Unit 06: Quadrilaterals, Polygons, and Triangle Similarity

Content Area: **Template**
Course(s):
Time Period: **Full Year**
Length: **7 Weeks**
Status: **Published**

Unit 6: Quadrilaterals, Polygons, and Triangle Similarity (Module 11 and 12)

Unit Rationale

In prior learning, proved and used properties of triangles. They worked with parallel lines and properties of parallelograms. They performed dilations and sequences of transformations. In this unit, students will prove and use properties of and conditions for parallelograms, rectangles, rhombuses, squares, kites, and trapezoids. Then, they will use corresponding parts of similar triangles to solve problems. They will prove and use the Triangle Proportionality Theorem. They will identify similar right triangles and develop the AA criterion for similar triangles. In later units, they will construct inscribed quadrilaterals and identify properties of their angles. They will explore trigonometry with right and oblique triangles.

Essential Questions

- What are the defining properties of a parallelogram?
- How are opposite sides, opposite angles, and diagonals related in a parallelogram?
- What are the unique properties of the special parallelograms, and how do these properties differ from those of general parallelograms?
- What are the distinguishing properties of trapezoids and kites?
- How can transformations establish similarity between figures?
- How can AA similarity be used to determine if two triangles are similar?
- How do proportions within and between triangles relate to similarity?
- How does similarity apply to right triangles, especially in contexts involving geometric figures and real-world applications?

Pre-Assessments

- Into Geometry: Are you ready? Diagnostic Assessment p. 320

Instructional Plan

Properties of Parallelograms (11.1)

Student Learning Intentions or We are learning to … (WALT)

- Students prove and use properties of parallelograms in order to solve problems involving parallelograms.

Student Success Criteria … "I can statements"

- I can use properties of parallelograms to solve problems to find unknown sides and angles.
- I can understand the proofs about properties of parallelograms, and I can use the properties to solve problems to find unknown sides and angles.
- I can prove and use properties of parallelograms.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 321
- Guided Notes
    - Explore quadrilaterals
    - Explore parallelograms
    - Theorems about quadrialterals and parallelograms
    - Prove diagonals bisect each other
    - Use properties of parallelograms
- DeltaMath practice assignment
- Into Geometry Practice p. 326

Formative Assessments

- Into Geometry Check Understanding- online assessment tool

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Conditions for Parallelograms (11.2)

Student Learning Intentions or We are learning to … (WALT)

- Prove and use conditions for parallelograms in order to solve geometric problems.

Student Success Criteria … "I can statements"

- I can identify if a quadrilateral is a parallelogram.
- I can prove and use conditions for parallelograms.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p, 329
- Guided Notes
    - Establish parallelogram criteria
    - Prove that a quadrilateral is a parallelogram
- DeltaMath practice assignment
- Into Geometry Practice p. 333

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Properties of Rectangles, Rhombuses, and Squares (11.3)

Student Learning Intentions or We are learning to … (WALT)

- Prove theorems about rectangles, rhombuses, and squares, discriminate among them based on their properties, and apply theorems to find measures of sides, diagonals, and interior angles.

Student Success Criteria … "I can statements"

- I can prove and use properties of parallelograms.
- I can define squares, rectangles, and rhombuses as special types of parallelograms.
- I can prove and use properties of squares, rectangles, and rhombuses.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 337
- Guided Notes
    - Special parallelograms
    - Diagonals of special parallelograms
    - Prove diagonals of a rectangle are congruent
    - Use properties of squares, rectangles, and rhombuses
- DeltaMath practice assignment
- Into Geometry Practice p. 342

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Conditions for Rectangles, Rhombuses, and Squares (11.4)

Student Learning Intentions or We are learning to … (WALT)

- Use the properties of diagonals for rectangles, rhombuses, and squares to identify special parallelograms in the coordinate plane and construct two-step proofs of theorems to prove conditions for rectangles and rhombuses.

Student Success Criteria … "I can statements"

- I can identify a rectangle, rhombus, or square based on what I know about its diagonals.

- I can state theorems on conditions for rectangles and rhombuses.
- I can prove and use conditions for rectangles, rhombuses, and squares.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 345
- Guided Notes
    - Determine conditions for special parallelograms
    - Prove conditions for rectangles
    - Prove conditions for rhombuses
    - Apply conditions for special parallelograms
    - Identify special parallelograms in the coordinate plane
- DeltaMath practice assignment
- Into Geometry Practice p. 350

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Properties and Conditions for Trapezoids and Kites (11.5)

Student Learning Intentions or We are learning to … (WALT)

- Use the exclusive and inclusive definitions of kite and trapezoid to classify quadrilaterals, prove and apply theorems about angle measures and diagonals of kites, prove and apply theorems about conditions on a trapezoid equivalent to it being isosceles, and apply the Trapezoid Midsegment Theorem.

Student Success Criteria … "I can statements"

- I can tell if a quadrilateral is a kite or a trapezoid using the definitions.
- I can explain how different kinds of quadrilaterals can be kites or trapezoids using the inclusive definitions.
- I can prove and apply theorems about trapezoids and kites.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 353
- Guided Notes
    - Understand definitions of kites and trapezoids
    - Theorems about kites
    - Theorems about trapezoids
    - Trapezoid Midsegment Theorem
    - Transformations and symmetry in trapezoids and kites
- DeltaMath practice assignment
- Into Geometry Practice p. 358

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Use Transformations to Prove Figures Are Similar (12.1)

Student Learning Intentions or We are learning to … (WALT)

- Determine when figures are similar using transformations and comparing corresponding side ratios in order to solve problems to prove figures are similar and to find missing values.

Student Success Criteria … "I can statements"

- I can identify when two figures are similar if I know the sides lengths and angle measures.
- I can solve for missing values when I know figures are similar.
- I can use similarity transformations to prove figures are similar.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 367
- Guided Notes
    - Investigate dilations
    - Find a sequence of similarity transformations
    - Prove all circles are similar
    - Apply properties of similar figures
- DeltaMath practice assignment

- Into Geometry Practice p. 372

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Develop AA Triangle Similarity (12.2)

Student Learning Intentions or We are learning to … (WALT)

- Prove the AA Triangle Similarity Theorem and use it to find missing dimensions of triangles. Use the SSS and SAS Triangle Similarity Theorems to prove triangles are similar and find missing dimensions.

Student Success Criteria … "I can statements"

- I can understand the proof of the similarity of two triangles using the AA Triangle Similarity Theorem.
- I can use given information to determine if triangles are similar.
- I can prove AA, SSS, and SAS Similarity Theorems.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 375
- Guided Notes
    - Prove the AA Triangle Similarity Theorem
    - Apply the AA Triangle Similarity Theorem
    - Apply the SSS and SAS Triangle Similarity Theorem
    - Use indirect measurement
- DeltaMath practice assignment
- Into Geometry Practice p. 380

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Develop and Prove Triangle Proportionality (12.3)

Student Learning Intentions or We are learning to … (WALT)

- Analyze a proof of the Triangle Proportionality Theorem, apply the theorem to solve for lengths of partitions of triangle sides, apply the converse of the theorem to determine partitions of triangle sides that give a line parallel to another side, and find the point on a directed line segment that partitions the segment in a given ratio.

Student Success Criteria … "I can statements"

- I can identify proportional parts of a triangle.
- I can use properties of parallel lines in triangles to identify proportional relationships.
- I can identify proportional relationships in tables and equations, identify the constant of proportionality, and write the associated equation.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 383
- Guided Notes
    - The Triangle Proportionality Theorem
    - Apply the Triangle Proportionality Theorem
    - Converse of the Triangle Proportionality Theorem
- DeltaMath practice assignment
- Into Geometry Practice p. 388

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Apply Similarity in Right Triangles (12.4)

Student Learning Intentions or We are learning to … (WALT)

- Identify similar right triangles created when an altitude is drawn to the hypotenuse of a right triangle, apply the Geometric Means Theorems, use geometric means to prove the Pythagorean Theorem, and recognize Pythagorean triples.

Student Success Criteria … "I can statements"

- I can find similar triangles after drawing an altitude to the hypotenuse of a right triangle.
- I can apply the Geometric Means Theorems.
- I can identify similar right triangles and recognize Pythagorean triples.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 391
- Guided Notes
    - Compare corresponding parts of similar triangles
    - Identify properties of similar right triangles
    - Apply a geometric means theorem
    - Prove the Pythagorean Theorem
    - Use Pythagorean triples
- DeltaMath practice assignment
- Into Geometry Practice p. 396

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Modifications and/or Accommodations

Suggested Modifications (ELL, Sp. Ed, Glfted, At-risk of Failure)

English Language Learners

Native language support: The teacher provides auditory or written content to students in their native language.

Adjusted Speech: The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.

Visuals: The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subjects at hand.

Front-Loading Vocabulary: The teacher front loads vocabulary. This means providing students with a list of important vocabulary words they will need to know for a book, lesson, etc. prior to the lesson being taught. Including pictures to go with the vocabulary words is also very beneficial for the students.

Special Education Students

Chunking: The teacher presents information in a way that makes it easy for students to understand and remember. Chunking is based on the presumption that our working memory is easily overloaded by excessive detail. The best way to deliver information is to organize it into meaningful units. Because students with special needs get overloaded easily, chunking is an effective strategy to use with them.

Checking for Understanding: It is important to constantly check for understanding, especially for students who have accommodations. Teachers want to make sure students understand the concepts being covered in a way that makes sense to them.

Extra time: The teacher provides students with special needs extra time to complete work or answer questions. It is important to give students enough time to process their thoughts.

Oral Reading: The teacher will read work orally to students. Class work such as tests and literature circles may need to be read aloud to the student.

Timers: The teacher will use timers as an instructional tool. The use of timers is beneficial for students who have trouble completing tasks. Timers can be helpful so the student is aware of how much time they have to complete an assignment.

Students with 504 Plans

Chunking: The teacher presents information in a way that makes it easy for students to understand and remember. Chunking is based on the presumption that our working memory is easily overloaded by excessive detail. The best way to deliver information is to organize it into meaningful

units. Because students with special needs get overloaded easily, chunking is an effective strategy to use with them.

Checking for Understanding: It is important to constantly check for understanding, especially for students who have accommodations. Teachers want to make sure students understand the concepts being covered in a way that makes sense to them.

Extra time: The teacher provides students with special needs extra time to complete work or answer questions. It is important to give students enough time to process their thoughts.

Gifted & Talented Strategies

Extensions/Enrichments: Teachers will provide gifted and talented students with extension/enrichment projects. Students will be challenged to further their understanding, to apply acquired knowledge, and/or to produce something in reference to acquired knowledge.

Modify/Change Activities: Teachers will monitor and modify activities to accommodate those students who need to be challenged further. Additional reading, problem-solving, writing, or project work is necessary for those students who are ready to move on at a rate more accelerated than their peers. In this way, G & T students are provided the same opportunity for support as special needs students.

Students at Risk of School Failure

Directions or Instructions: Make sure directions and/or instructions are given in limited numbers. Give directions/instructions verbally and in simple written format. Ask students to repeat the instructions or directions to ensure understanding occurs. Check back with the student to ensure he/she hasn't forgotten.

Peer Support: Peers can help build confidence in other students by assisting in peer learning. Many teachers use the 'ask 3 before me' approach. This is fine, however, a student at risk may have to have a specific student or two to ask. Set this up for the student so he/she knows who to ask for clarification before going to you.

Alternate or Modified Assignments: Always ask yourself, "How can I modify this assignment to ensure the students at risk are able to complete it?" Sometimes you'll simplify the task, reduce the length of the assignment or allow for a different mode of delivery. For instance, many students may hand something in, the at-risk student may jot notes and give you the information verbally. Or, it just may be that you will need to assign an alternate assignment.

Increase One to One Time: When other students are working, always touch base with your students at risk and find out if they're on track or needing some additional support. A few minutes here and there will go a long way to intervene as the need presents itself.

Contracts: It helps to have a working contract between you and your students at risk. This helps prioritize the tasks that need to be done and ensure completion happens. Each day write down what needs to be completed, as the tasks are done, provide a checkmark or happy face. The goal of using contracts is to eventually have the student come to you for completion sign-offs.

Hands On: As much as possible, think in concrete terms and provide hands-on tasks. This means a

child doing math may require a calculator or counters. The child may need to tape record comprehension activities instead of writing them. A child may have to listen to a story being read instead of reading it him/herself.

Tests/Assessments: Tests can be done orally if need be. Break tests down in smaller increments by having a portion of the test in the morning, another portion after lunch and the final part the next day.

Seating: Seat students near a helping peer or with quick access to the teacher. Those with hearing or sight issues need to be close to the instruction which often means near the front.

Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media Literacy

Diversity, Equity, and Inclusion

NCTM: Access and Equity in Mathematics Education

A Pathway to Equitable Math Instruction

Provide students with opportunities to give feedback to teachers about the classroom and instruction.

• Verbal Example: Fist to five, How well do you understand what we talked about today? Fist to five, How well did I teach this today?

• Classroom Activity: Exit tickets or surveys that ask students to identify how well teachers taught, what helped them learn, what got in the way of their learning, etc.

Treat mathematics as a language that everyone is learning while authentically centering students home languages.

• Classroom Strategies: Color-coding ideas, learning vocabulary in student languages, visual and kinesthetic learning, representations of learning without words.

• Classroom Activity: Multilingual Frayer Models for
definitions or concepts

Incorporate true culturally relevant pedagogy, practice, and curriculum.

• Verbal Example: What are some of your family traditions that you are proud of? Would you be okay if we brought some of those into the classroom?

• Classroom Activity: Use Ankara fabric to teach mathematical concepts such as tessellations, fractions, area, percentages, etc.

Incorporate the history of mathematics into lessons.
• Verbal Example: Why do you think we call it Pythagorean's theorem, when it was used before he was even born? What should we call it instead?
• Classroom Activity: Learn about different bases and numerical ideas: Base 2, binary and connections to computer programming, how the Yoruba of Nigeria used base 20, and how the Mayans conceptualized the number 0 before the first recording of it

Solicit student ways of thinking and processing.
• Verbal Example: How might you all go about this? What do you notice?
• Classroom Activity: Incorporate explorations, where students interact with mathematics in a way that allows them to "discover" or experience mathematics.

Reorganize your classroom teaching around concepts, and teach them more like a web rather than discrete sets of knowledge.
• Verbal Example: How does this connect to what you've learned in the past? How can you use that knowledge today?
• Classroom Activity: Learning webs that connect content

Start with more complex math problems and scaffold as necessary.
• Verbal Example: If we wanted to build a rocket, what are all the things we might need to know before we get started? Along the way, we decided that we want the rocket to reach the moon. What do we need to consider now?
• Classroom Activity: When solving equations, start with the most complex problem, generate ideas for how to solve it, and use the simpler equations as examples to support those ideas.

Offer a variety of ways to demonstrate thinking and knowledge.
• Verbal Example: Show your thinking with words, pictures, symbols.

Ask other questions that will demonstrate learning when it is not clear to you how students know the answer.
• Verbal Example: If you were working with a fellow mathematician who was absent this day, what might you tell them to help them learn it?

Learn about, engage with, and incorporate ethnomathematics.
• Verbal Example: Reflect on your day so far. What math have you already used today?
• Classroom Activity: Community walks to engage with slope.

Co-construct knowledge in the classroom.

• Verbal Example: Let's get into partners and do a think pair-share. We will incorporate everyone's ideas and try to synthesize them.
• Classroom Activity: Have students create mathematical definitions in their own words in groups, and bring the groups together to co-construct mathematical definitions as a class

Choose problems that have complex, competing, or multiple answers.
• Verbal Example: Come up with at least two answers that might solve this problem.
• Classroom Activity: Challenge standardized test questions by getting the "right" answer, but justify other answers by unpacking the assumptions that are made in the problem.
• Classroom Activity: Deconstructed Multiple Choice
- given a set of multiple choice answers, students discuss why these answers may have been included (can also be used to highlight common mistakes).

Identify what is right about the thinking, and highlight the mistake in what is factually or procedurally accepted.
• Verbal Example: You recognized that you had to combine the constants 27 and 9, could you explain your thinking?
• Classroom Activity: Error Analysis worksheets that highlight what is the right idea behind the mistake.

Using thoughtful questioning to solicit mathematical thoughts rather than telling.

• Verbal Example: What would a mathematician who is confused ask about this question?

• Classroom Activity: After students demonstrate knowledge of a topic, have them play a game where they have to explain their topic to a fellow mathematician and a skeptic. Develop their own reflective questioning/explaining in all three roles.

Create multiple ways of participating that honor myriad ways of thinking and being.
• Verbal Example: For this section, feel free to work alone, in pairs, trios, or quads (let them choose).
• Classroom Activity: Community circles or storytelling circles, incorporating dance, music, song, call and response, and other cultural ways of communicating.

Climate Change

Math Climate Change Companion Guide

New Jersey Student Learning Standards: Content Area

MATH.9-12.G.C.A.1	Prove that all circles are similar.

MATH.9-12.G.C.A.3	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
MATH.9-12.G.CO.A.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
MATH.9-12.G.CO.C.10	Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
MATH.9-12.G.CO.C.11	Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.
MATH.9-12.G.CO.D.13	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
MATH.9-12.G.GPE.B.4	Use coordinates to prove simple geometric theorems algebraically.
MATH.9-12.G.GPE.B.5	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
MATH.9-12.G.SRT.A.1	Verify experimentally the properties of dilations given by a center and a scale factor:
MATH.9-12.G.SRT.A.2	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
MATH.9-12.G.SRT.A.3	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
MATH.9-12.G.SRT.B.5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Integration of Career Readiness, Life Literacies and Key Skills

TECH.9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
TECH.9.4.12.CI.2	Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).
TECH.9.4.12.CI.3	Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1).
TECH.9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
TECH.9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
TECH.9.4.12.TL.2	Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.
TECH.9.4.12.TL.3	Analyze the effectiveness of the process and quality of collaborative environments.

Integration of Computer Science and Design Thinking

CS.9-12.8.1.12.AP.1	Design algorithms to solve computational problems using a combination of original and existing algorithms.

CS.9-12.8.1.12.AP.5	Decompose problems into smaller components through systematic analysis, using constructs such as procedures, modules, and/or objects.

Interdisciplinary Connections: NJSLS for ELA, Social Studies, Science and/or Math

ELA.RI.MF.9–10.6	Analyze, integrate, and evaluate multiple interpretations (e.g., charts, graphs, diagrams, videos) of a single text or text/s presented in different formats (visually, quantitatively) as well as in words in order to address a question or solve a problem.
ELA.W.AW.9–10.1.E	Provide a concluding paragraph or section that supports the argument presented.
	Mathematical and computational thinking in 9–12 builds on K–8 experiences and progresses to using algebraic thinking and analysis, a range of linear and nonlinear functions including trigonometric functions, exponentials and logarithms, and computational tools for statistical analysis to analyze, represent, and model data. Simple computational simulations are created and used based on mathematical models of basic assumptions.

21st Century Life and Career

TECH.9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
TECH.9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
TECH.9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
TECH.9.4.12.CT.3	Enlist input from a variety of stakeholders (e.g., community members, experts in the field) to design a service learning activity that addresses a local or global issue (e.g., environmental justice).
TECH.9.4.12.CT.4	Participate in online strategy and planning sessions for course-based, school-based, or other project and determine the strategies that contribute to effective outcomes.