

Unit 10: Probability (week 37-39)

Content Area: **Template**
Course(s):
Time Period: **Full Year**
Length: **3 Weeks**
Status: **Published**

Unit 10: Probability (Modules 20 and 21)

Unit Rationale

In prior learning, students expressed the likelihood of an events as a positive number between 0 and 1. They summarized categorical data in two way frequency tables, and interpreted relative frequencies in the context of other data. In this unit, students will understand how elements and subsets relate to a larger universal set, and they will describe and represent sets, their characteristics, and their relationships. They will calculate theoretical probabilities, employing the addition rule for the probability of the union of overlapping events. Later, they will find conditional probabilities from a two-way table, then derive and apply the Conditional Probability Formula. They will use the multiplication rule to find the probability of events. In later learning, they will describe events as subsets of a sample space using characteristics of the outcomes, or as unions, intersections, or complements of other events. They will look more deeply into independent events and conditional probabilities.

Essential Questions

- What is the relationship between probability and set theory, and how are they used together in solving problems?
- What are disjoint and overlapping events, and how do their probabilities relate to each other?
- What is conditional probability, and how is it calculated?
- How do dependent events differ from independent events, and how are their probabilities determined?
- Why do real-world scenarios often involve independent and dependent events, and how does this influence decision-making and risk assessment?

Pre-Assessments

- Into Geometry: Are you ready? p. 614

Instructional Plan

Probability and Set Theory (20.1)

Student Learning Intentions or We are learning to ... (WALT)

- Describe sets and their relationships, including the universal set and complement, calculate theoretical probabilities and outcomes of events, and use the complement of an event to calculate probability.

Student Success Criteria ... “I can statements”

- I can understand and calculate the probabilities of simple events.
- I can use sets and their relationships to understand and calculate probabilities.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 615
- Guided Notes
 - Work with sets
 - Calculate theoretical probability
 - Use the complement of an event
 - Calculate probability in a real world scenario
- DeltaMath practice assignment
- Into Geometry Practice p. 620

Formative Assessments

- Into Geometry Check Understanding online interactive lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Disjoint and Overlapping Events (20.2)

Student Learning Intentions or We are learning to ... (WALT)

- Recognize disjoint and overlapping events and understand the Addition Rule to find the probability of the union of overlapping events.

Student Success Criteria ... “I can statements”

- I can explain what are disjoint events and overlapping events.
- I can use a two way frequency table to tell whether two events are disjoint or overlapping.
- I can calculate probabilities for both disjoint and overlapping events.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 623
- Guided Notes
 - Recognize disjoint events
 - Recognize overlapping events
 - Find the probability of disjoint events
 - Find a probability from a two way table of data
- DeltaMath practice assignment
- Into Geometry Practice p. 628

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Conditional Probability (21.1)

Student Learning Intentions or We are learning to ... (WALT)

- Calculate conditional probability and use it to solve real-world problems.

Student Success Criteria ... “I can statements”

- I can calculate the probability of an event.
- I can use a frequency table to represent the number of outcomes in an event and calculate the probability of each event.
- I can calculate conditional probability and use it to solve real-world problems.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 635
- Guided Notes
 - Find conditional probabilities from a two way frequency table
 - Derive the Conditional Probability Theorem
 - Use the conditional probability formula
 - Use the conditional probability formula to solve a real world problem
- DeltaMath practice assignment
- Into Geometry Practice p. 640

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Independent Events (21.2)

Student Learning Intentions or We are learning to ... (WALT)

- Interpret independence and its definition in terms of conditional probability. Derive and apply the Multiplication Rule for the probability of the intersection of independent events and use the rule to test for independence of events. Use the concept of independence to solve real-world problems.

Student Success Criteria ... “I can statements”

- I can explain what independent events are.
- I can determine whether two events are independent and find their probabilities.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 643
- Guided Notes
 - Understand independence of events
 - Find the probability of independent events
- DeltaMath practice assignment
- Into Geometry Practice p. 646

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Dependent Events (21.3)

Student Learning Intentions or We are learning to ... (WALT)

- Determine whether two events are dependent and find their probabilities.

Student Success Criteria ... “I can statements”

- I can find the probability of simple events.
- I can determine whether two events are dependent and find their probabilities.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 649
- Guided Notes
 - Derive a formula for the probability of the occurrence of two dependent events
 - Find the probability of a dependent event
 - Find the probability of three or more dependent events
- DeltaMath practice assignment
- Into Geometry Practice p. 653

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Modifications and/or Accommodations

Suggested Modifications (ELL, Sp. Ed, Gifted, At-risk of Failure)

English Language Learners

Native language support: The teacher provides auditory or written content to students in their native language.

Adjusted Speech: The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.

Visuals: The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subjects at hand.

Front-Loading Vocabulary: The teacher front loads vocabulary. This means providing students with a list of important vocabulary words they will need to know for a book, lesson, etc. prior to the lesson being taught. Including pictures to go with the vocabulary words is also very beneficial for the students.

Special Education Students

Chunking: The teacher presents information in a way that makes it easy for students to understand and remember. Chunking is based on the presumption that our working memory is easily overloaded by excessive detail. The best way to deliver information is to organize it into meaningful units. Because students with special needs get overloaded easily, chunking is an effective strategy to use with them.

Checking for Understanding: It is important to constantly check for understanding, especially for students who have accommodations. Teachers want to make sure students understand the concepts being covered in a way that makes sense to them.

Extra time: The teacher provides students with special needs extra time to complete work or answer questions. It is important to give students enough time to process their thoughts.

Oral Reading: The teacher will read work orally to students. Class work such as tests and literature circles may need to be read aloud to the student.

Timers: The teacher will use timers as an instructional tool. The use of timers is beneficial for students who have trouble completing tasks. Timers can be helpful so the student is aware of how much time they have to complete an assignment.

Students with 504 Plans

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Gifted & Talented Strategies

Extensions/Enrichments: Teachers will provide gifted and talented students with extension/enrichment projects. Students will be challenged to further their understanding, to apply acquired knowledge, and/or to produce something in reference to acquired knowledge.

Modify/Change Activities: Teachers will monitor and modify activities to accommodate those students who need to be challenged further. Additional reading, problem-solving, writing, or project work is necessary for those students who are ready to move on at a rate more accelerated than their peers. In this way, G & T students are provided the same opportunity for support as special needs students.

Students at Risk of School Failure

Directions or Instructions: Make sure directions and/or instructions are given in limited numbers. Give directions/instructions verbally and in simple written format. Ask students to repeat the instructions or directions to ensure understanding occurs. Check back with the student to ensure he/she hasn't forgotten.

Peer Support: Peers can help build confidence in other students by assisting in peer learning. Many teachers use the 'ask 3 before me' approach. This is fine, however, a student at risk may have to have a specific student or two to ask. Set this up for the student so he/she knows who to ask for

clarification before going to you.

Alternate or Modified Assignments: Always ask yourself, "How can I modify this assignment to ensure the students at risk are able to complete it?" Sometimes you'll simplify the task, reduce the length of the assignment or allow for a different mode of delivery. For instance, many students may hand something in, the at-risk student may jot notes and give you the information verbally. Or, it just may be that you will need to assign an alternate assignment.

Increase One to One Time: When other students are working, always touch base with your students at risk and find out if they're on track or needing some additional support. A few minutes here and there will go a long way to intervene as the need presents itself.

Contracts: It helps to have a working contract between you and your students at risk. This helps prioritize the tasks that need to be done and ensure completion happens. Each day write down what needs to be completed, as the tasks are done, provide a checkmark or happy face. The goal of using contracts is to eventually have the student come to you for completion sign-offs.

Hands On: As much as possible, think in concrete terms and provide hands-on tasks. This means a child doing math may require a calculator or counters. The child may need to tape record comprehension activities instead of writing them. A child may have to listen to a story being read instead of reading it him/herself.

Tests/Assessments: Tests can be done orally if need be. Break tests down in smaller increments by having a portion of the test in the morning, another portion after lunch and the final part the next day.

Seating: Seat students near a helping peer or with quick access to the teacher. Those with hearing or sight issues need to be close to the instruction which often means near the front.

Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media Literacy

Diversity, Equity, and Inclusion

[NCTM: Access and Equity in Mathematics Education](#)

[A Pathway to Equitable Math Instruction](#)

Provide students with opportunities to give feedback to teachers about the classroom and instruction.

- **Verbal Example:** Fist to five, How well do you understand what we talked about today? Fist to five, How well did I teach this today?
- **Classroom Activity:** Exit tickets or surveys that ask students to identify how well teachers taught, what helped them learn, what got in the way of their learning, etc.

Treat mathematics as a language that everyone is learning while authentically centering students home

languages.

- Classroom Strategies: Color-coding ideas, learning vocabulary in student languages, visual and kinesthetic learning, representations of learning without words.
- Classroom Activity: Multilingual Frayer Models for definitions or concepts

Incorporate true culturally relevant pedagogy, practice, and curriculum.

- Verbal Example: What are some of your family traditions that you are proud of? Would you be okay if we brought some of those into the classroom?
- Classroom Activity: Use Ankara fabric to teach mathematical concepts such as tessellations, fractions, area, percentages, etc.

Incorporate the history of mathematics into lessons.

- Verbal Example: Why do you think we call it Pythagorean's theorem, when it was used before he was even born? What should we call it instead?
- Classroom Activity: Learn about different bases and numerical ideas: Base 2, binary and connections to computer programming, how the Yoruba of Nigeria used base 20, and how the Mayans conceptualized the number 0 before the first recording of it

Solicit student ways of thinking and processing.

- Verbal Example: How might you all go about this? What do you notice?
- Classroom Activity: Incorporate explorations, where students interact with mathematics in a way that allows them to "discover" or experience mathematics.

Reorganize your classroom teaching around concepts, and teach them more like a web rather than discrete sets of knowledge.

- Verbal Example: How does this connect to what you've learned in the past? How can you use that knowledge today?
- Classroom Activity: Learning webs that connect content

Start with more complex math problems and scaffold as necessary.

- Verbal Example: If we wanted to build a rocket, what are all the things we might need to know before we get started? Along the way, we decided that we want the rocket to reach the moon. What do we need to consider now?
- Classroom Activity: When solving equations, start with the most complex problem, generate ideas for how to solve it, and use the simpler equations as examples to support those ideas.

Offer a variety of ways to demonstrate thinking and knowledge.

- Verbal Example: Show your thinking with words, pictures, symbols.

Ask other questions that will demonstrate learning when it is not clear to you how students know the answer.

- Verbal Example: If you were working with a fellow mathematician who was absent this day, what might you tell them to help them learn it?

Learn about, engage with, and incorporate ethnomathematics.

- Verbal Example: Reflect on your day so far. What math have you already used today?
- Classroom Activity: Community walks to engage with slope.

Co-construct knowledge in the classroom.

- Verbal Example: Let's get into partners and do a think pair-share. We will incorporate everyone's ideas and try to synthesize them.
- Classroom Activity: Have students create mathematical definitions in their own words in groups, and bring the groups together to co-construct mathematical definitions as a class

Choose problems that have complex, competing, or multiple answers.

- Verbal Example: Come up with at least two answers that might solve this problem.
- Classroom Activity: Challenge standardized test questions by getting the "right" answer, but justify other answers by unpacking the assumptions that are made in the problem.
- Classroom Activity: Deconstructed Multiple Choice
- given a set of multiple choice answers, students discuss why these answers may have been included (can also be used to highlight common mistakes).

Identify what is right about the thinking, and highlight the mistake in what is factually or procedurally accepted.

- Verbal Example: You recognized that you had to combine the constants 27 and 9, could you explain your thinking?
- Classroom Activity: Error Analysis worksheets that highlight what is the right idea behind the mistake.

Using thoughtful questioning to solicit mathematical thoughts rather than telling.

- Verbal Example: What would a mathematician who is confused ask about this question?
- Classroom Activity: After students demonstrate knowledge of a topic, have them play a game where they have to explain their topic to a fellow mathematician and a skeptic. Develop their own reflective questioning/explaining in all three roles.

Create multiple ways of participating that honor myriad ways of thinking and being.

- Verbal Example: For this section, feel free to work alone, in pairs, trios, or quads (let them choose).
- Classroom Activity: Community circles or storytelling circles, incorporating dance, music, song, call and response, and other cultural ways of communicating.

Climate Change

[Math Climate Change Companion Guide](#)

New Jersey Student Learning Standards: Content Area

MATH.9-12.G.CO.A.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
MATH.9-12.G.CO.C.9	Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
MATH.9-12.G.CO.D.12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
MATH.9-12.G.GPE.B.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.
MATH.9-12.G.MG.A.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
MATH.9-12.G.MG.A.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

Integration of Career Readiness, Life Literacies and Key Skills

TECH.9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
TECH.9.4.12.CI.2	Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).
TECH.9.4.12.CI.3	Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1).
TECH.9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
TECH.9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
TECH.9.4.12.TL.2	Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.

Integration of Computer Science and Design Thinking

CS.9-12.8.1.12.AP.1	Design algorithms to solve computational problems using a combination of original and existing algorithms.
CS.9-12.8.1.12.AP.5	Decompose problems into smaller components through systematic analysis, using constructs such as procedures, modules, and/or objects.

Interdisciplinary Connections: NJSL for ELA, Social Studies, Science and/or Math

ELA.RI.MF.9–10.6	Analyze, integrate, and evaluate multiple interpretations (e.g., charts, graphs, diagrams, videos) of a single text or text/s presented in different formats (visually, quantitatively) as well as in words in order to address a question or solve a problem.
ELA.W.AW.9–10.1.E	Provide a concluding paragraph or section that supports the argument presented. Mathematical and computational thinking in 9–12 builds on K–8 experiences and progresses to using algebraic thinking and analysis, a range of linear and nonlinear functions including trigonometric functions, exponentials and logarithms, and computational tools for statistical analysis to analyze, represent, and model data. Simple computational simulations are created and used based on mathematical models of basic assumptions.

21st Century Life and Career

CRP.K-12.CRP1.1	Career-ready individuals understand the obligations and responsibilities of being a member of a community, and they demonstrate this understanding every day through their interactions with others. They are conscientious of the impacts of their decisions on others and the environment around them. They think about the near-term and long-term consequences of their actions and seek to act in ways that contribute to the betterment of their teams, families, community and workplace. They are reliable and consistent in going beyond the minimum expectation and in participating in activities that serve the greater good.
CRP.K-12.CRP2.1	Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation.
CRP.K-12.CRP3.1	Career-ready individuals understand the relationship between personal health, workplace performance and personal well-being; they act on that understanding to regularly practice healthy diet, exercise and mental health activities. Career-ready individuals also take regular action to contribute to their personal financial well-being, understanding that personal financial security provides the peace of mind required to contribute more fully to their own career success.
CRP.K-12.CRP4.1	Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others' time. They are excellent writers; they master conventions, word choice, and organization, and use effective tone and presentation skills to articulate ideas. They are skilled at interacting with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to

ensure the desired outcome.

CRP.K-12.CRP5.1

Career-ready individuals understand the interrelated nature of their actions and regularly make decisions that positively impact and/or mitigate negative impact on other people, organization, and the environment. They are aware of and utilize new technologies, understandings, procedures, materials, and regulations affecting the nature of their work as it relates to the impact on the social condition, the environment and the profitability of the organization.

CRP.K-12.CRP6.1

Career-ready individuals regularly think of ideas that solve problems in new and different ways, and they contribute those ideas in a useful and productive manner to improve their organization. They can consider unconventional ideas and suggestions as solutions to issues, tasks or problems, and they discern which ideas and suggestions will add greatest value. They seek new methods, practices, and ideas from a variety of sources and seek to apply those ideas to their own workplace. They take action on their ideas and understand how to bring innovation to an organization.

CRP.K-12.CRP7.1

Career-ready individuals are discerning in accepting and using new information to make decisions, change practices or inform strategies. They use reliable research process to search for new information. They evaluate the validity of sources when considering the use and adoption of external information or practices in their workplace situation.

CRP.K-12.CRP8.1

Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.

CRP.K-12.CRP9.1

Career-ready individuals consistently act in ways that align personal and community-held ideals and principles while employing strategies to positively influence others in the workplace. They have a clear understanding of integrity and act on this understanding in every decision. They use a variety of means to positively impact the directions and actions of a team or organization, and they apply insights into human behavior to change others' action, attitudes and/or beliefs. They recognize the near-term and long-term effects that management's actions and attitudes can have on productivity, morals and organizational culture.

CRP.K-12.CRP10.1

Career-ready individuals take personal ownership of their own education and career goals, and they regularly act on a plan to attain these goals. They understand their own career interests, preferences, goals, and requirements. They have perspective regarding the pathways available to them and the time, effort, experience and other requirements to pursue each, including a path of entrepreneurship. They recognize the value of each step in the education and experiential process, and they recognize that nearly all career paths require ongoing education and experience. They seek counselors, mentors, and other experts to assist in the planning and execution of career and personal goals.

CRP.K-12.CRP11.1

Career-ready individuals find and maximize the productive value of existing and new technology to accomplish workplace tasks and solve workplace problems. They are flexible and adaptive in acquiring new technology. They are proficient with ubiquitous technology applications. They understand the inherent risks-personal and organizational-of technology applications, and they take actions to prevent or mitigate these risks.

CRP.K-12.CRP12.1

Career-ready individuals positively contribute to every team, whether formal or informal. They apply an awareness of cultural difference to avoid barriers to productive and positive interaction. They find ways to increase the engagement and contribution of all team members. They plan and facilitate effective team meetings.

