Plans

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Gifted & Talented Strategies

Extensions/Enrichments: Teachers will provide gifted and talented students with extension/enrichment projects. Students will be challenged to further their understanding, to apply acquired knowledge, and/or to produce something in reference to acquired knowledge.

Modify/Change Activities: Teachers will monitor and modify activities to accommodate those students who need to be challenged further. Additional reading, problem-solving, writing, or project work is necessary for those students who are ready to move on at a rate more accelerated than their peers. In this way, G & T students are provided the same opportunity for support as special needs students.

Students at Risk of School Failure

Directions or Instructions: Make sure directions and/or instructions are given in limited numbers. Give directions/instructions verbally and in simple written format. Ask students to repeat the instructions or directions to ensure understanding occurs. Check back with the student to ensure he/she hasn't forgotten.

Peer Support: Peers can help build confidence in other students by assisting in peer learning. Many teachers use the 'ask 3 before me' approach. This is fine, however, a student at risk may have to have a specific student or two to ask. Set this up for the student so he/she knows who to ask for clarification before going to you.

Alternate or Modified Assignments: Always ask yourself, "How can I modify this assignment to ensure the students at risk are able to complete it?" Sometimes you'll simplify the task, reduce the length of the assignment or allow for a different mode of delivery. For instance, many students may hand something in, the at-risk student may jot notes and give you the information verbally. Or, it just may be that you will need to assign an alternate assignment.

Increase One to One Time: When other students are working, always touch base with your students at risk and find out if they're on track or needing some additional support. A few minutes here and there will go a long way to intervene as the need presents itself.

Contracts: It helps to have a working contract between you and your students at risk. This helps prioritize the tasks that need to be done and ensure completion happens. Each day write down what needs to be completed, as the tasks are done, provide a checkmark or happy face. The goal of using contracts is to eventually have the student come to you for completion sign-offs.

Hands On: As much as possible, think in concrete terms and provide hands-on tasks. This means a child doing math may require a calculator or counters. The child may need to tape record comprehension activities instead of writing them. A child may have to listen to a story being read instead of reading it him/herself.

Tests/Assessments: Tests can be done orally if need be. Break tests down in smaller increments by having a portion of the test in the morning, another portion after lunch and the final part the next day.

Seating: Seat students near a helping peer or with quick access to the teacher. Those with hearing or sight issues need to be close to the instruction which often means near the front.