Unit 01: Quadratic Equations

Content Area: **Template**
Course(s):
Time Period: **Full Year**
Length: **FY**
Status: **Published**

Standards Alignment

New Jersey Student Learning Standards

MA.F-IF	Interpreting Functions
MA.A-SSE	Seeing Structure in Expressions
MA.F-IF.A	Understand the concept of a function and use function notation
MA.A-SSE.A	Interpret the structure of expressions
MA.F-IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
MA.A-SSE.A.1	Interpret expressions that represent a quantity in terms of its context.
MA.A-SSE.A.1a	Interpret parts of an expression, such as terms, factors, and coefficients.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.A-SSE.A.2	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
MA.F-IF.B	Interpret functions that arise in applications in terms of the context
MA.A-SSE.B	Write expressions in equivalent forms to solve problems
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
MA.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MA.A-SSE.B.3a	Factor a quadratic expression to reveal the zeros of the function it defines.
MA.A-SSE.B.3b	Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
MA.N-CN	The Complex Number System
MA.F-IF.C	Analyze functions using different representations
MA.N-CN.A	Perform arithmetic operations with complex numbers.
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

MA.F-IF.C.7a	Graph linear and quadratic functions and show intercepts, maxima, and minima.
MA.N-CN.A.1	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
MA.N-CN.A.2	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
MA.N-CN.A.3	Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
MA.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
MA.F-IF.C.8a	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
MA.N-CN.C	Use complex numbers in polynomial identities and equations.
MA.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MA.N-CN.C.7	Solve quadratic equations with real coefficients that have complex solutions.
MA.A-CED	Creating Equations
MA.A-CED.A	Create equations that describe numbers or relationships
MA.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MA.A-REI	Reasoning with Equations and Inequalities
MA.A-REI.B	Solve equations and inequalities in one variable
MA.A-REI.B.4b	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Integration of Career Readiness, Life Literacies and Key Skills

CRP.K-12.CRP1	Act as a responsible and contributing citizen and employee.
CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP3	Attend to personal health and financial well-being.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP5	Consider the environmental, social and economic impacts of decisions.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP9	Model integrity, ethical leadership and effective management.
CRP.K-12.CRP10	Plan education and career paths aligned to personal goals.
CRP.K-12.CRP11	Use technology to enhance productivity.
CRP.K-12.CRP12	Work productively in teams while using cultural global competence.

Technology / Integration of Computer Science and Design Thinking

TECH.8.1.12	Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.
TECH.8.1.12.A	Technology Operations and Concepts: Students demonstrate a sound understanding of technology concepts, systems and operations.
TECH.8.1.12.A.3	Collaborate in online courses, learning communities, social networks or virtual worlds to discuss a resolution to a problem or issue.
TECH.8.1.12.C	Communication and Collaboration: Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
TECH.8.1.12.C.1	Develop an innovative solution to a real world problem or issue in collaboration with peers and experts, and present ideas for feedback through social media or in an online community.

Interdisciplinary Connections: NJSLS for ELA, Social Studies, Science and/or Math Section

Capacities of the Literate Individual
Students Who are College and Career Ready in Reading, Writing, Speaking, Listening, & Language

They demonstrate independence.

They build strong content knowledge.

They comprehend as well as critique.

They use technology and digital media strategically and capably.

LA.K-12.NJSLSA.R	Reading
MATH.K-12.1	Make sense of problems and persevere in solving them
	Key Ideas and Details
LA.K-12.NJSLSA.R1	Read closely to determine what the text says explicitly and to make logical inferences and relevant connections from it; cite specific textual evidence when writing or speaking to support conclusions drawn from the text.
MATH.K-12.2	Reason abstractly and quantitatively
LA.K-12.NJSLSA.R2	Determine central ideas or themes of a text and analyze their development; summarize the key supporting details and ideas.
MATH.K-12.3	Construct viable arguments and critique the reasoning of others

MATH.K-12.4	Model with mathematics
MATH.K-12.5	Use appropriate tools strategically
MATH.K-12.6	Attend to precision
MATH.K-12.7	Look for and make use of structure
MATH.K-12.8	Look for and express regularity in repeated reasoning
LA.RI.11-12.1	Accurately cite strong and thorough textual evidence, (e.g., via discussion, written response, etc.), to support analysis of what the text says explicitly as well as inferentially, including determining where the text leaves matters uncertain.
LA.RI.11-12.2	Determine two or more central ideas of a text, and analyze their development and how they interact to provide a complex analysis; provide an objective summary of the text.
LA.K-12.NJSLSA.W1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
LA.K-12.NJSLSA.W2	Write informative/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.
LA.K-12.NJSLSA.W6	Use technology, including the Internet, to produce and publish writing and to interact and collaborate with others.
	Research to Build and Present Knowledge
LA.K-12.NJSLSA.W7	Conduct short as well as more sustained research projects, utilizing an inquiry-based research process, based on focused questions, demonstrating understanding of the subject under investigation.
LA.W.11-12.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
LA.K-12.NJSLSA.SL	Speaking and Listening
	Comprehension and Collaboration
LA.K-12.NJSLSA.SL1	Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.
LA.K-12.NJSLSA.SL2	Integrate and evaluate information presented in diverse media and formats, including visually, quantitatively, and orally.
LA.W.11-12.2	Write informative/explanatory texts to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.
	Presentation of Knowledge and Ideas
LA.K-12.NJSLSA.SL4	Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.
LA.K-12.NJSLSA.SL5	Make strategic use of digital media and visual displays of data to express information and enhance understanding of presentations.
LA.K-12.NJSLSA.L	Language
	Conventions of Standard English
LA.K-12.NJSLSA.L1	Demonstrate command of the conventions of standard English grammar and usage when writing or speaking.
	Knowledge of Language
LA.K-12.NJSLSA.L3	Apply knowledge of language to understand how language functions in different contexts, to make effective choices for meaning or style, and to comprehend more fully when

	reading or listening.
LA.W.11-12.6	Use technology, including the Internet, to produce, share, and update individual or shared writing products in response to ongoing feedback, including new arguments or information.
LA.W.11-12.7	Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.
LA.SL.11-12.1	Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.
LA.SL.11-12.1.A	Come to discussions prepared, having read and researched material under study; explicitly draw on that preparation by referring to evidence from texts and other research on the topic or issue to stimulate a thoughtful, well reasoned exchange of ideas.
LA.SL.11-12.1.B	Collaborate with peers to promote civil, democratic discussions and decision-making, set clear goals and assessments (e.g., student developed rubrics), and establish individual roles as needed.
LA.SL.11-12.2	Integrate multiple sources of information presented in diverse media or formats (e.g., visually, quantitatively, qualitatively, orally) evaluating the credibility and accuracy of each source.
LA.SL.11-12.4	Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
LA.SL.11-12.5	Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.
LA.L.11-12	Language
	Conventions of Standard English
LA.L.11-12.1	Demonstrate command of the conventions of standard English grammar and usage when writing or speaking.
LA.L.11-12.3	Apply knowledge of language to understand how language functions in different contexts, to make effective choices for meaning or style, and to comprehend more fully when reading or listening.

Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media LiteracyNew Section

see Crosswalks

21st Century Life and Careers

Stage I: Desired Results

Transfer/Overview/Rationale

Transfer / Overview / Rationale
Unit Rationale The purpose of this unit… **From throwing a ball into the air, to launching a rocket, and to calculating the maximum volume of a box that can be created from a single piece of paper, quadratic functions feature predominantly in mathematics. Students will discover how to manipulate quadratic functions using various techniques, including factoring and quadratic formula, to solve application problems and to solve problems that were once widely believed to have no solution. Students will develop several ways to solve quadratic equations and be able to decide which way is appropriate to the situation. Students will perform operations on complex numbers and express the results in simplest form using patterns of the powers of i.**

Meaning

Essential Questions

Essential Questions

- **What are the key characteristics of quadratic functions and their graphs?**
- **How are the key characteristics of quadratic functions similar and different to the key characteristics of linear functions?**
- **How do you identify a situation where a quadratic model would be most appropriate?**
- **How do we know when to use a particular method to solve a Quadratic equation?**
- **When can we determine if a quadratic equation has a Real solution or a Complex solution?**

Enduring Understanding/Indicators of Understanding

Enduring Understanding/Indicators of Understanding

- **Factoring techniques and the quadratic formula are used to solve any quadratic equation.**
- **The imaginary number i is used to represent the value of $\sqrt{-1}$.**
- **The shape of a quadratic equation is parabolic in nature and can have a maximum or minimum.**

- **Quadratic models can be used to describe and quantify many real life relationships.**
- **Algebraic procedures and graphical representations are interconnected and build on one another to produce a coherent whole.**

Acquisition (Student Learning Objectives)

Knowledge

Students will know…

- **The graph of a quadratic function is the shape of a parabola.**
- **The vertex is the intersection of the axis of symmetry and the quadratic function, and can either be a minimum or maximum point of the parabola.**
- **The graph of any quadratic function is a transformation of the graph of the parent quadratic function, $y = x^2$.**
- **All equations of a function can be manipulated into different forms; for a quadratic function the different forms of an equation are standard form, factored form, and vertex form.**
- **Domain of a function is determined by the context of the function given and what values make sense to be included in the particular problem**
- **A real life context that compares two quantities that increase to a peak and then decrease or decrease to a low and then increase are represented by quadratic functions**
- **The zeros of a quadratic function are where the function intersects the x-axis**
- **The vertex of a quadratic function is the highest or lowest y-value the function reaches**
- **A graphing calculator can be used to calculate all points of a quadratic function by using the table or**

looking at the graph

- **A quadratic function can have real or imaginary solutions**
- **The discriminant of a quadratic function can be used to determine what type of solutions the function has**
- **The quadratic formula can be used to solve any quadratic function**
- **Factoring a quadratic expression is based on the number of terms (2, 3, or 4).**
- **A quadratic equation can be written in standard form ($f(x)=ax^2+bx+c$) or vertex form ($f(x)=a(x-h)^2+k$)**
- **The roots or solutions of a quadratic equation are the x-intercepts or the values of x where the function is equal to zero.**
- **There is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real.**
- **Complex numbers form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication.**
- **Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find the value(s) of the solution to the equation f(x) = g(x).**

Skills

Student will be skilled at …

- **Factor using GCF, binomial vs. trinomial, difference of two squares, and grouping methods.**
- **Use factoring techniques, completing the square, and the quadratic formula to solve quadratic equations.**
- **Determine roots (or solutions) of quadratic equations.**

- Write quadratic equations given specific characteristics.
- Identify the domain/ range of a quadratic function given the context of the application
- Identify types of real life problems that could be modeled by a quadratic
- Use a graphing calculator to graph quadratic functions and identify the key features
- Convert standard form to vertex form
- Graph quadratic equations that are written in vertex or standard form.
- Construct quadratic equations to model real world applications given specific information
- Simplify expressions involving imaginary numbers.

Stage 3: Learning Plan

Resource and Mentor Texts

- Algebra 2: Common Core, Pearson 2012
- TI-83 calculator
- Internet Resources (Sample Problem Based Tasks in Learning Activities Section)
- Bellringers
- Whiteboards (individual and group)
- Desmos
- Parcc Website

Formative Assessment Strategies

- Prepared Quizzes
- Daily Warm - Up Problems
- Homework
- Exit Tickets
- Open Ended Short Answer Questions
- Sample Parcc Questions

Learning Activities/Unit of Study

- Do Now--problem of the day related to previous learned skills or bellringers problems

- **Review/Check Homework - (group check, partner check, whiteboard check)- Lecture**
- **Lecture**
- **Work together to understand and practice the skill - partner work/larger group work to read lesson, and practice skills through "On Your Own" problems incorporated throughout each lesson**
- **Stations - (Small group instruction, skills practice - scavenger hunts, online games, board work)**
- **Board/White Board Work - (solve problems/practice skills at board, or at seat with individual white boards)**
- **Kahoot to reinforce skills**
- **Review and practice skills using a variety of materials - (text, workbook, chromebook, games, activities, discussion)**
- **Scavenger hunts--self-checking, out of seats activity**
- **Jeopardy style review games**
- **Relay races--each student does one part of a problem, hands it to the next student to check then completes the next part, etc.**
- **Partner/Group investigation where students must create a formula, method, or strategy to solve a problem.**

Sample Problem-Based Tasks:

- **A Real Learning Curve: Catapults Demonstrate the Quadratic Equation: http://www.edutopia.org/impact-academy-catapults-quadratic-equation**
- **Egg Launch: http://illuminations.nctm.org/Lesson.aspx?id=2650**
- **Bacterial Growth and Quadratics: https://www.scribd.com/doc/139559000/Quadratics-Vertex-Form-and-Bacterial-Growth**
- **Garden Problem- Solving Quadratic Application Activity: http://www.scribd.com/doc/68802196/Garden-Problem-Michelle-Sweeney**
- **Hot Rod - Constructing Quadratic Equation: https://emergentmath.com/2013/04/23/hot-rod-quadratics-lets-jump-this-jump/**
- **Solving Quadratic Equations Cutting Corners Activity: http://map.mathshell.org/lessons.php?unit=9250&collection=8**
- **Two Squares are Equal Activity: https://www.illustrativemathematics.org/content-standards/tasks/618**
- **Seeing Dots Activity: https://www.illustrativemathematics.org/content-standards/tasks/**
- **Circle and Line Activity: https://www.illustrativemathematics.org/content-standards/tasks/223**
- **Desmos- Online Graphing Interactive**

Modifications and/or Accommodations

Suggested Modifications (ELL, Sp. Ed, GIfted, At-risk of Failure)

English Language Learners

Native language support: The teacher provides auditory or written content to students in their native language.

Adjusted Speech: The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.

Visuals: The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subjects at hand.

Front-Loading Vocabulary: The teacher front loads vocabulary. This means providing students with a list of important vocabulary words they will need to know for a book, lesson, etc. prior to the lesson being taught. Including pictures to go with the vocabulary words is also very beneficial for the students.

Special Education Students

Chunking: The teacher presents information in a way that makes it easy for students to understand and remember. Chunking is based on the presumption that our working memory is easily overloaded by excessive detail. The best way to deliver information is to organize it into meaningful units. Because students with special needs get overloaded easily, chunking is an effective strategy to use with them.

Checking for Understanding: It is important to constantly check for understanding, especially for students who have accommodations. Teachers want to make sure students understand the concepts being covered in a way that makes sense to them.

Extra time: The teacher provides students with special needs extra time to complete work or answer questions. It is important to give students enough time to process their thoughts.

Oral Reading: The teacher will read work orally to students. Class work such as tests and literature circles may need to be read aloud to the student.

Timers: The teacher will use timers as an instructional tool. The use of timers is beneficial for students who have trouble completing tasks. Timers can be helpful so the student is aware of how much time they have to complete an assignment.

Students with 504 Plans

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Gifted & Talented Strategies

Extensions/Enrichments: Teachers will provide gifted and talented students with extension/enrichment projects. Students will be challenged to further their understanding, to apply acquired knowledge, and/or to produce something in reference to acquired knowledge.

Modify/Change Activities: Teachers will monitor and modify activities to accommodate those students who need to be challenged further. Additional reading, problem-solving, writing, or project work is necessary for those students who are ready to move on at a rate more accelerated than their peers. In this way, G & T students are provided the same opportunity for support as special needs students.

Students at Risk of School Failure

Directions or Instructions: Make sure directions and/or instructions are given in limited numbers. Give directions/instructions verbally and in simple written format. Ask students to repeat the instructions or directions to ensure understanding occurs. Check back with the student to ensure he/she hasn't forgotten.

Peer Support: Peers can help build confidence in other students by assisting in peer learning. Many teachers use the 'ask 3 before me' approach. This is fine, however, a student at risk may have to have a specific student or two to ask. Set this up for the student so he/she knows who to ask for clarification before going to you.

Alternate or Modified Assignments: Always ask yourself, "How can I modify this assignment to ensure the students at risk are able to complete it?" Sometimes you'll simplify the task, reduce the length of the assignment or allow for a different mode of delivery. For instance, many students may hand something in, the at-risk student may jot notes and give you the information verbally. Or, it just may be that you will need to assign an alternate assignment.

Increase One to One Time: When other students are working, always touch base with your students at risk and find out if they're on track or needing some additional support. A few minutes here and there will go a long way to intervene as the need presents itself.

Contracts: It helps to have a working contract between you and your students at risk. This helps prioritize the tasks that need to be done and ensure completion happens. Each day write down what needs to be completed, as the tasks are done, provide a checkmark or happy face. The goal of using contracts is to eventually have the student come to you for completion sign-offs.

Hands On: As much as possible, think in concrete terms and provide hands-on tasks. This means a child doing math may require a calculator or counters. The child may need to tape record comprehension activities instead of writing them. A child may have to listen to a story being read instead of reading it him/herself.

Tests/Assessments: Tests can be done orally if need be. Break tests down in smaller increments by having a portion of the test in the morning, another portion after lunch and the final part the next day.

Seating: Seat students near a helping peer or with quick access to the teacher. Those with hearing

or sight issues need to be close to the instruction which often means near the front.