Unit 08: Properties of Circles

Content Area: **Template**
Course(s):
Time Period: **Full Year**
Length: **5 Weeks**
Status: **Published**

Unit 8: Properties of Circles (Modules 15, 16, and 17)

Unit Rationale

In prior learning, students applied the Pythagorean Theorem to determine unknown side lengths in right triangles They completed the square in a quadratic expression. They wrote coordinate proofs about triangle relationships. They used the formulas for the circumference and area of a circle to solve problems. In this unit, students will define and determine the measures of central angles, inscribed angles, and arcs of a circle. The will use the properties of angles of quadrilaterals inscribed in a circle to prove theorems and solve problems. They will prove theorems about tangents to a circle and use them to solve mathematical and real world problems. Then, they will prove theorems about the relationships of chords, secants, and tangents of circles and angle relationships of circles. They will use the formulas for the circumference and area of a circle to solve mathematical and real world problems. They will derive and use the formula for arc length and area of a sector to solve problems. Lastly, they will convert between degree and radian measure. In later units, they will use geometric shapes, their measures, and their properties to describe three-dimensional objects. They will find volume and surface area of cylinders, cones, pyramids, and spheres. They will explain how the unit circle in the coordinate plane can be used to extend the trigonometric functions to all real numbers.

Essential Questions

- What are central and inscribed angles and how are they related to the measure of the arc they intercept?
- What are the special lines and segments in relation to circles, and what are the relationships among the segments and angles that they form when they intersect?
- What are the geometric properties of circles that can be observed or calculated using coordinate geometry?
- What are the formulas for the circumference and area of a circle, and how are they derived?
- Why are circumference and area calculations important in fields such as engineering, architecture, and physics?
- How are arc length and sector area calculated, and how do they relate to the radius and central angle?
- Why are radians preferred over degrees in calculus and physics applications involving circular motion?

Pre-Assessments

- Into Geometry: Are you ready? p. 464

Instructional Plan

Central and Inscribed Angles (15.1)

Student Learning Intentions or We are learning to … (WALT)

- Identify the relationships between the measures of associated central angles, inscribed angles, and intercepted arcs; apply the Arc Addition Postulate; apply the Inscribed Angle Theorem to solve for unknown arc and angle measures.

Student Success Criteria … "I can statements"

- I can find measures of a major arc and a minor arc from their central angle.
- I can determine the measure of an inscribed angle from its central angle.
- I can determine the measures of central angles from its inscribed angles.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 465
- Guided Notes
    - Investigate central angle and inscribed angle
    - Understand arc and arc measure
    - Construct a regular hexagon
    - Prove the Inscribed Angle Theorem
    - Use Inscribed Angle Theorem
- DeltaMath practice assignment
- Into Geometry Practice p. 471

Formative Assessments

- Into Geometry Check Understanding

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Angles in Inscribed Quadrilaterals (15.2)

Student Learning Intentions or We are learning to … (WALT)

- Prove the Inscribed Quadrilateral Theorem and the Congruent Corresponding Chords Theorem and apply them to quadrilaterals inscribed in circles.

Student Success Criteria … "I can statements"

- I can understand the proof of the Inscribed Quadrilateral Theorem.
- I can use the Arc Addition Postulate and the Inscribed Angle Theorem to prove steps in the Inscribed Quadrilateral Theorem.
- I can use the properties of angles of quadrilaterals inscribed in a circle to prove theorems and solve problems.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 475
- Guided Notes
    - Investigate inscribed quadrilaterals
    - Prove the Inscribed Quadrilaterals Theorem
    - Prove the Congruent Corresponding Chords Theorem
    - Apply the Inscribed Quadrilateral Theorem
- DeltaMath practice assignment
- Into Geometry Practice p. 480

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Tangents and Circumscribed Angles (15.3)

Student Learning Intentions or We are learning to … (WALT)

- Solve problems where tangent lines form angles with other tangent lines and with the radius of the circle. Prove that two tangent segments that share an exterior point are congruent. Prove that a circumscribed angle is supplementary to its related central angle. Prove that a tangent is perpendicular to the radius it intersects..

Student Success Criteria … "I can statements"

- I can identify tangents, radii, and circumscribed angles related to a triangle.
- I can use theorems about tangents to a circle to solve problems.
- I can prove theorems about tangents to a circle and use them to solve mathematical and real-world problems..

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 483
- Guided Notes
  - Investigate the Tangent-Radius Theorem
  - Prove the Tangent-Radius Theorem
  - Prove the Circumscribed Angle Theorem
  - Construct a tangent to a circle
  - Prove the Two-Tangent Theorem
  - Apply the Two-Tangent Theorem
- DeltaMath practice assignment
- Into Geometry Practice p. 488

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Segment Relationships in Circles (16.1)

Student Learning Intentions or We are learning to … (WALT)

- Use proportional relationships in circles to prove the Chord-Chord, Secant-Secant, and Secant-Tangent Product Theorems in order to apply the theorems to solve for segment lengths in mathematical and real-world problems.

Student Success Criteria … "I can statements"

- I can identify chords, tangent segments, secant segments, and external secant segments.
- I can find the length of a tangent segment when I know the length of a secant segment and external secant segment sharing an endpoint with the tangent segment.
- I can use segment relationships in circles to solve mathematical and real-world problems.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 503
- Guided Notes
  - Prove the Chord-Chord Product Theorem
  - Investigate segment relationships in circles
  - Use the Secant Tangent Product Theorem
  - Apply segment relationships
- DeltaMath practice assignment
- Into Geometry Practice p. 507

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Angle Relationships in Circles (16.2)

Student Learning Intentions or We are learning to … (WALT)

- Determine the relationships that exist between secants, tangents, and chords in a circle and the angles and arcs formed by them in order to prove and use theorems about these relationships to solve mathematical and real-world problems.

Student Success Criteria … "I can statements"

- I can identify tangents, secants, and chords in circles.
- I can use equations that relate angles and arcs in circles.
- I can use angle relationships in circles to solve mathematical and real-world problems.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 511
- Guided Notes
    - Prove the Intersecting Chords Angle Measure Theorem
    - Explore the Tangent-Secant Interior Angle Measure Theorem
    - Prove the Tangent-Secant Interior Angle Measure Theorem
    - Prove the TangentSecant Exterior Angle Measure Theorem
    - Apply Angle Relationships in Circles
- DeltaMath practice assignment
- Into Geometry Practice p. 516

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Measure Circumference and Area of a Circle (17.1)

Student Learning Intentions or We are learning to … (WALT)

- Justify and use the formulas for the circumference and area of a circle to solve real-world and mathematical problems.

Student Success Criteria … "I can statements"

- I can write the formulas for area and circumference of a circle.
- I can sue the circumference and area of a circle formulas to solve mathematical problems.

- I can justify and use the circumference and area of a circle formulas to solve real-world problems.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 523
- Guided Notes
    - Justify the formula for the circumference of a circle
    - Justify the formula for the area of a circle
    - Justify pi
    - Apply the circumference formula
- DeltaMath practice assignment
- Into Geometry Practice p. 528

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Measure Arc Length and Use Radians (17.2)

Student Learning Intentions or We are learning to … (WALT)

- Use the arc length formula and apply it to real-world problems, and convert between degree and radian measure.

Student Success Criteria … "I can statements"

- I can find the fraction of a circle represented by a central angle.
- I can convert between degree and radian measure.
- I can use similarity of circles to find arc length.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 531
- Guided Notes
    - Derive the formula for arc length
    - Derive an expression for radian measure
    - Apply the formula for arc length

    - Convert between radian measure and degree measure
- DeltaMath practice assignment
- Into Geometry Practice p. 536

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Measure Sector Area (17.3)

Student Learning Intentions or We are learning to … (WALT)

- I can find the area of a sector of a circle.
- I can use sector area to solve real-world problems.

Student Success Criteria … "I can statements"

- Derive the formula for the area of a sector of a circle and use that formula to compute the area of sectors of circles having different central angles and radii.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 539
- Guided Notes
    - Derive the formula for sector area.
    - Use the formula for sector area
- DeltaMath practice assignment
- Into Geometry Practice p. 542

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Modifications and/or Accommodations

Suggested Modifications (ELL, Sp. Ed, GIfted, At-risk of Failure)

English Language Learners

Native language support: The teacher provides auditory or written content to students in their native language.

Adjusted Speech: The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.

Visuals: The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subjects at hand.

Front-Loading Vocabulary: The teacher front loads vocabulary. This means providing students with a list of important vocabulary words they will need to know for a book, lesson, etc. prior to the lesson being taught. Including pictures to go with the vocabulary words is also very beneficial for the students.

Special Education Students

Chunking: The teacher presents information in a way that makes it easy for students to understand and remember. Chunking is based on the presumption that our working memory is easily overloaded by excessive detail. The best way to deliver information is to organize it into meaningful units. Because students with special needs get overloaded easily, chunking is an effective strategy to use with them.

Checking for Understanding: It is important to constantly check for understanding, especially for students who have accommodations. Teachers want to make sure students understand the concepts being covered in a way that makes sense to them.

Extra time: The teacher provides students with special needs extra time to complete work or answer questions. It is important to give students enough time to process their thoughts.

Oral Reading: The teacher will read work orally to students. Class work such as tests and literature circles may need to be read aloud to the student.

Timers: The teacher will use timers as an instructional tool. The use of timers is beneficial for students who have trouble completing tasks. Timers can be helpful so the student is aware of how much time they have to complete an assignment.

Students with 504 Plans

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Gifted & Talented Strategies

Extensions/Enrichments: Teachers will provide gifted and talented students with extension/enrichment projects. Students will be challenged to further their understanding, to apply acquired knowledge, and/or to produce something in reference to acquired knowledge.

Modify/Change Activities: Teachers will monitor and modify activities to accommodate those students who need to be challenged further. Additional reading, problem-solving, writing, or project work is necessary for those students who are ready to move on at a rate more accelerated than their peers. In this way, G & T students are provided the same opportunity for support as special needs students.

Students at Risk of School Failure

Directions or Instructions: Make sure directions and/or instructions are given in limited numbers. Give directions/instructions verbally and in simple written format. Ask students to repeat the instructions or directions to ensure understanding occurs. Check back with the student to ensure he/she hasn't forgotten.

Peer Support: Peers can help build confidence in other students by assisting in peer learning. Many teachers use the 'ask 3 before me' approach. This is fine, however, a student at risk may have to have a specific student or two to ask. Set this up for the student so he/she knows who to ask for clarification before going to you.

Alternate or Modified Assignments: Always ask yourself, "How can I modify this assignment to ensure the students at risk are able to complete it?" Sometimes you'll simplify the task, reduce the length of the assignment or allow for a different mode of delivery. For instance, many students may hand something in, the at-risk student may jot notes and give you the information verbally. Or, it just may be that you will need to assign an alternate assignment.

Increase One to One Time: When other students are working, always touch base with your students at risk and find out if they're on track or needing some additional support. A few minutes here and there will go a long way to intervene as the need presents itself.

Contracts: It helps to have a working contract between you and your students at risk. This helps prioritize the tasks that need to be done and ensure completion happens. Each day write down what needs to be completed, as the tasks are done, provide a checkmark or happy face. The goal of using contracts is to eventually have the student come to you for completion sign-offs.

Hands On: As much as possible, think in concrete terms and provide hands-on tasks. This means a child doing math may require a calculator or counters. The child may need to tape record comprehension activities instead of writing them. A child may have to listen to a story being read instead of reading it him/herself.

Tests/Assessments: Tests can be done orally if need be. Break tests down in smaller increments by having a portion of the test in the morning, another portion after lunch and the final part the next day.

Seating: Seat students near a helping peer or with quick access to the teacher. Those with hearing or sight issues need to be close to the instruction which often means near the front.

Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media Literacy

Diversity, Equity, and Inclusion

NCTM: Access and Equity in Mathematics Education

A Pathway to Equitable Math Instruction

Provide students with opportunities to give feedback to teachers about the classroom and instruction.

• Verbal Example: Fist to five, How well do you understand what we talked about today? Fist to five, How well did I teach this today?

• Classroom Activity: Exit tickets or surveys that ask students to identify how well teachers taught, what helped them learn, what got in the way of their learning, etc.

Treat mathematics as a language that everyone is learning while authentically centering students home languages.

• Classroom Strategies: Color-coding ideas, learning vocabulary in student languages, visual and kinesthetic learning, representations of learning without words.

• Classroom Activity: Multilingual Frayer Models for definitions or concepts

Incorporate true culturally relevant pedagogy, practice, and curriculum.

• Verbal Example: What are some of your family traditions that you are proud of? Would you be okay if we brought some of those into the classroom?

• Classroom Activity: Use Ankara fabric to teach mathematical concepts such as tessellations, fractions, area, percentages, etc.

Incorporate the history of mathematics into lessons.
• Verbal Example: Why do you think we call it Pythagorean's theorem, when it was used before he was even born? What should we call it instead?
• Classroom Activity: Learn about different bases and numerical ideas: Base 2, binary and connections to computer programming, how the Yoruba of Nigeria used base 20, and how the Mayans conceptualized the number 0 before the first recording of it

Solicit student ways of thinking and processing.
• Verbal Example: How might you all go about this? What do you notice?
• Classroom Activity: Incorporate explorations, where students interact with mathematics in a way that allows them to "discover" or experience mathematics.

Reorganize your classroom teaching around concepts, and teach them more like a web rather than discrete sets of knowledge.
• Verbal Example: How does this connect to what you've learned in the past? How can you use that knowledge today?
• Classroom Activity: Learning webs that connect content

Start with more complex math problems and scaffold as necessary.
• Verbal Example: If we wanted to build a rocket, what are all the things we might need to know before we get started? Along the way, we decided that we want the rocket to reach the moon. What do we need to consider now?
• Classroom Activity: When solving equations, start with the most complex problem, generate ideas for how to solve it, and use the simpler equations as examples to support those ideas.

Offer a variety of ways to demonstrate thinking and knowledge.
• Verbal Example: Show your thinking with words, pictures, symbols.

Ask other questions that will demonstrate learning when it is not clear to you how students know the answer.
• Verbal Example: If you were working with a fellow mathematician who was absent this day, what might you tell them to help them learn it?

Learn about, engage with, and incorporate ethnomathematics.
• Verbal Example: Reflect on your day so far. What math have you already used today?
• Classroom Activity: Community walks to engage with slope.

Co-construct knowledge in the classroom.
• Verbal Example: Let's get into partners and do a think pair-share. We will incorporate everyone's ideas and try to synthesize them.
• Classroom Activity: Have students create mathematical definitions in their own words in groups, and bring the groups together to co-construct mathematical definitions as a class

Choose problems that have complex, competing, or multiple answers.
• Verbal Example: Come up with at least two answers that might solve this problem.
• Classroom Activity: Challenge standardized test questions by getting the "right" answer, but justify other answers by unpacking the assumptions that are made in the problem.
• Classroom Activity: Deconstructed Multiple Choice
- given a set of multiple choice answers, students discuss why these answers may have been included (can also be used to highlight common mistakes).

Identify what is right about the thinking, and highlight the mistake in what is factually or procedurally accepted.
• Verbal Example: You recognized that you had to combine the constants 27 and 9, could you explain your thinking?
• Classroom Activity: Error Analysis worksheets that highlight what is the right idea behind the mistake.

Using thoughtful questioning to solicit mathematical thoughts rather than telling.

• Verbal Example: What would a mathematician who is confused ask about this question?

• Classroom Activity: After students demonstrate knowledge of a topic, have them play a game where they have to explain their topic to a fellow mathematician and a skeptic. Develop their own reflective questioning/explaining in all three roles.

Create multiple ways of participating that honor myriad ways of thinking and being.
• Verbal Example: For this section, feel free to work alone, in pairs, trios, or quads (let them choose).
• Classroom Activity: Community circles or storytelling circles, incorporating dance, music, song, call and response, and other cultural ways of communicating.

Climate Change

Math Climate Change Companion Guide

- G.MG.A.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
  Climate Change Example: Students may use circles, their measures, and their properties to describe the cross section of a tree and compare changes in radial diameter or circumference variations of tree trunks when considering changes in seasonal weather patterns over time.

New Jersey Student Learning Standards: Content Area

MATH.9-12.G.C.A.2	Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
MATH.9-12.G.C.A.3	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
MATH.9-12.G.C.A.4	Construct a tangent line from a point outside a given circle to the circle.
MATH.9-12.G.C.B.5	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
MATH.9-12.G.CO.A.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
MATH.9-12.G.CO.A.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
MATH.9-12.G.CO.D.13	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
MATH.9-12.G.GMD.A.1	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
MATH.9-12.G.GPE.A.1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
MATH.9-12.G.GPE.B.4	Use coordinates to prove simple geometric theorems algebraically.

Integration of Career Readiness, Life Literacies and Key Skills

TECH.9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
TECH.9.4.12.CI.2	Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).
TECH.9.4.12.CI.3	Investigate new challenges and opportunities for personal growth, advancement, and

	transition (e.g., 2.1.12.PGD.1).
TECH.9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
TECH.9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
TECH.9.4.12.TL.2	Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.
TECH.9.4.12.TL.3	Analyze the effectiveness of the process and quality of collaborative environments.

Integration of Computer Science and Design Thinking

CS.9-12.8.1.12.AP.1	Design algorithms to solve computational problems using a combination of original and existing algorithms.
CS.9-12.8.1.12.AP.5	Decompose problems into smaller components through systematic analysis, using constructs such as procedures, modules, and/or objects.

Interdisciplinary Connections: NJSLS for ELA, Social Studies, Science and/or Math

ELA.RI.MF.9–10.6	Analyze, integrate, and evaluate multiple interpretations (e.g., charts, graphs, diagrams, videos) of a single text or text/s presented in different formats (visually, quantitatively) as well as in words in order to address a question or solve a problem.
ELA.W.AW.9–10.1.E	Provide a concluding paragraph or section that supports the argument presented.
	Mathematical and computational thinking in 9–12 builds on K–8 experiences and progresses to using algebraic thinking and analysis, a range of linear and nonlinear functions including trigonometric functions, exponentials and logarithms, and computational tools for statistical analysis to analyze, represent, and model data. Simple computational simulations are created and used based on mathematical models of basic assumptions.

21st Century Life and Career

TECH.9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
TECH.9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
TECH.9.4.12.CT.3	Enlist input from a variety of stakeholders (e.g., community members, experts in the field) to design a service learning activity that addresses a local or global issue (e.g., environmental justice).
TECH.9.4.12.CT.4	Participate in online strategy and planning sessions for course-based, school-based, or other project and determine the strategies that contribute to effective outcomes.