

Unit 09: Surface Area and Volume

Content Area: **Template**
Course(s):
Time Period: **Full Year**
Length: **4 Weeks**
Status: **Published**

Unit 9: Surface Area and Volume (Modules 18 and 19)

Unit Rationale

In prior learning, students represented three-dimensional figures using nets and used them to find surface area. They solved real world mathematical problems involving the areas of two-dimensional objects composed of known shapes. In this unit, students will identify solids of rotation and cross sections of solids. They will use nets to develop formulas for surface area, and use formulas to find surface area of hemispheres and composite figures. Then, they will develop, relate, and apply formulas for the volume of right and oblique prisms and cylinders. They will find the volume of composite figures involving spheres and estimate volume in real world situations. In future learning, students will translate between the geometric description and the equation for a conic section. They will interpret key features for a function that models relationship between two quantities. They will sketch graphs showing key features of a function given a verbal description of the relationship.

Essential Questions

- How can three-dimensional figures be classified based on their geometric properties and dimensions?
- How is the surface area of a prism, cylinder, cone, pyramid, or sphere calculated?
- Why is understanding surface area crucial in determining material requirements and cost estimations in construction and manufacturing?
- What are the formulas for calculating the volume of a prism, cylinder, cone, pyramid, or sphere, and how do these formulas relate to the geometric properties and dimensions of each shape?
- Why is volume important in measuring capacity and storage in real-world applications?

Pre-Assessments

- Into Geometry: Are you ready? p. 550

Instructional Plan

Three-Dimensional Figures (18.1)

Student Learning Intentions or We are learning to ... (WALT)

- Identify and classify three-dimensional solids by name, identify solids of rotation with plane figures rotated about axes, and identify cross sections of solids in planes parallel and not parallel to bases.

Student Success Criteria ... “I can statements”

- I can identify three-dimensional figures by name.
- I can describe a cross section of cylinder solid of rotation when the plane of the cross section intersects a base and when it does not.
- I can identify the characteristics of three-dimensional figures and represent them using drawings.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 551
- Guided Notes
 - Identify solids of rotation
 - Cross sections of solids
 - Apply cross sections
- DeltaMath practice assignment
- Into Geometry Practice p. 555

Formative Assessments

- Into Geometry Check Understanding

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Surface Area of Prisms and Cylinders (18.2)

Student Learning Intentions or We are learning to ... (WALT)

- Develop the formulas for the surface areas of right prisms and right cylinders and use the formulas to solve mathematical problems, such as applying surface area to population density problems in the real world.

Student Success Criteria ... “I can statements”

- I can identify if the figure is a prism or cylinder and determine which figure to use.
- I can find the surface area of a prism or cylinder.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 557
- Guided Notes
 - Develop surface area formula for a right prism
 - Develop a surface area formula for a right cylinder
 - Find the surface area of a composite figure
 - Apply a surface area formula
- DeltaMath practice assignment
- Into Geometry Practice p. 562

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Surface Area of Pyramids and Cones (18.3)

Student Learning Intentions or We are learning to ... (WALT)

- Use formulas for the surface area of pyramids and cones to solve real-world problems.

Student Success Criteria ... “I can statements”

- I can write formulas for the surface area of right cones and regular pyramids.
- I can use formulas for the surface area of pyramids and cones to solve real-world problems.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 565
- Guided Notes
 - Develop a surface area formula for a regular prism
 - Develop a surface area formula for a right cone
 - Find the surface area of a composite formula
 - Apply a surface area formula
- DeltaMath practice assignment
- Into Geometry Practice p. 570

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Surface Area of Spheres (18.4)

Student Learning Intentions or We are learning to ... (WALT)

- Find the surface area of a sphere and use the formula to find the surface area of hemispheres and composite figures in real-world problems.

Student Success Criteria ... “I can statements”

- I can define the variables of the formula for the surface area of a sphere.
- I can use the formula for the surface area of a sphere to calculate the surface areas of composite figures.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 573
- Guided Notes
 - Investigate the formula for the surface area of a sphere
 - Use the surface area of a sphere
- DeltaMath practice assignment
- Into Geometry Practice p. 576

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Volume of Prisms and Cylinders (19.1)

Student Learning Intentions or We are learning to ... (WALT)

- Develop, relate, and use formulas for the volumes of prisms and cylinders to be able to use algebraic models and a graphing calculator to maximize the volumes of rectangular prisms.

Student Success Criteria ... “I can statements”

- I can use the formula for the volume of a rectangular prism
- I can develop and use formulas for the volume of a prism and a cylinder.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 583
- Guided Notes
 - Develop a basic volume formula
 - Justify the procedure for finding volumes of oblique prisms and cylinders
 - Investigate the volume of a solid formed by rotation
 - Maximize the volume of a rectangular prism
- DeltaMath practice assignment
- Into Geometry Practice p. 587

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Volume of Pyramids and Cones (19.2)

Student Learning Intentions or We are learning to ... (WALT)

- Derive formulas for the volume of a cone and a pyramid, relating the formulas to the volumes of cylinders and prisms; and solve real-world and mathematical problems by finding the volumes of pyramids, cones, and composite figures.

Student Success Criteria ... “I can statements”

- I can identify whether the shape is a pyramid or a cone and determine the dimensions.
- I can use the formulas to find the volume of a pyramid or a cone.
- I can show the relationship between the volume formulas for pyramids and cones.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 591
- Guided Notes
 - Develop a formula for the volume of a pyramid
 - Develop a formula for the volume of a cone
 - Model a real world structure to estimate volume
 - Apply a volume formula to find density
- DeltaMath practice assignment
- Into Geometry Practice p. 596

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Volume of Spheres (19.3)

Student Learning Intentions or We are learning to ... (WALT)

- Derive and use a formula for the volume of a sphere. Use the volume formula to solve real-world problems including calculating capacity and dimensions. Calculate the volume of composite figures involving hemispheres and other known solids.

Student Success Criteria ... “I can statements”

- I can identify and name the characteristics of a sphere and its volume.
- I can use the formula for the volume of a sphere to calculate the volumes of composite figures.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 601
- Guided Notes
 - Develop a formula for the volume of a sphere
 - Use a volume formula to solve a real world problem
 - Estimate volume in real world problems
 - Find the volume of a composite figure
- DeltaMath practice assignment
- Into Geometry Practice p. 606

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Modifications and/or Accommodations

Suggested Modifications (ELL, Sp. Ed, Gifted, At-risk of Failure)

English Language Learners

Native language support: The teacher provides auditory or written content to students in their native language.

Adjusted Speech: The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.

Visuals: The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subjects at hand.

Front-Loading Vocabulary: The teacher front loads vocabulary. This means providing students with a list of important vocabulary words they will need to know for a book, lesson, etc. prior to the lesson being taught. Including pictures to go with the vocabulary words is also very beneficial for the students.

Special Education Students

Chunking: The teacher presents information in a way that makes it easy for students to understand and remember. Chunking is based on the presumption that our working memory is easily overloaded by excessive detail. The best way to deliver information is to organize it into meaningful units. Because students with special needs get overloaded easily, chunking is an effective strategy to use with them.

Checking for Understanding: It is important to constantly check for understanding, especially for students who have accommodations. Teachers want to make sure students understand the concepts being covered in a way that makes sense to them.

Extra time: The teacher provides students with special needs extra time to complete work or answer questions. It is important to give students enough time to process their thoughts.

Oral Reading: The teacher will read work orally to students. Class work such as tests and literature circles may need to be read aloud to the student.

Timers: The teacher will use timers as an instructional tool. The use of timers is beneficial for students who have trouble completing tasks. Timers can be helpful so the student is aware of how much time they have to complete an assignment.

Students with 504 Plans

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Gifted & Talented Strategies

Extensions/Enrichments: Teachers will provide gifted and talented students with extension/enrichment projects. Students will be challenged to further their understanding, to apply acquired knowledge, and/or to produce something in reference to acquired knowledge.

Modify/Change Activities: Teachers will monitor and modify activities to accommodate those students who need to be challenged further. Additional reading, problem-solving, writing, or project work is necessary for those students who are ready to move on at a rate more accelerated than their peers. In this way, G & T students are provided the same opportunity for support as special needs students.

Students at Risk of School Failure

Directions or Instructions: Make sure directions and/or instructions are given in limited numbers. Give directions/instructions verbally and in simple written format. Ask students to repeat the instructions or directions to ensure understanding occurs. Check back with the student to ensure he/she hasn't forgotten.

Peer Support: Peers can help build confidence in other students by assisting in peer learning. Many teachers use the 'ask 3 before me' approach. This is fine, however, a student at risk may have to have a specific student or two to ask. Set this up for the student so he/she knows who to ask for clarification before going to you.

Alternate or Modified Assignments: Always ask yourself, "How can I modify this assignment to ensure the students at risk are able to complete it?" Sometimes you'll simplify the task, reduce the length of the assignment or allow for a different mode of delivery. For instance, many students may hand something in, the at-risk student may jot notes and give you the information verbally. Or, it just may be that you will need to assign an alternate assignment.

Increase One to One Time: When other students are working, always touch base with your students at risk and find out if they're on track or needing some additional support. A few minutes here and there will go a long way to intervene as the need presents itself.

Contracts: It helps to have a working contract between you and your students at risk. This helps

prioritize the tasks that need to be done and ensure completion happens. Each day write down what needs to be completed, as the tasks are done, provide a checkmark or happy face. The goal of using contracts is to eventually have the student come to you for completion sign-offs.

Hands On: As much as possible, think in concrete terms and provide hands-on tasks. This means a child doing math may require a calculator or counters. The child may need to tape record comprehension activities instead of writing them. A child may have to listen to a story being read instead of reading it him/herself.

Tests/Assessments: Tests can be done orally if need be. Break tests down in smaller increments by having a portion of the test in the morning, another portion after lunch and the final part the next day.

Seating: Seat students near a helping peer or with quick access to the teacher. Those with hearing or sight issues need to be close to the instruction which often means near the front.

Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media Literacy

Diversity, Equity, and Inclusion

[NCTM: Access and Equity in Mathematics Education](#)

[A Pathway to Equitable Math Instruction](#)

Provide students with opportunities to give feedback to teachers about the classroom and instruction.

- Verbal Example: Fist to five, How well do you understand what we talked about today? Fist to five, How well did I teach this today?
- Classroom Activity: Exit tickets or surveys that ask students to identify how well teachers taught, what helped them learn, what got in the way of their learning, etc.

Treat mathematics as a language that everyone is learning while authentically centering students home languages.

- Classroom Strategies: Color-coding ideas, learning vocabulary in student languages, visual and kinesthetic learning, representations of learning without words.
- Classroom Activity: Multilingual Frayer Models for definitions or concepts

Incorporate true culturally relevant pedagogy, practice, and curriculum.

- Verbal Example: What are some of your family traditions that you are proud of? Would you be okay if we

brought some of those into the classroom?

- Classroom Activity: Use Ankara fabric to teach mathematical concepts such as tessellations, fractions, area, percentages, etc.

Incorporate the history of mathematics into lessons.

- Verbal Example: Why do you think we call it Pythagorean's theorem, when it was used before he was even born? What should we call it instead?
- Classroom Activity: Learn about different bases and numerical ideas: Base 2, binary and connections to computer programming, how the Yoruba of Nigeria used base 20, and how the Mayans conceptualized the number 0 before the first recording of it

Solicit student ways of thinking and processing.

- Verbal Example: How might you all go about this? What do you notice?
- Classroom Activity: Incorporate explorations, where students interact with mathematics in a way that allows them to "discover" or experience mathematics.

Reorganize your classroom teaching around concepts, and teach them more like a web rather than discrete sets of knowledge.

- Verbal Example: How does this connect to what you've learned in the past? How can you use that knowledge today?
- Classroom Activity: Learning webs that connect content

Start with more complex math problems and scaffold as necessary.

- Verbal Example: If we wanted to build a rocket, what are all the things we might need to know before we get started? Along the way, we decided that we want the rocket to reach the moon. What do we need to consider now?
- Classroom Activity: When solving equations, start with the most complex problem, generate ideas for how to solve it, and use the simpler equations as examples to support those ideas.

Offer a variety of ways to demonstrate thinking and knowledge.

- Verbal Example: Show your thinking with words, pictures, symbols.

Ask other questions that will demonstrate learning when it is not clear to you how students know the answer.

- Verbal Example: If you were working with a fellow mathematician who was absent this day, what might you tell them to help them learn it?

Learn about, engage with, and incorporate ethnomathematics.

- Verbal Example: Reflect on your day so far. What math have you already used today?

- Classroom Activity: Community walks to engage with slope.

Co-construct knowledge in the classroom.

- Verbal Example: Let's get into partners and do a think pair-share. We will incorporate everyone's ideas and try to synthesize them.
- Classroom Activity: Have students create mathematical definitions in their own words in groups, and bring the groups together to co-construct mathematical definitions as a class

Choose problems that have complex, competing, or multiple answers.

- Verbal Example: Come up with at least two answers that might solve this problem.
- Classroom Activity: Challenge standardized test questions by getting the "right" answer, but justify other answers by unpacking the assumptions that are made in the problem.
- Classroom Activity: Deconstructed Multiple Choice
 - given a set of multiple choice answers, students discuss why these answers may have been included (can also be used to highlight common mistakes).

Identify what is right about the thinking, and highlight the mistake in what is factually or procedurally accepted.

- Verbal Example: You recognized that you had to combine the constants 27 and 9, could you explain your thinking?
- Classroom Activity: Error Analysis worksheets that highlight what is the right idea behind the mistake.

Using thoughtful questioning to solicit mathematical thoughts rather than telling.

- Verbal Example: What would a mathematician who is confused ask about this question?
- Classroom Activity: After students demonstrate knowledge of a topic, have them play a game where they have to explain their topic to a fellow mathematician and a skeptic. Develop their own reflective questioning/explaining in all three roles.

Create multiple ways of participating that honor myriad ways of thinking and being.

- Verbal Example: For this section, feel free to work alone, in pairs, trios, or quads (let them choose).
- Classroom Activity: Community circles or storytelling circles, incorporating dance, music, song, call and response, and other cultural ways of communicating.

Climate Change

[Math Climate Change Companion Guide](#)

- G.MG.A.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
Climate Change Example: Students may apply geometric methods to solve design problems such as increasing

access to green spaces in cities given physical and cost constraints.

New Jersey Student Learning Standards: Content Area

MATH.9-12.G.GMD.A.1	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
MATH.9-12.G.GMD.A.2	Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
MATH.9-12.G.GMD.A.3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
MATH.9-12.G.GMD.B.4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
MATH.9-12.G.GPE.A.2	Derive the equation of a parabola given a focus and directrix.
MATH.9-12.G.GPE.A.3	Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.
MATH.9-12.G.MG.A.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
MATH.9-12.G.MG.A.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
MATH.9-12.G.MG.A.3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Integration of Career Readiness, Life Literacies and Key Skills

TECH.9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
TECH.9.4.12.CI.2	Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).
TECH.9.4.12.CI.3	Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1).
TECH.9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
TECH.9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
TECH.9.4.12.TL.2	Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.
TECH.9.4.12.TL.3	Analyze the effectiveness of the process and quality of collaborative environments.

Integration of Computer Science and Design Thinking

CS.9-12.8.1.12.AP.1	Design algorithms to solve computational problems using a combination of original and
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existing algorithms.

CS.9-12.8.1.12.AP.5

Decompose problems into smaller components through systematic analysis, using constructs such as procedures, modules, and/or objects.

Interdisciplinary Connections: NJSLS for ELA, Social Studies, Science and/or Math

ELA.RI.MF.9–10.6

Analyze, integrate, and evaluate multiple interpretations (e.g., charts, graphs, diagrams, videos) of a single text or text/s presented in different formats (visually, quantitatively) as well as in words in order to address a question or solve a problem.

ELA.W.AW.9–10.1.E

Provide a concluding paragraph or section that supports the argument presented.

Mathematical and computational thinking in 9–12 builds on K–8 experiences and progresses to using algebraic thinking and analysis, a range of linear and nonlinear functions including trigonometric functions, exponentials and logarithms, and computational tools for statistical analysis to analyze, represent, and model data. Simple computational simulations are created and used based on mathematical models of basic assumptions.

21st Century Life and Career

TECH.9.4.12.CT.1

Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).

TECH.9.4.12.CT.2

Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).

TECH.9.4.12.CT.3

Enlist input from a variety of stakeholders (e.g., community members, experts in the field) to design a service learning activity that addresses a local or global issue (e.g., environmental justice).

TECH.9.4.12.CT.4

Participate in online strategy and planning sessions for course-based, school-based, or other project and determine the strategies that contribute to effective outcomes.