

Unit 04: Exponential and Logarithmic Functions and Equations

Content Area: **Template**

Course(s):

Time Period: **Full Year**

Length: **5 Weeks**

Status: **Published**

Unit 4: Exponential and Logarithmic Functions and Equations (Module 8, 9, and 10)

Unit Rationale

In prior learning, students have analyzed a variety of functions and their graphs, including linear, quadratic, cubic, and absolute value. They have used those functions to model real world situations. They have calculated simple interest. They have found inverse functions. In this unit, students will examine exponential growth and decay models and their graphs. They will compare simple and compound interest and model interest compounded over different periods of time. Next, they will learn about logarithmic functions and their inverse relationship with exponential functions. They will evaluate, graph, and transform logarithmic functions. Later, they will investigate and prove properties of logarithms and use them to evaluate logarithmic functions. They will solve logarithmic and exponential equations. In future learning, they will solve more complicated types of equations involving logarithms.

Essential Questions

- How can we model real-world phenomena using exponential growth and decay functions?
- What is the significance of the natural base e in exponential functions, and why is it important in continuous growth and decay?
- How does compound interest differ from simple interest, and how is it modeled using exponential functions?
- What is the relationship between logarithms and exponential functions, and how do logarithms help solve exponential equations?
- How does the graph of a logarithmic function compare to the graphs of exponential functions?
- How can we create exponential and logarithmic functions that model given data or scenarios?
- What are the fundamental properties of logarithms, such as the product, quotient, and power rules?
- What methods and strategies can be employed to solve equations involving exponential and logarithmic functions, including equations with base e ?

Pre-Assessments

- Into Geometry: Are you ready? Online activity- TE p. 238

Instructional Plan

Exponential Growth and Decay Functions (8.1)

Student Learning Intentions or We are learning to ... (WALT)

- We are learning to identify the effect of transformations on exponential functions to model situations of growth and decay.

Student Success Criteria ... “I can statements”

- I can recognize when an exponential function has been transformed from a parent function.
- I can identify the effect of transformations on exponential functions to model situations of growth and decay.

Instructional Strategies and Activities

- Into Algebra 2 Spark Your Learning- TE p. 239
- Guided Notes
 - Analyze exponential growth and decay
 - Transform exponential growth functions
 - Transform exponential decay functions
 - Create an exponential function from a graph
- Into Algebra 2 Practice- TE p. 244
- DeltaMath practice assignment

Formative Assessments

- Into Algebra 2: Check Understanding- online learning activity

Instructional Materials and Resources

- Into Math resources
- DeltaMath

- Desmos

Reflections and Suggested Modifications

The Natural Base e (8.2)

Student Learning Intentions or We are learning to ... (WALT)

- We are learning to graph transformations of exponential functions having base e and use the graphs to solve real world problems.

Student Success Criteria ... "I can statements"

- I can recognize transformations of exponential functions having base e.
- I can use the graph of transformations of exponential functions having base e to solve real world problems.
- I can graph transformations of exponential functions having base e and use the graphs to solve real world problems.

Instructional Strategies and Activities

- Into Algebra 2 Spark Your Learning- TE p. 247
- Guided Notes
 - Graph and analyze the function $f(x) = e^x$
 - Transformations of the graph $f(x) = e^x$
 - Create an equation for a transformation of $f(x) = e^x$
 - Model a real world situation using an exponential function with base e
- Into Algebra 2 Practice- TE p. 252
- DeltaMath practice assignment

Formative Assessments

- Into Algebra 2: Check Understanding- online learning activity

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Compound Interest (8.3)

Student Learning Intentions or We are learning to ... (WALT)

- We are learning to rewrite exponential functions to better model real-world situations, including situations involving interest.

Student Success Criteria ... "I can statements"

- I can recognize which models represent the value of an investment that earns interest compounded yearly, monthly, and continuously.
- I can use an investment model to calculate the value of an investment that earns compound interest.
- I can model the value of an investment that earns compound interest.

Instructional Strategies and Activities

- Into Algebra 2 Spark Your Learning- TE p. 255
- Guided Notes
 - Compare simple and compound interest
 - Model interest compounded yearly

- Model interest compounded more than once per year
- Model interest compounded continuously
- Into Algebra 2 Practice- TE p. 260
- DeltaMath practice assignment

Formative Assessments

- Into Algebra 2: Check Understanding- online learning activity

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Logarithms and Logarithmic Functions (9.1)

Student Learning Intentions or We are learning to ... (WALT)

- We are learning to create, graph, and evaluate logarithmic functions in mathematical and real world contexts.

Student Success Criteria ... "I can statements"

- I can identify an exponential equation and a logarithmic equation.
- I can convert exponential and logarithmic equations to their inverse forms.
- I can define and evaluate logarithms.

Instructional Strategies and Activities

- Into Algebra 2 Spark Your Learning- TE p. 267
- Guided Notes
 - Understand logarithmic functions as inverses of exponential functions
 - Convert between exponential and logarithmic equations
 - Evaluate logarithmic functions by converting to exponential form
 - Use a logarithmic model
- Into Algebra 2 Practice- TE p. 272
- DeltaMath practice assignment

Formative Assessments

- Into Algebra 2: Check Understanding- online learning activity

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Graph Logarithmic Functions (9.2)

Student Learning Intentions or We are learning to ... (WALT)

- We are learning to use the inverse relationship of parent logarithmic functions with exponential functions to determine points on their graphs, graph transformations of parent logarithmic functions, describe properties of graphs of logarithmic functions, and describe effects of transformations on properties of the parent function such as domain and range.

Student Success Criteria ... “I can statements”

- I can find the reference points and asymptote of a parent logarithmic function.
- I can tell how transformations move the reference points of a parent logarithmic function.
- I can graph logarithmic functions.

Instructional Strategies and Activities

- Into Algebra 2 Spark Your Learning- TE p. 275
- Guided Notes
 - Graph logarithmic functions
 - Transformations of logarithmic functions
 - Graph logarithmic models
- Into Algebra 2 Practice- TE p. 280
- DeltaMath practice assignment

Formative Assessments

- Into Algebra 2: Check Understanding- online learning activity

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Student Learning Intentions or We are learning to ... (WALT)

- We are learning to create logarithmic and exponential equations to represent relationships between quantities.

Student Success Criteria ... “I can statements”

- I can perform some of the steps needed to write a logarithmic and an exponential equation.
- I can create logarithmic and exponential equations to represent relationships between quantities.

Instructional Strategies and Activities

- Into Algebra 2 Spark Your Learning- TE p. 283
- Guided Notes
 - Create an exponential model from a data set
 - Create an exponential model using technology
 - Create a logarithmic model
- Into Algebra 2 Practice- TE p. 288
- DeltaMath practice assignment

Formative Assessments

- Into Algebra 2: Check Understanding- online learning activity

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Properties of Logarithms (10.1)

Student Learning Intentions or We are learning to ... (WALT)

- We are learning to apply properties of logarithms.

Student Success Criteria ... “I can statements”

- I can develop and apply the properties of logarithms to simplify expressions.

Instructional Strategies and Activities

- Into Algebra 2 Spark Your Learning- TE p. 295
- Guided Notes
 - Investigate properties of logarithms numerically
 - Investigate properties of logarithms graphically
 - Prove the properties of logarithms
 - Use properties of logarithms to evaluate expressions
- Into Algebra 2 Practice- TE p. 300
- DeltaMath practice assignment

Formative Assessments

- Into Algebra 2: Check Understanding- online learning activity

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Solve Exponential Equations (10.2)

Student Learning Intentions or We are learning to ... (WALT)

- We are learning to use logarithms to find missing values for exponential models.

Student Success Criteria ... “I can statements”

- I can solve exponential equations in the form of $b^x = b^y$ after isolating the base and exponent.
- I can use logarithms to find missing values for exponential models.

Instructional Strategies and Activities

- Into Algebra 2 Spark Your Learning- TE p. 303
- Guided Notes
 - Estimate solutions of exponential equations from tables.
 - Solve an exponential equation graphically
 - Solve an exponential equation algebraically
 - Real world problems modeled by exponential equations
- Into Algebra 2 Practice- TE p. 308
- DeltaMath practice assignment

Formative Assessments

- Into Algebra 2: Check Understanding- online learning activity

Instructional Materials and Resources

- Into Math resources
- DeltaMath

- Desmos

Reflections and Suggested Modifications

Solve Logarithmic Equations (10.3)

Student Learning Intentions or We are learning to ... (WALT)

- We are learning to solve logarithmic equations graphically with help from a graphing calculator and algebraically by using the properties of logarithms, and use them to solve real world problems.

Student Success Criteria ... "I can statements"

- I can use graphs to solve a logarithmic equation.
- I can use properties of logarithms to solve a logarithmic equation algebraically.
- I can solve logarithmic equations.

Instructional Strategies and Activities

- Into Algebra 2 Spark Your Learning- TE p. 311
- Guided Notes
 - Solve a logarithmic equation graphically
 - Solve a logarithmic equation algebraically
 - Real world problems modeled by logarithmic equations
- Into Algebra 2 Practice- TE p. 315
- DeltaMath practice assignment

Formative Assessments

- Into Algebra 2: Check Understanding- online learning activity

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Modifications and/or Accommodations

Suggested Modifications (ELL, Sp. Ed, Gifted, At-risk of Failure)

English Language Learners

Native language support: The teacher provides auditory or written content to students in their native language.

Adjusted Speech: The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.

Visuals: The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subjects at hand.

Front-Loading Vocabulary: The teacher front loads vocabulary. This means providing students with a list of important vocabulary words they will need to know for a book, lesson, etc. prior to the lesson being taught. Including pictures to go with the vocabulary words is also very beneficial for the students.

Special Education Students

Chunking: The teacher presents information in a way that makes it easy for students to understand and remember. Chunking is based on the presumption that our working memory is easily overloaded by excessive detail. The best way to deliver information is to organize it into meaningful units. Because students with special needs get overloaded easily, chunking is an effective strategy to use with them.

Checking for Understanding: It is important to constantly check for understanding, especially for students who have accommodations. Teachers want to make sure students understand the concepts being covered in a way that makes sense to them.

Extra time: The teacher provides students with special needs extra time to complete work or answer questions. It is important to give students enough time to process their thoughts.

Oral Reading: The teacher will read work orally to students. Class work such as tests and literature circles may need to be read aloud to the student.

Timers: The teacher will use timers as an instructional tool. The use of timers is beneficial for students who have trouble completing tasks. Timers can be helpful so the student is aware of how much time they have to complete an assignment.

Students with 504 Plans

Chunking: The teacher presents information in a way that makes it easy for students to understand and remember. Chunking is based on the presumption that our working memory is easily overloaded by excessive detail. The best way to deliver information is to organize it into meaningful units. Because students with special needs get overloaded easily, chunking is an effective strategy to use with them.

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Gifted & Talented Strategies

Extensions/Enrichments: Teachers will provide gifted and talented students with extension/enrichment projects. Students will be challenged to further their understanding, to apply acquired knowledge, and/or to produce something in reference to acquired knowledge.

Modify/Change Activities: Teachers will monitor and modify activities to accommodate those students who need to be challenged further. Additional reading, problem-solving, writing, or project work is necessary for those students who are ready to move on at a rate more accelerated than their peers. In this way, G & T students are provided the same opportunity for support as special needs students.

Students at Risk of School Failure

Directions or Instructions: Make sure directions and/or instructions are given in limited numbers. Give directions/instructions verbally and in simple written format. Ask students to repeat the instructions or directions to ensure understanding occurs. Check back with the student to ensure he/she hasn't forgotten.

Peer Support: Peers can help build confidence in other students by assisting in peer learning. Many

teachers use the 'ask 3 before me' approach. This is fine, however, a student at risk may have to have a specific student or two to ask. Set this up for the student so he/she knows who to ask for clarification before going to you.

Alternate or Modified Assignments: Always ask yourself, "How can I modify this assignment to ensure the students at risk are able to complete it?" Sometimes you'll simplify the task, reduce the length of the assignment or allow for a different mode of delivery. For instance, many students may hand something in, the at-risk student may jot notes and give you the information verbally. Or, it just may be that you will need to assign an alternate assignment.

Increase One to One Time: When other students are working, always touch base with your students at risk and find out if they're on track or needing some additional support. A few minutes here and there will go a long way to intervene as the need presents itself.

Contracts: It helps to have a working contract between you and your students at risk. This helps prioritize the tasks that need to be done and ensure completion happens. Each day write down what needs to be completed, as the tasks are done, provide a checkmark or happy face. The goal of using contracts is to eventually have the student come to you for completion sign-offs.

Hands On: As much as possible, think in concrete terms and provide hands-on tasks. This means a child doing math may require a calculator or counters. The child may need to tape record comprehension activities instead of writing them. A child may have to listen to a story being read instead of reading it him/herself.

Tests/Assessments: Tests can be done orally if need be. Break tests down in smaller increments by having a portion of the test in the morning, another portion after lunch and the final part the next day.

Seating: Seat students near a helping peer or with quick access to the teacher. Those with hearing or sight issues need to be close to the instruction which often means near the front.

Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media Literacy

Diversity, Equity, and Inclusion

[NCTM: Access and Equity in Mathematics Education](#)

[A Pathway to Equitable Math Instruction](#)

Provide students with opportunities to give feedback to teachers about the classroom and instruction.

- Verbal Example: Fist to five, How well do you understand what we talked about today? Fist to five, How well did I teach this today?
- Classroom Activity: Exit tickets or surveys that ask students to identify how well teachers taught, what helped them learn, what got in the way of their learning, etc.

Treat mathematics as a language that everyone is learning while authentically centering students home languages.

- Classroom Strategies: Color-coding ideas, learning vocabulary in student languages, visual and kinesthetic learning, representations of learning without words.
- Classroom Activity: Multilingual Frayer Models for definitions or concepts

Incorporate true culturally relevant pedagogy, practice, and curriculum.

- Verbal Example: What are some of your family traditions that you are proud of? Would you be okay if we brought some of those into the classroom?
- Classroom Activity: Use Ankara fabric to teach mathematical concepts such as tessellations, fractions, area, percentages, etc.

Incorporate the history of mathematics into lessons.

- Verbal Example: Why do you think we call it Pythagorean's theorem, when it was used before he was even born? What should we call it instead?
- Classroom Activity: Learn about different bases and numerical ideas: Base 2, binary and connections to computer programming, how the Yoruba of Nigeria used base 20, and how the Mayans conceptualized the number 0 before the first recording of it

Solicit student ways of thinking and processing.

- Verbal Example: How might you all go about this? What do you notice?
- Classroom Activity: Incorporate explorations, where students interact with mathematics in a way that allows them to "discover" or experience mathematics.

Reorganize your classroom teaching around concepts, and teach them more like a web rather than discrete sets of knowledge.

- Verbal Example: How does this connect to what you've learned in the past? How can you use that knowledge today?
- Classroom Activity: Learning webs that connect content

Start with more complex math problems and scaffold as necessary.

- Verbal Example: If we wanted to build a rocket, what are all the things we might need to know before we get started? Along the way, we decided that we want the rocket to reach the moon. What do we need to consider now?
- Classroom Activity: When solving equations, start with the most complex problem, generate ideas for how to solve it, and use the simpler equations as examples to support those ideas.

Offer a variety of ways to demonstrate thinking and knowledge.

- Verbal Example: Show your thinking with words, pictures, symbols.

Ask other questions that will demonstrate learning when it is not clear to you how students know the answer.

- Verbal Example: If you were working with a fellow mathematician who was absent this day, what might you tell them to help them learn it?

Learn about, engage with, and incorporate ethnomathematics.

- Verbal Example: Reflect on your day so far. What math have you already used today?
- Classroom Activity: Community walks to engage with slope.

Co-construct knowledge in the classroom.

- Verbal Example: Let's get into partners and do a think pair-share. We will incorporate everyone's ideas and try to synthesize them.
- Classroom Activity: Have students create mathematical definitions in their own words in groups, and bring the groups together to co-construct mathematical definitions as a class

Choose problems that have complex, competing, or multiple answers.

- Verbal Example: Come up with at least two answers that might solve this problem.
- Classroom Activity: Challenge standardized test questions by getting the "right" answer, but justify other answers by unpacking the assumptions that are made in the problem.
- Classroom Activity: Deconstructed Multiple Choice
- given a set of multiple choice answers, students discuss why these answers may have been included (can also be used to highlight common mistakes).

Identify what is right about the thinking, and highlight the mistake in what is factually or procedurally accepted.

- Verbal Example: You recognized that you had to combine the constants 27 and 9, could you explain your thinking?
- Classroom Activity: Error Analysis worksheets that highlight what is the right idea behind the mistake.

Using thoughtful questioning to solicit mathematical thoughts rather than telling.

- Verbal Example: What would a mathematician who is confused ask about this question?
- Classroom Activity: After students demonstrate knowledge of a topic, have them play a game where they have to explain their topic to a fellow mathematician and a skeptic. Develop their own reflective questioning/explaining in all three roles.

Create multiple ways of participating that honor myriad ways of thinking and being.

- Verbal Example: For this section, feel free to work alone, in pairs, trios, or quads (let them choose).
- Classroom Activity: Community circles or storytelling circles, incorporating dance, music, song, call and response, and other cultural ways of communicating.

Climate Change

[Math Climate Change Companion Guide](#)

• S.ID.B.6a Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.

Climate Change Example: Students may use linear or exponential functions fitted to geoscience data to solve problems and analyze the results from global climate models to make an evidence-based forecast of the current rate of global climate change.

• F.IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Climate Change Example: Students may use function notation to determine the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline), m , where $c(m)$ is the number of molecules of carbon dioxide.

• F.IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

Climate Change Example: Students may relate the domain of a function $c(m)$ representing the amount of carbon dioxide produced by burning m molecules of ethane (gasoline), to its graph in order to determine the appropriate domain for $c(m)$.

• F.IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Climate Change Example: Students may calculate the average rate of change of a function $c(m)$ presented symbolically or as a table, where $c(m)$ represents the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline).

• A.CED.A.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

Climate Change Example: Students may create equations and/or inequalities to represent the economic impact of climate change.

• A.CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

Climate Change Example: Students may represent constraints describing the economic impact of climate change by equations, inequalities, and/or by systems of inequalities, and interpret solutions as viable or nonviable options.

• A.CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law to highlight resistance R .

Climate Change Example: Students may rearrange formulas related to the economic impact of climate change to highlight a quantity of interest, using the same reasoning as in solving equations.

• N.Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

Climate Change Example: Students may use units to guide the solution of multi-step problems about how variations in the flow of energy into and out of the Earth's systems result in climate change. Note: Changes in climate are limited to changes in surface temperatures, precipitation patterns, glacial ice volumes, sea levels, and biosphere distribution.

- N.Q.A.2 Define appropriate quantities for the purpose of descriptive modeling.

Climate Change Example: Students may define appropriate quantities for a descriptive model of how variations in the flow of energy into and out of Earth's systems result in climate change. Note: changes in climate are limited to changes in surface temperatures, precipitation patterns, glacial ice volumes, sea levels, and biosphere distribution.

- S.ID.B.6a Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.

Climate Change Example: Students may use linear or exponential functions fitted to geoscience data to solve problems and analyze the results from global climate models to make an evidence-based forecast of the current rate of global climate change.

New Jersey Student Learning Standards: Content Area

MATH.9-12.F.BF.A.1	Write a function that describes a relationship between two quantities.
MATH.9-12.F.BF.A.1.a	Determine an explicit expression, a recursive process, or steps for calculation from a context.
MATH.9-12.S.ID.B.6	Represent data on two quantitative variables on a scatter plot and describe how the variables are related.
MATH.9-12.S.ID.B.6.a	Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.
MATH.9-12.N.Q.A.2	Define appropriate quantities for the purpose of descriptive modeling.
MATH.9-12.F.BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MATH.9-12.F.BF.B.4	Find inverse functions.
MATH.9-12.F.BF.B.4.a	Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.
MATH.9-12.F.BF.B.4.c	Read values of an inverse function from a graph or a table, given that the function has an inverse.
MATH.9-12.A.CED.A.1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
MATH.9-12.A.CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MATH.9-12.A.CED.A.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

MATH.9-12.F.IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MATH.9-12.A.REI.A.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
MATH.9-12.F.IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MATH.9-12.F.IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MATH.9-12.F.IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
MATH.9-12.F.IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MATH.9-12.F.IF.C.7.e	Graph exponential and logarithmic functions, showing intercepts and end behavior.
MATH.9-12.F.IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
MATH.9-12.A.REI.D.11	Explain why the x -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
MATH.9-12.F.IF.C.8.b	Use the properties of exponents to interpret expressions for exponential functions.
MATH.9-12.F.IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MATH.9-12.A.SSE.A.1	Interpret expressions that represent a quantity in terms of its context.
MATH.9-12.A.SSE.A.1.a	Interpret parts of an expression, such as terms, factors, and coefficients.
MATH.9-12.F.LE.A.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
MATH.9-12.A.SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
MATH.9-12.F.LE.A.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
MATH.9-12.F.LE.A.4	Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a , c , and d are numbers and the base b is 2, 10, or e ; evaluate the logarithm using technology.
MATH.9-12.A.SSE.B.3.c	Use the properties of exponents to transform expressions for exponential functions.
MATH.9-12.F.LE.B.5	Interpret the parameters in a linear or exponential function in terms of a context.

Integration of Career Readiness, Life Literacies and Key Skills

TECH.9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
TECH.9.4.12.CI.2	Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).

TECH.9.4.12.CI.3	Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1).
TECH.9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
TECH.9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
TECH.9.4.12.TL.2	Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.
TECH.9.4.12.TL.3	Analyze the effectiveness of the process and quality of collaborative environments.

Integration of Computer Science and Design Thinking

CS.9-12.8.1.12.AP.1	Design algorithms to solve computational problems using a combination of original and existing algorithms.
CS.9-12.8.1.12.AP.5	Decompose problems into smaller components through systematic analysis, using constructs such as procedures, modules, and/or objects.

Interdisciplinary Connections: NJSLs for ELA, Social Studies, Science and/or Math

ELA.RI.MF.9–10.6	Analyze, integrate, and evaluate multiple interpretations (e.g., charts, graphs, diagrams, videos) of a single text or text/s presented in different formats (visually, quantitatively) as well as in words in order to address a question or solve a problem.
ELA.W.AW.9–10.1.E	Provide a concluding paragraph or section that supports the argument presented. Mathematical and computational thinking in 9–12 builds on K–8 experiences and progresses to using algebraic thinking and analysis, a range of linear and nonlinear functions including trigonometric functions, exponentials and logarithms, and computational tools for statistical analysis to analyze, represent, and model data. Simple computational simulations are created and used based on mathematical models of basic assumptions.

21st Century Life and Career

CRP.K-12.CRP1.1	Career-ready individuals understand the obligations and responsibilities of being a member of a community, and they demonstrate this understanding every day through their interactions with others. They are conscientious of the impacts of their decisions on others and the environment around them. They think about the near-term and long-term consequences of their actions and seek to act in ways that contribute to the betterment of their teams, families, community and workplace. They are reliable and consistent in going beyond the minimum expectation and in participating in activities that serve the greater good.
CRP.K-12.CRP2.1	Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation.
CRP.K-12.CRP3.1	Career-ready individuals understand the relationship between personal health, workplace performance and personal well-being; they act on that understanding to regularly practice healthy diet, exercise and mental health activities. Career-ready individuals also take

regular action to contribute to their personal financial well-being, understanding that personal financial security provides the peace of mind required to contribute more fully to their own career success.

CRP.K-12.CRP4.1

Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others' time. They are excellent writers; they master conventions, word choice, and organization, and use effective tone and presentation skills to articulate ideas. They are skilled at interacting with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to ensure the desired outcome.

CRP.K-12.CRP5.1

Career-ready individuals understand the interrelated nature of their actions and regularly make decisions that positively impact and/or mitigate negative impact on other people, organization, and the environment. They are aware of and utilize new technologies, understandings, procedures, materials, and regulations affecting the nature of their work as it relates to the impact on the social condition, the environment and the profitability of the organization.

CRP.K-12.CRP6.1

Career-ready individuals regularly think of ideas that solve problems in new and different ways, and they contribute those ideas in a useful and productive manner to improve their organization. They can consider unconventional ideas and suggestions as solutions to issues, tasks or problems, and they discern which ideas and suggestions will add greatest value. They seek new methods, practices, and ideas from a variety of sources and seek to apply those ideas to their own workplace. They take action on their ideas and understand how to bring innovation to an organization.

CRP.K-12.CRP7.1

Career-ready individuals are discerning in accepting and using new information to make decisions, change practices or inform strategies. They use reliable research process to search for new information. They evaluate the validity of sources when considering the use and adoption of external information or practices in their workplace situation.

CRP.K-12.CRP8.1

Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.

CRP.K-12.CRP9.1

Career-ready individuals consistently act in ways that align personal and community-held ideals and principles while employing strategies to positively influence others in the workplace. They have a clear understanding of integrity and act on this understanding in every decision. They use a variety of means to positively impact the directions and actions of a team or organization, and they apply insights into human behavior to change others' action, attitudes and/or beliefs. They recognize the near-term and long-term effects that management's actions and attitudes can have on productivity, morals and organizational culture.

CRP.K-12.CRP10.1

Career-ready individuals take personal ownership of their own education and career goals, and they regularly act on a plan to attain these goals. They understand their own career interests, preferences, goals, and requirements. They have perspective regarding the pathways available to them and the time, effort, experience and other requirements to pursue each, including a path of entrepreneurship. They recognize the value of each step in the education and experiential process, and they recognize that nearly all career paths require ongoing education and experience. They seek counselors, mentors, and other experts to assist in the planning and execution of career and personal goals.

CRP.K-12.CRP11.1

Career-ready individuals find and maximize the productive value of existing and new technology to accomplish workplace tasks and solve workplace problems. They are flexible and adaptive in acquiring new technology. They are proficient with ubiquitous technology

applications. They understand the inherent risks-personal and organizational-of technology applications, and they take actions to prevent or mitigate these risks.

CRP.K-12.CRP12.1

Career-ready individuals positively contribute to every team, whether formal or informal. They apply an awareness of cultural difference to avoid barriers to productive and positive interaction. They find ways to increase the engagement and contribution of all team members. They plan and facilitate effective team meetings.