Unit 07: Right Triangle Trigonometry (week 24-27)

Content Area: **Template**
Course(s):
Time Period: **Full Year**
Length: **4 Weeks**
Status: **Published**

Unit 7: Right Triangle Trigonometry (Modules 13 and 14)

Unit Rationale

In prior learning, students used the definition of similarity to decide if triangles are similar, and they used congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. They applied Pythagorean Theorem to determine missing side lengths. In this unit, students will use the trigonometric ratios and their inverses to find side lengths and angle measures in right triangles. They will use the trigonometric ratios and the Pythagorean Theorem to find the side lengths and angle measures involved in special right triangles. Then, they will use the Law of Sines and the Law of Cosines to find side lengths and angle measures of non-right triangles and solve real world problems. In later units, they will identify and describe relationships among inscribed angles, radii, and chords. They will construct the inscribed and circumscribed circles of a triangle and prove properties of angles for a quadrilateral inscribed in a circle.

Essential Questions

- What are tangent, sine, and cosine ratios, and how do they relate to the sides of a right triangle?
- How can trigonometric ratios be used to find missing side lengths and angle measures in right triangles, and how can they be applied in solving real world problems?
- How do these special triangles simplify trigonometric calculations?
- What strategies and steps are involved in solving trigonometric problems?
- What is the Law of Sines and Law of Cosines and how are they used to solve triangles?
- When is the Law of Sines applicable, and what are its limitations?
- How can the Law of Cosines be applied to solve triangles, including cases involving obtuse angles?
- How does trigonometry extend beyond right triangles into other geometric shapes and practical applications?

Pre-Assessments

- Into Geometry: Are you ready? p. 404

Instructional Plan

Tangent Ratio (13.1)

Student Learning Intentions or We are learning to … (WALT)

- Solve for missing side lengths and angle measures using the tangent, inverse tangent, and properties of similar triangles.

Student Success Criteria … "I can statements"

- I can find the opposite and adjacent sides related to a given angle in a right triangle.
- I can write the tangent ratio and find an unknown side length.
- I can use the tangent ratio and the inverse tangent to find side lengths and angle measures in right triangles so that I can solve geometric and real world problems.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 405
- Guided Notes
    - Investigate a ratio in a right triangle
    - Understand tangent ratios
    - Apply tangent to find a length
    - Apply inverse tangent to find an angle measure.
- DeltaMath practice assignment
- Into Geometry Practice p. 410

Formative Assessments

- Into Geometry Check Understanding

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Sine and Cosine Ratios (13.2)

Student Learning Intentions or We are learning to … (WALT)

- Use sine and cosine ratios and the inverses to find side lengths and angle measures in right triangles in order to solve geometric and real world problems.

Student Success Criteria … "I can statements"

- I can find the opposite side, adjacent side, and the hypotenuse related to a given angle in a right triangle.
- I can write the sine and cosine ratio and find an unknown side length.
- I can use sine and cosine ratios and the inverses to find side lengths and angle measures in right triangles.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 413
- Guided Notes
    - Investigate ratios in a right triangle
    - Find the sine and cosine of an angle
    - Find side lengths and perimeter using sine and cosine
    - Find an angle measure using inverse sine and inverse cosine
- DeltaMath practice assignment
- Into Geometry Practice p. 418

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Special Right Triangles (13.3)

Student Learning Intentions or We are learning to … (WALT)

- Examine and use trigonometric ratios for special right triangles, and use the Pythagorean Theorem to find the side lengths and angle measures of special right triangles.

Student Success Criteria … "I can statements"

- I know the ratio of the sides in special right triangles.
- I can use the special right triangles to solve problems.
- I can use trigonometric ratios and the Pythagorean Theorem to find the side lengths and angle measures of special right triangles.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 421
- Guided Notes
    - Investigate 45-45-90 triangles
    - Investigate 30-60-90 triangles
    - Find trigonometric ratios of special right triangles
    - Model real world measurements
- DeltaMath practice assignment
- Into Geometry Practice p. 425

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Solve Problems Using Trigonometry (13.4)

Student Learning Intentions or We are learning to … (WALT)

- Apply the sine, cosine, and tangent ratios and the inverses to find the areas of triangles and measures of sides and acute angles in right triangles both in the coordinate plane and represented in real-world

situations.

Student Success Criteria … "I can statements"

- I can use the Pythagorean Theorem to solve problems in right triangles.
- I can define the trigonometric ratios for the acute angles in right triangles and use them to solve problems.
- I can use trigonometric ratios, the area formula for a triangle in terms of its side lengths, and the Pythagorean Theorem to solve right triangles in applied problems

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 429
- Guided Notes
    - Derive an area formula
    - Solve a right triangle
    - Find an angle of elevation or depression
    - Solve a right triangle in the coordinate plane
- DeltaMath practice assignment
- Into Geometry Practice p. 434

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Law of Sines (14.1)

Student Learning Intentions or We are learning to … (WALT)

- Prove the Law of Sines, determine which combinations of given triangle measures are sufficient to apply the Law of Sines, determine how many triangles can exist with an ambiguous (SSA) combination of given measures, and apply the Law of Sines to determine unknown triangle measures in real world problems.

Student Success Criteria … "I can statements"

- I can tell which triangles the Law of Sines will help me find missing side lengths and angle measures.

- I can use the Law of Sines to find side lengths and angle measures of non-right triangles and solve real-world problems.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 441
- Guided Notes
    - Derive the Law of Sines
    - Determine when to apply the law of Sines
    - Explore the SSA case
    - Apply the Law of Sines to the SSA case
- DeltaMath practice assignment
- Into Geometry Practice p. 447

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Law of Cosines (14.2)

Student Learning Intentions or We are learning to … (WALT)

- Derive the Law of Cosines, recognize the SAS and SSS cases where the Law of Cosines is applicable and apply it to find unknown measures, derive the area formula A = 1/2ab sin C for a triangle, and find the area of triangles when given an SAS or SSS combination of measures in real world problems.

Student Success Criteria … "I can statements"

- I can tell when to use the Law of Cosines to find missing measures of parts of a triangle.
- I can use the Law of Cosines to find side lengths and angle measures of non-right triangles and solve real-world problems.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 451
- Guided Notes
    - Derive the Law of Cosines

    - Investigate cosine values of obtuse angles
    - Derive a formula for the area of a triangle
    - Solve a triangle using the Law of Cosines
    - Use the Law of Cosines to solve a real world problem
    - Find the area of a triangle
- DeltaMath practice assignment
- Into Geometry Practice p. 456

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Modifications and/or Accommodations

Suggested Modifications (ELL, Sp. Ed, GIfted, At-risk of Failure)

English Language Learners

Native language support: The teacher provides auditory or written content to students in their native language.

Adjusted Speech: The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.

Visuals: The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subjects at hand.

Front-Loading Vocabulary: The teacher front loads vocabulary. This means providing students with a list of important vocabulary words they will need to know for a book, lesson, etc. prior to the lesson being taught. Including pictures to go with the vocabulary words is also very beneficial for the students.

Special Education Students

Chunking: The teacher presents information in a way that makes it easy for students to understand and remember. Chunking is based on the presumption that our working memory is easily overloaded by excessive detail. The best way to deliver information is to organize it into meaningful units. Because students with special needs get overloaded easily, chunking is an effective strategy to use with them.

Checking for Understanding: It is important to constantly check for understanding, especially for students who have accommodations. Teachers want to make sure students understand the concepts being covered in a way that makes sense to them.

Extra time: The teacher provides students with special needs extra time to complete work or answer questions. It is important to give students enough time to process their thoughts.

Oral Reading: The teacher will read work orally to students. Class work such as tests and literature circles may need to be read aloud to the student.

Timers: The teacher will use timers as an instructional tool. The use of timers is beneficial for students who have trouble completing tasks. Timers can be helpful so the student is aware of how much time they have to complete an assignment.

Students with 504 Plans

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Gifted & Talented Strategies

Extensions/Enrichments: Teachers will provide gifted and talented students with extension/enrichment projects. Students will be challenged to further their understanding, to apply acquired knowledge, and/or to produce something in reference to acquired knowledge.

Modify/Change Activities: Teachers will monitor and modify activities to accommodate those students who need to be challenged further. Additional reading, problem-solving, writing, or project work is necessary for those students who are ready to move on at a rate more accelerated than their peers. In this way, G & T students are provided the same opportunity for support as special needs

students.

Students at Risk of School Failure

Directions or Instructions: Make sure directions and/or instructions are given in limited numbers. Give directions/instructions verbally and in simple written format. Ask students to repeat the instructions or directions to ensure understanding occurs. Check back with the student to ensure he/she hasn't forgotten.

Peer Support: Peers can help build confidence in other students by assisting in peer learning. Many teachers use the 'ask 3 before me' approach. This is fine, however, a student at risk may have to have a specific student or two to ask. Set this up for the student so he/she knows who to ask for clarification before going to you.

Alternate or Modified Assignments: Always ask yourself, "How can I modify this assignment to ensure the students at risk are able to complete it?" Sometimes you'll simplify the task, reduce the length of the assignment or allow for a different mode of delivery. For instance, many students may hand something in, the at-risk student may jot notes and give you the information verbally. Or, it just may be that you will need to assign an alternate assignment.

Increase One to One Time: When other students are working, always touch base with your students at risk and find out if they're on track or needing some additional support. A few minutes here and there will go a long way to intervene as the need presents itself.

Contracts: It helps to have a working contract between you and your students at risk. This helps prioritize the tasks that need to be done and ensure completion happens. Each day write down what needs to be completed, as the tasks are done, provide a checkmark or happy face. The goal of using contracts is to eventually have the student come to you for completion sign-offs.

Hands On: As much as possible, think in concrete terms and provide hands-on tasks. This means a child doing math may require a calculator or counters. The child may need to tape record comprehension activities instead of writing them. A child may have to listen to a story being read instead of reading it him/herself.

Tests/Assessments: Tests can be done orally if need be. Break tests down in smaller increments by having a portion of the test in the morning, another portion after lunch and the final part the next day.

Seating: Seat students near a helping peer or with quick access to the teacher. Those with hearing or sight issues need to be close to the instruction which often means near the front.

Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media Literacy

Diversity, Equity, and Inclusion

NCTM: Access and Equity in Mathematics Education

A Pathway to Equitable Math Instruction

Provide students with opportunities to give feedback to teachers about the classroom and instruction.

• Verbal Example: Fist to five, How well do you understand what we talked about today? Fist to five, How well did I teach this today?

• Classroom Activity: Exit tickets or surveys that ask students to identify how well teachers taught, what helped them learn, what got in the way of their learning, etc.

Treat mathematics as a language that everyone is learning while authentically centering students home languages.

• Classroom Strategies: Color-coding ideas, learning vocabulary in student languages, visual and kinesthetic learning, representations of learning without words.

• Classroom Activity: Multilingual Frayer Models for
definitions or concepts

Incorporate true culturally relevant pedagogy, practice, and curriculum.

• Verbal Example: What are some of your family traditions that you are proud of? Would you be okay if we brought some of those into the classroom?

• Classroom Activity: Use Ankara fabric to teach mathematical concepts such as tessellations, fractions, area, percentages, etc.

Incorporate the history of mathematics into lessons.
• Verbal Example: Why do you think we call it Pythagorean's theorem, when it was used before he was even born? What should we call it instead?
• Classroom Activity: Learn about different bases and numerical ideas: Base 2, binary and connections to computer programming, how the Yoruba of Nigeria used base 20, and how the Mayans conceptualized the number 0 before the first recording of it

Solicit student ways of thinking and processing.
• Verbal Example: How might you all go about this? What do you notice?
• Classroom Activity: Incorporate explorations, where students interact with mathematics in a way that allows them to "discover" or experience mathematics.

Reorganize your classroom teaching around concepts, and teach them more like a web rather than discrete sets of knowledge.
• Verbal Example: How does this connect to what you've learned in the past? How can you use that knowledge today?

• Classroom Activity: Learning webs that connect content

Start with more complex math problems and scaffold as necessary.
• Verbal Example: If we wanted to build a rocket, what are all the things we might need to know before we get started? Along the way, we decided that we want the rocket to reach the moon. What do we need to consider now?
• Classroom Activity: When solving equations, start with the most complex problem, generate ideas for how to solve it, and use the simpler equations as examples to support those ideas.

Offer a variety of ways to demonstrate thinking and knowledge.
• Verbal Example: Show your thinking with words, pictures, symbols.

Ask other questions that will demonstrate learning when it is not clear to you how students know the answer.
• Verbal Example: If you were working with a fellow mathematician who was absent this day, what might you tell them to help them learn it?

Learn about, engage with, and incorporate ethnomathematics.
• Verbal Example: Reflect on your day so far. What math have you already used today?
• Classroom Activity: Community walks to engage with slope.

Co-construct knowledge in the classroom.
• Verbal Example: Let's get into partners and do a think pair-share. We will incorporate everyone's ideas and try to synthesize them.
• Classroom Activity: Have students create mathematical definitions in their own words in groups, and bring the groups together to co-construct mathematical definitions as a class

Choose problems that have complex, competing, or multiple answers.
• Verbal Example: Come up with at least two answers that might solve this problem.
• Classroom Activity: Challenge standardized test questions by getting the "right" answer, but justify other answers by unpacking the assumptions that are made in the problem.
• Classroom Activity: Deconstructed Multiple Choice
- given a set of multiple choice answers, students discuss why these answers may have been included (can also be used to highlight common mistakes).

Identify what is right about the thinking, and highlight the mistake in what is factually or procedurally accepted.
• Verbal Example: You recognized that you had to combine the constants 27 and 9, could you explain your thinking?

• Classroom Activity: Error Analysis worksheets that highlight what is the right idea behind the mistake.

Using thoughtful questioning to solicit mathematical thoughts rather than telling.

• Verbal Example: What would a mathematician who is confused ask about this question?

• Classroom Activity: After students demonstrate knowledge of a topic, have them play a game where they have to explain their topic to a fellow mathematician and a skeptic. Develop their own reflective questioning/explaining in all three roles.

Create multiple ways of participating that honor myriad ways of thinking and being.
• Verbal Example: For this section, feel free to work alone, in pairs, trios, or quads (let them choose).
• Classroom Activity: Community circles or storytelling circles, incorporating dance, music, song, call and response, and other cultural ways of communicating.

Climate Change

Math Climate Change Companion Guide

New Jersey Student Learning Standards: Content Area

MATH.9-12.G.SRT.C.6	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
MATH.9-12.G.SRT.C.7	Explain and use the relationship between the sine and cosine of complementary angles.
MATH.9-12.G.SRT.C.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
MATH.9-12.G.SRT.D.9	Derive the formula $A = 1/2\ ab\ \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
MATH.9-12.G.SRT.D.10	Prove the Laws of Sines and Cosines and use them to solve problems.
MATH.9-12.G.SRT.D.11	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Integration of Career Readiness, Life Literacies and Key Skills

TECH.9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
TECH.9.4.12.CI.2	Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).
TECH.9.4.12.CI.3	Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1).

TECH.9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
TECH.9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
TECH.9.4.12.TL.2	Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.
TECH.9.4.12.TL.3	Analyze the effectiveness of the process and quality of collaborative environments.

Integration of Computer Science and Design Thinking

CS.9-12.8.1.12.AP.1	Design algorithms to solve computational problems using a combination of original and existing algorithms.
CS.9-12.8.1.12.AP.5	Decompose problems into smaller components through systematic analysis, using constructs such as procedures, modules, and/or objects.

Interdisciplinary Connections: NJSLS for ELA, Social Studies, Science and/or Math

ELA.RI.MF.9–10.6	Analyze, integrate, and evaluate multiple interpretations (e.g., charts, graphs, diagrams, videos) of a single text or text/s presented in different formats (visually, quantitatively) as well as in words in order to address a question or solve a problem.
ELA.W.AW.9–10.1.E	Provide a concluding paragraph or section that supports the argument presented.
	Mathematical and computational thinking in 9–12 builds on K–8 experiences and progresses to using algebraic thinking and analysis, a range of linear and nonlinear functions including trigonometric functions, exponentials and logarithms, and computational tools for statistical analysis to analyze, represent, and model data. Simple computational simulations are created and used based on mathematical models of basic assumptions.

21st Century Life and Career

CRP.K-12.CRP1.1	Career-ready individuals understand the obligations and responsibilities of being a member of a community, and they demonstrate this understanding every day through their interactions with others. They are conscientious of the impacts of their decisions on others and the environment around them. They think about the near-term and long-term consequences of their actions and seek to act in ways that contribute to the betterment of their teams, families, community and workplace. They are reliable and consistent in going beyond the minimum expectation and in participating in activities that serve the greater good.
CRP.K-12.CRP2.1	Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation.
CRP.K-12.CRP3.1	Career-ready individuals understand the relationship between personal health, workplace performance and personal well-being; they act on that understanding to regularly practice healthy diet, exercise and mental health activities. Career-ready individuals also take regular action to contribute to their personal financial well-being, understanding that personal financial security provides the peace of mind required to contribute more fully to their own career success.

CRP.K-12.CRP4.1	Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others' time. They are excellent writers; they master conventions, word choice, and organization, and use effective tone and presentation skills to articulate ideas. They are skilled at interacting with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to ensure the desired outcome.
CRP.K-12.CRP5.1	Career-ready individuals understand the interrelated nature of their actions and regularly make decisions that positively impact and/or mitigate negative impact on other people, organization, and the environment. They are aware of and utilize new technologies, understandings, procedures, materials, and regulations affecting the nature of their work as it relates to the impact on the social condition, the environment and the profitability of the organization.
CRP.K-12.CRP6.1	Career-ready individuals regularly think of ideas that solve problems in new and different ways, and they contribute those ideas in a useful and productive manner to improve their organization. They can consider unconventional ideas and suggestions as solutions to issues, tasks or problems, and they discern which ideas and suggestions will add greatest value. They seek new methods, practices, and ideas from a variety of sources and seek to apply those ideas to their own workplace. They take action on their ideas and understand how to bring innovation to an organization.
CRP.K-12.CRP7.1	Career-ready individuals are discerning in accepting and using new information to make decisions, change practices or inform strategies. They use reliable research process to search for new information. They evaluate the validity of sources when considering the use and adoption of external information or practices in their workplace situation.
CRP.K-12.CRP8.1	Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.
CRP.K-12.CRP9.1	Career-ready individuals consistently act in ways that align personal and community-held ideals and principles while employing strategies to positively influence others in the workplace. They have a clear understanding of integrity and act on this understanding in every decision. They use a variety of means to positively impact the directions and actions of a team or organization, and they apply insights into human behavior to change others' action, attitudes and/or beliefs. They recognize the near-term and long-term effects that management's actions and attitudes can have on productivity, morals and organizational culture.
CRP.K-12.CRP10.1	Career-ready individuals take personal ownership of their own education and career goals, and they regularly act on a plan to attain these goals. They understand their own career interests, preferences, goals, and requirements. They have perspective regarding the pathways available to them and the time, effort, experience and other requirements to pursue each, including a path of entrepreneurship. They recognize the value of each step in the education and experiential process, and they recognize that nearly all career paths require ongoing education and experience. They seek counselors, mentors, and other experts to assist in the planning and execution of career and personal goals.
CRP.K-12.CRP11.1	Career-ready individuals find and maximize the productive value of existing and new technology to accomplish workplace tasks and solve workplace problems. They are flexible and adaptive in acquiring new technology. They are proficient with ubiquitous technology applications. They understand the inherent risks-personal and organizational-of technology applications, and they take actions to prevent or mitigate these risks.
CRP.K-12.CRP12.1	Career-ready individuals positively contribute to every team, whether formal or informal.

They apply an awareness of cultural difference to avoid barriers to productive and positive interaction. They find ways to increase the engagement and contribution of all team members. They plan and facilitate effective team meetings.