

Unit 05: Relationships within Triangles

Content Area: **Template**
Course(s):
Time Period: **Full Year**
Length: **3 Weeks**
Status: **Published**

Unit 5: Relationships within Triangles (Module 9 and 10)

Unit Rationale

In prior learning, students have completed proofs on triangle congruence criteria, showing lines are perpendicular, and on theorems about perpendicular bisectors. They investigated angles in triangles and used the Triangle Sum Theorem. In algebra, they wrote and solved inequalities. In this unit, they will prove theorems about perpendicular bisectors and angles bisectors of triangles. They will construct angle bisectors and use them to find the incenter and circumcenter of a triangle. They will prove theorems about altitudes and medians, and they will prove and use the Triangle Midsegment Theorem. Then, they will learn how to apply inequalities to relate the sides and angles in one triangle and in two triangles. In later units, they will prove theorems about triangle inequalities and quadrilaterals. They will investigate segment length and angle measure relationships among lines intersecting circles.

Essential Questions

- How are the interior and exterior angles of a triangle related?
- How does the perpendicular bisector theorem contribute to triangle congruence proofs?
- What is the angle bisector theorem and how is it used in triangle congruence proofs?
- What are the properties and uses of medians in triangles?
- What is the midsegment of a triangle and what are its properties?
- What are the relationships between the sides and angles of a triangle?
- How do inequalities like the triangle inequality theorem and the relationship between sides and angles affect triangle congruence?

Pre-Assessments

- Into Geometry: Are you ready? Diagnostic Assessment p. 256

Instructional Plan

Angle Relationships in Triangles (9.1)

Student Learning Intentions or We are learning to ... (WALT)

- Prove theorems about triangle angles and apply the theorems in solving problems.

Student Success Criteria ... “I can statements”

- I can apply the Triangle Sum Theorem to solve problems.
- I can apply the triangle theorems to solve problems.
- I can prove theorems about triangle angles.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 257
- Guided Notes
 - Prove the Triangle Sum Theorem
 - Prove the Exterior Angle Theorem
 - Prove the Isosceles Triangle Theorem
 - Apply triangle theorems
- DeltaMath practice assignment
- Into Geometry Practice p. 261

Formative Assessments

- Into Geometry Check Understanding online assessment tool

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Perpendicular Bisectors in Triangles (9.2)

Student Learning Intentions or We are learning to ... (WALT)

- Use the Circumcenter Theorem to solve problems in the coordinate plane, and use the Circumcenter Theorem to determine the lengths of the segments that connect the circumcenter to the vertices of a triangle.

Student Success Criteria ... “I can statements”

- I can identify the perpendicular bisector of a side of a triangle.
- I can show that the perpendicular bisectors of a triangle are concurrent lines.
- I can construct perpendicular bisectors and use the point of concurrency to circumscribe triangles with circles.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 265
- Guided Notes
 - Investigate perpendicular bisectors
 - Prove that perpendicular bisectors are concurrent
 - Prove the Circumcenter Theorem
 - Use circumcenter to solve problems
- DeltaMath practice assignment
- Into Geometry Practice p. 269

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Angle Bisectors in Triangles (9.3)

Student Learning Intentions or We are learning to ... (WALT)

- Prove the Angle Bisector Theorem, the converse of the Angle Bisector Theorem, and the Incenter Theorem, and students construct the incenters and inscribed circles of triangles.

Student Success Criteria ... “I can statements”

- I can construct the angle bisector of a triangle.
- I can prove that if a point lies on the bisector of an angle, the point is equidistant from the sides of the angle.
- I can prove that angle bisectors are concurrent and inscribe circles in triangles.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 273
- Guided Notes
 - Prove the Angle Bisector Theorem
 - Prove the converse of the Angle Bisector Theorem
 - Prove the Incenter Theorem
 - Use the properties of angle bisectors
- DeltaMath practice assignment
- Into Geometry Practice p. 278

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Medians and Altitudes in Triangles (9.4)

Student Learning Intentions or We are learning to ... (WALT)

- Construct medians and altitudes to find centroids and orthocenters in order to understand the properties and relationships of these key triangle centers and solve geometric problems.

Student Success Criteria ... “I can statements”

- I can distinguish between the orthocenter and centroid of a triangle.

- I can find the orthocenter of a triangle drawn on a coordinate grid.
- I can construct medians and altitudes to find centroids and orthocenters.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 281
- Guided Notes
 - Median and the centroid theorem
 - Altitudes and the orthocenter
 - Find the center of gravity
 - Find the orthocenter of a triangle on the coordinate plane
- DeltaMath practice assignment
- Into Geometry Practice p. 285

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

The Triangle Midsegment Theorem (9.5)

Student Learning Intentions or We are learning to ... (WALT)

- Construct medians and altitudes to find centroids and orthocenters in order to understand the properties and relationships of these key triangle centers, which are essential for solving geometric problems.

Student Success Criteria ... “I can statements”

- I can use the side lengths of a triangle to find the length of a midsegment.
- I can construct midsegments.
- I can construct medians and altitudes to find centroids and orthocenters.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 289
- Guided Notes
 - Investigate midsegments of a triangle
 - Use the Triangle Midsegment Theorem

- DeltaMath practice assignment
- Into Geometry Practice p. 292

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Inequalities Within A Triangle (10.1)

Student Learning Intentions or We are learning to ... (WALT)

- Decide when three lengths can form a triangle, find the possible ranges of side lengths for the third side of a triangle, and order and compare the side lengths and angle measures in a triangle.

Student Success Criteria ... “I can statements”

- I can identify properties of sides and angles in triangles.
- I can describe the relationships between sides and angles in a triangle.
- I can determine the relative sizes of angles and sides in a triangle.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 299
- Guided Notes
 - Explore triangle inequalities
 - Use the Triangle Inequality Theorem
 - Use side-angle relationships in triangles
- DeltaMath practice assignment
- Into Geometry Practice p. 303

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Inequalities Between Two Triangles (10.2)

Student Learning Intentions or We are learning to ... (WALT)

- Apply known information about a triangle(s) in a pair of triangles to determine relative lengths and angle measures.

Student Success Criteria ... “I can statements”

- I can recognize that the measure of an angle in a triangle is related to the length of the opposite side.
- I can understand the Hinge Theorem and its converse.
- I can determine the relative sizes of angles and sides in two triangles.

Instructional Strategies and Activities

- Into Geometry Spark Your Learning p. 307
- Guided Notes
 - Explore inequalities in two triangles
 - Hinge Theorem and its converse
 - Use the Hinge Theorem in a proof
 - Write an indirect proof
- DeltaMath practice assignment
- Into Geometry Practice p. 312

Formative Assessments

- Into Geometry Check Your Understanding Interactive Lesson

Instructional Materials and Resources

- Into Math resources
- DeltaMath
- Desmos

Reflections and Suggested Modifications

Modifications and/or Accommodations

Suggested Modifications (ELL, Sp. Ed, Gifted, At-risk of Failure)

English Language Learners

Native language support: The teacher provides auditory or written content to students in their native language.

Adjusted Speech: The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.

Visuals: The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subjects at hand.

Front-Loading Vocabulary: The teacher front loads vocabulary. This means providing students with a list of important vocabulary words they will need to know for a book, lesson, etc. prior to the lesson being taught. Including pictures to go with the vocabulary words is also very beneficial for the students.

Special Education Students

Chunking: The teacher presents information in a way that makes it easy for students to understand and remember. Chunking is based on the presumption that our working memory is easily overloaded by excessive detail. The best way to deliver information is to organize it into meaningful units. Because students with special needs get overloaded easily, chunking is an effective strategy to use with them.

Checking for Understanding: It is important to constantly check for understanding, especially for students who have accommodations. Teachers want to make sure students understand the concepts being covered in a way that makes sense to them.

Extra time: The teacher provides students with special needs extra time to complete work or answer questions. It is important to give students enough time to process their thoughts.

Oral Reading: The teacher will read work orally to students. Class work such as tests and literature circles may need to be read aloud to the student.

Timers: The teacher will use timers as an instructional tool. The use of timers is beneficial for students who have trouble completing tasks. Timers can be helpful so the student is aware of how much time they have to complete an assignment.

Students with 504 Plans

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Gifted & Talented Strategies

Extensions/Enrichments: Teachers will provide gifted and talented students with extension/enrichment projects. Students will be challenged to further their understanding, to apply acquired knowledge, and/or to produce something in reference to acquired knowledge.

Modify/Change Activities: Teachers will monitor and modify activities to accommodate those students who need to be challenged further. Additional reading, problem-solving, writing, or project work is necessary for those students who are ready to move on at a rate more accelerated than their peers. In this way, G & T students are provided the same opportunity for support as special needs students.

Students at Risk of School Failure

Directions or Instructions: Make sure directions and/or instructions are given in limited numbers. Give directions/instructions verbally and in simple written format. Ask students to repeat the instructions or directions to ensure understanding occurs. Check back with the student to ensure he/she hasn't forgotten.

Peer Support: Peers can help build confidence in other students by assisting in peer learning. Many teachers use the 'ask 3 before me' approach. This is fine, however, a student at risk may have to have a specific student or two to ask. Set this up for the student so he/she knows who to ask for clarification before going to you.

Alternate or Modified Assignments: Always ask yourself, "How can I modify this assignment to ensure the students at risk are able to complete it?" Sometimes you'll simplify the task, reduce the length of the assignment or allow for a different mode of delivery. For instance, many students may hand something in, the at-risk student may jot notes and give you the information verbally. Or, it just may be that you will need to assign an alternate assignment.

Increase One to One Time: When other students are working, always touch base with your students at risk and find out if they're on track or needing some additional support. A few minutes here and there will go a long way to intervene as the need presents itself.

Contracts: It helps to have a working contract between you and your students at risk. This helps

prioritize the tasks that need to be done and ensure completion happens. Each day write down what needs to be completed, as the tasks are done, provide a checkmark or happy face. The goal of using contracts is to eventually have the student come to you for completion sign-offs.

Hands On: As much as possible, think in concrete terms and provide hands-on tasks. This means a child doing math may require a calculator or counters. The child may need to tape record comprehension activities instead of writing them. A child may have to listen to a story being read instead of reading it him/herself.

Tests/Assessments: Tests can be done orally if need be. Break tests down in smaller increments by having a portion of the test in the morning, another portion after lunch and the final part the next day.

Seating: Seat students near a helping peer or with quick access to the teacher. Those with hearing or sight issues need to be close to the instruction which often means near the front.

Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media Literacy

Diversity, Equity, and Inclusion

[NCTM: Access and Equity in Mathematics Education](#)

[A Pathway to Equitable Math Instruction](#)

Provide students with opportunities to give feedback to teachers about the classroom and instruction.

- Verbal Example: Fist to five, How well do you understand what we talked about today? Fist to five, How well did I teach this today?
- Classroom Activity: Exit tickets or surveys that ask students to identify how well teachers taught, what helped them learn, what got in the way of their learning, etc.

Treat mathematics as a language that everyone is learning while authentically centering students home languages.

- Classroom Strategies: Color-coding ideas, learning vocabulary in student languages, visual and kinesthetic learning, representations of learning without words.
- Classroom Activity: Multilingual Frayer Models for definitions or concepts

Incorporate true culturally relevant pedagogy, practice, and curriculum.

- Verbal Example: What are some of your family traditions that you are proud of? Would you be okay if we

brought some of those into the classroom?

- Classroom Activity: Use Ankara fabric to teach mathematical concepts such as tessellations, fractions, area, percentages, etc.

Incorporate the history of mathematics into lessons.

- Verbal Example: Why do you think we call it Pythagorean's theorem, when it was used before he was even born? What should we call it instead?
- Classroom Activity: Learn about different bases and numerical ideas: Base 2, binary and connections to computer programming, how the Yoruba of Nigeria used base 20, and how the Mayans conceptualized the number 0 before the first recording of it

Solicit student ways of thinking and processing.

- Verbal Example: How might you all go about this? What do you notice?
- Classroom Activity: Incorporate explorations, where students interact with mathematics in a way that allows them to "discover" or experience mathematics.

Reorganize your classroom teaching around concepts, and teach them more like a web rather than discrete sets of knowledge.

- Verbal Example: How does this connect to what you've learned in the past? How can you use that knowledge today?
- Classroom Activity: Learning webs that connect content

Start with more complex math problems and scaffold as necessary.

- Verbal Example: If we wanted to build a rocket, what are all the things we might need to know before we get started? Along the way, we decided that we want the rocket to reach the moon. What do we need to consider now?
- Classroom Activity: When solving equations, start with the most complex problem, generate ideas for how to solve it, and use the simpler equations as examples to support those ideas.

Offer a variety of ways to demonstrate thinking and knowledge.

- Verbal Example: Show your thinking with words, pictures, symbols.

Ask other questions that will demonstrate learning when it is not clear to you how students know the answer.

- Verbal Example: If you were working with a fellow mathematician who was absent this day, what might you tell them to help them learn it?

Learn about, engage with, and incorporate ethnomathematics.

- Verbal Example: Reflect on your day so far. What math have you already used today?

- Classroom Activity: Community walks to engage with slope.

Co-construct knowledge in the classroom.

- Verbal Example: Let's get into partners and do a think pair-share. We will incorporate everyone's ideas and try to synthesize them.
- Classroom Activity: Have students create mathematical definitions in their own words in groups, and bring the groups together to co-construct mathematical definitions as a class

Choose problems that have complex, competing, or multiple answers.

- Verbal Example: Come up with at least two answers that might solve this problem.
- Classroom Activity: Challenge standardized test questions by getting the "right" answer, but justify other answers by unpacking the assumptions that are made in the problem.
- Classroom Activity: Deconstructed Multiple Choice
 - given a set of multiple choice answers, students discuss why these answers may have been included (can also be used to highlight common mistakes).

Identify what is right about the thinking, and highlight the mistake in what is factually or procedurally accepted.

- Verbal Example: You recognized that you had to combine the constants 27 and 9, could you explain your thinking?
- Classroom Activity: Error Analysis worksheets that highlight what is the right idea behind the mistake.

Using thoughtful questioning to solicit mathematical thoughts rather than telling.

- Verbal Example: What would a mathematician who is confused ask about this question?
- Classroom Activity: After students demonstrate knowledge of a topic, have them play a game where they have to explain their topic to a fellow mathematician and a skeptic. Develop their own reflective questioning/explaining in all three roles.

Create multiple ways of participating that honor myriad ways of thinking and being.

- Verbal Example: For this section, feel free to work alone, in pairs, trios, or quads (let them choose).
- Classroom Activity: Community circles or storytelling circles, incorporating dance, music, song, call and response, and other cultural ways of communicating.

Climate Change

[Math Climate Change Companion Guide](#)

New Jersey Student Learning Standards: Content Area

MATH.9-12.G.C.A.3	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
MATH.9-12.G.CO.D.12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
MATH.9-12.G.CO.D.13	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
MATH.9-12.G.GPE.B.6	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
MATH.9-12.G.MG.A.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
MATH.9-12.G.SRT.B.4	Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

Integration of Career Readiness, Life Literacies and Key Skills

TECH.9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
TECH.9.4.12.CI.2	Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).
TECH.9.4.12.CI.3	Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1).
TECH.9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
TECH.9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
TECH.9.4.12.TL.2	Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.
TECH.9.4.12.TL.3	Analyze the effectiveness of the process and quality of collaborative environments.

Integration of Computer Science and Design Thinking

CS.9-12.8.1.12.AP.1	Design algorithms to solve computational problems using a combination of original and existing algorithms.
CS.9-12.8.1.12.AP.5	Decompose problems into smaller components through systematic analysis, using constructs such as procedures, modules, and/or objects.

Interdisciplinary Connections: NJSL for ELA, Social Studies, Science and/or Math

ELA.RI.MF.9–10.6	Analyze, integrate, and evaluate multiple interpretations (e.g., charts, graphs, diagrams,
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videos) of a single text or text/s presented in different formats (visually, quantitatively) as well as in words in order to address a question or solve a problem.

ELA.W.AW.9–10.1.E

Provide a concluding paragraph or section that supports the argument presented.

Mathematical and computational thinking in 9–12 builds on K–8 experiences and progresses to using algebraic thinking and analysis, a range of linear and nonlinear functions including trigonometric functions, exponentials and logarithms, and computational tools for statistical analysis to analyze, represent, and model data. Simple computational simulations are created and used based on mathematical models of basic assumptions.

21st Century Life and Career

TECH.9.4.12.CT.1

Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).

TECH.9.4.12.CT.2

Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).

TECH.9.4.12.CT.3

Enlist input from a variety of stakeholders (e.g., community members, experts in the field) to design a service learning activity that addresses a local or global issue (e.g., environmental justice).

TECH.9.4.12.CT.4

Participate in online strategy and planning sessions for course-based, school-based, or other project and determine the strategies that contribute to effective outcomes.

Collaboration with individuals with diverse experiences can aid in the problem-solving process, particularly for global issues where diverse solutions are needed.