Unit 3: Exponents and Exponential Functions

Content Area: **Template**
Course(s):
Time Period: **Full Year**
Length: **3-4 Weeks**
Status: **Published**

Unit Rationale

1. Foundation of Algebraic Operations

- **Understanding Exponential Notation:** Exponents provide a compact way to represent repeated multiplication, which is a key concept in algebra. Mastery of exponents is essential for performing operations with polynomials and other algebraic expressions.
- **Rules of Exponents:** Learning the rules of exponents (product rule, quotient rule, power rule, etc.) helps students simplify and manipulate algebraic expressions more efficiently. These rules are foundational for more advanced algebraic operations.

2. Development of Problem-Solving Skills

- **Simplification:** Exponents are used to simplify complex expressions, making it easier to solve equations and inequalities. This skill is critical for solving higher-level algebraic problems and for understanding functions and their behaviors.
- **Application of Rules:** Applying exponent rules to various problems helps students develop a systematic approach to problem-solving and strengthens their algebraic reasoning.

3. Real-World Applications

- **Modeling Growth and Decay:** Exponents are used to model real-world phenomena such as population growth, radioactive decay, and compound interest. Understanding exponents allows students to analyze and solve problems involving exponential growth and decay.
- **Scientific Notation:** Exponents are used in scientific notation to represent very large or very small numbers. This is important for understanding and working with data in scientific and engineering contexts.

4. Preparation for Advanced Topics

- **Introduction to Functions:** Exponential functions, which involve exponents, are fundamental for studying more advanced mathematical concepts, including logarithms and calculus. Understanding exponents provides a foundation for exploring these topics.
- **Algebraic Manipulation:** Mastery of exponents is crucial for solving equations involving exponential functions and for manipulating algebraic expressions in more complex scenarios.

5. Development of Critical Thinking and Analytical Skills

- **Reasoning and Justification:** Working with exponents requires students to justify their steps and apply rules systematically, which enhances their critical thinking and analytical skills.
- **Error Analysis:** Learning to work with exponents helps students develop the ability to identify and correct errors in their calculations, fostering persistence and attention to detail.

6. Enhancing Mathematical Literacy

- **Understanding Mathematical Language:** Exponents are a key part of mathematical notation. Learning how to read, interpret, and use exponent notation helps students become more fluent in mathematical language and concepts.
- **Building Confidence:** Mastery of exponents builds students' confidence in their algebraic abilities and prepares them for more complex mathematical problems.

7. Preparation for Standardized Testing

- **Test Preparation:** Exponents and exponential functions are commonly tested on standardized exams. Proficiency in these topics is essential for performing well on tests such as the SAT, ACT, and state assessments.

Including a unit on exponents helps students develop essential algebraic skills, understand mathematical notation, and apply concepts to real-world problems. This unit lays the groundwork for more advanced mathematical topics and provides students with the tools they need for success in algebra and beyond.

Topic 5 Exponents and Exponential Functions

Essential Questions

How do you use exponential functions to model situations and solve problems?

- What are the properties of rational exponents and how are they used to solve problems?
- What are the characteristics of exponential functions?
- What kinds of situations can be modeled with exponential growth or exponential decay functions?
- How are geometric sequences related to exponential functions?
- How do changes in an exponential function relate to translations of its graph?

Pre-Assessments

Topic Readiness Assessments available at SavvasRealize.com

Instructional Plan

Lesson 5.1 Rational Exponents and Properties of Exponents

Student Learning Intentions or We are learning to … (WALT)

- We are learning to extend the properties of integer exponents to rational exponents to rewrite radical expressions using rational exponents.
- We are learning to solve equations with rational exponents using the properties of exponents.

Student Success Criteria … "I can statements"

- I can use the properties of exponents to solve equations with rational exponents.

Instructional Strategies and Activities

- Habits of Mind: How do you know when exponential expressions are equivalent? When is the value of an expression undefined? Can you use the same properties of exponents for expressions with rational exponents as you do when computing with integers?
- Student textbook page 177-183
- Vocabulary: rational exponent, integer exponent, power of a power property, product of powers property, quotient of powers property

Formative Assessments

- 5-1 Lesson Quiz (printable or available online)

Instructional Materials and Resources

- enVisions Math
- Reteach to Build Understanding 5-1
- Additional Practice 5-1
- Enrichment 5-1
- Math Literacy and Vocabulary
- Digital Resources and Video Tutorials
- Assignment Guide: Basic 19-38, 43-50; Advanced 19-28, 33-50

Vocabulary: equivalent equations, inverse operations, isolate, solution of an equation, variable.

Common Error: review of the distributive property, multiply all terms by the number outside the parenthesis

Lesson 5.2 Exponential Functions

Student Learning Intentions or We are learning to … (WALT)

- We are learning to sketch graphs showing key features of exponential functions.
- We are learning to write exponential functions using tables and graphs.
- We are learning to compare linear and exponential functions.

Student Success Criteria … "I can statements"

- I can describe and graph exponential functions.

Instructional Strategies and Activities

- Habits of Mind: Describe situations that you could represent using an exponential function.
- Student textbook page 184-190
- Vocabulary: asymptote, constant ratio, exponential function, constant rate versus constant ratio

Formative Assessments

- 5-2 Lesson Quiz (printable or available online)

Instructional Materials and Resources

- enVisions Math
- Reteach to Build Understanding 5-2
- Additional Practice 5-2
- Enrichment 5-2
- Math Literacy and Vocabulary
- Digital Resources and Video Tutorials
- Assignment Guide: Basic 9-12, 23, 25-31; Advanced 9-15, 17, 19-31

Reflections and Suggested Modifications

Common Error: Remind student to check x and y values into the equations to confirm values; Substitute 0 into the function for x to find the y intercept.

Lesson 5.3 Exponential Growth and Decay

Student Learning Intentions or We are learning to … (WALT)

- We are learning to construct exponential growth and decay functions given a description of a relationship.
- We are learning to recognize if a situation can be modeled with exponential growth or exponential decay, and interpret the parameters of the model in context.

Student Success Criteria … "I can statements"

- I can use exponential functions to model situations and make predictions

Instructional Strategies and Activities

- Habits of Mind: What is the constant ratio for declining values? Why does the total value increase more times a value is compounded? What are the key differences in algebraic representations of exponential growth and decay?
- Student textbook page 191-198
- Vocabulary: compound interest, decay factor, exponential decay, exponential growth, growth factor

Formative Assessments

- 5-3 Lesson Quiz (printable or available online)

Instructional Materials and Resources

- enVisions Math
- Reteach to Build Understanding 5-3
- Additional Practice 5-3
- Enrichment 5-3
- Math Literacy and Vocabulary
- Digital Resources and Video Tutorials
- Assignment Guide: Basic 10-21, 24-25, 27-32; Advanced 10-14, 16-17, 20-32

Reflections and Suggested Modifications

Common Error: Students may add the percent rate of change to 1 instead of subtracting it. Decay decreases therefore the decay factor should be less than 1.

Lesson 5.4 Geometric Sequences

Student Learning Intentions or We are learning to … (WALT)

- We are learning to find explicit and recursive formulas for geometric sequences.
- We are learning to translate between recursive and explicit formulas for geometric sequences.
- We are learning to construct exponential functions to represent geometric sequences.

Student Success Criteria … "I can statements"

- I can identify geometric sequences.

Instructional Strategies and Activities

- Habits of Mind: When is using a diagram the best tool to determine information? Why can a common ratio in a geometric sequence not be zero? What is the relationship between the explicit formula and the recursive formula?
- Student textbook page 199-205
- Vocabulary: geometric sequence, arithmetic sequences, explicit formula, recursive formula

Formative Assessments

- 5-4 Lesson Quiz (printable or available online)

Instructional Materials and Resources

- enVisions Math
- Reteach to Build Understanding 5-4
- Additional Practice 5-4
- Enrichment 5-4
- Math Literacy and Vocabulary
- Digital Resources and Video Tutorials
- Assignment Guide: Basic 11-32, 37-39, 41-46; Advanced 11-22, 28-46

Reflections and Suggested Modifications

Common Error: When finding the common ratio, some students may divide each term by the next term in the sequence. Students should check the common ratio by multiplying each term of the sequence the common ratios to see if they get the next term.

Lesson 5.5 Transformations of Exponential Functions

Student Learning Intentions or We are learning to … (WALT)

- We are learning to translate the graph of an exponential function vertically and horizontally, identifying the effect different values of h and k have on the graph of the function.
- We are learning to compare characteristics of two exponential functions represented in different ways, such as tables and graphs.

Student Success Criteria … "I can statements"

- I can perform, analyze, and use transformations of exponential functions.

Instructional Strategies and Activities

- Habits of Mind: Is it possible for the graph of the function to have a negative value for x? How does a function that results in a vertical translation differ from a function that results in a horizontal translation? What effect does inserting a different constant into an exponential function have on the transformation of a function?
- Student textbook page 206-212
- Vocabulary: constant, exponential functions, translation

Formative Assessments

- 5-5 Lesson Quiz (printable or available online)

Instructional Materials and Resources

- enVisions Math
- Reteach to Build Understanding 5-5
- Additional Practice 5-5
- Enrichment 5-5
- Math Literacy and Vocabulary
- Digital Resources and Video Tutorials
- Assignment Guide: Basic 15-25, 28-35; Advanced 15-19; 22-35

Reflections and Suggested Modifications

Common Error: Have students always rewrite exponents with subtraction. If the constant is subtracted from x it does not always mean the translation is horizontal. Emphasize the sign of the number being subtracted indicated the translation's direction. Encourage students to express functions in the same form- either algebraically or in a graph, so they can compare the key features easily.

Modifications and/or Accommodations

Suggested Modifications (ELL, Sp. Ed, Gifted, At-risk of Failure)

English Language Learners

*SCIOP/WIDA levels will allow the teacher to determine what supports are appropriate. Reach out to the MLL teacher for suggestions.

Native language support: The teacher provides auditory or written content to students in their native language. Sentence Stems

Adjusted Speech: The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.

Visuals: The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subjects at hand.

Front-Loading Vocabulary: The teacher front-loads vocabulary. This means providing students with a list of important vocabulary words they will need to know for a book, lesson, etc. prior to the lesson being taught. Including pictures to go with the vocabulary words is also very beneficial for the students.

Special Education Students

*Always reference the students IEP for specific accommodations or modification per student need.

Chunking: The teacher presents information in a way that makes it easy for students to understand and remember. Chunking is based on the presumption that our working memory is easily overloaded by excessive detail. The best way to deliver information is to organize it into meaningful units. Because students with special needs get overloaded easily, chunking is an effective strategy to use with them.

Checking for Understanding: It is important to constantly check for understanding, especially for students who have accommodations. Teachers want to make sure students understand the concepts being covered in a way that makes sense to them.

Extra time: The teacher provides students with special needs extra time to complete work or answer questions. It is important to give students enough time to process their thoughts.

Oral Reading: The teacher will read work orally to students. Class work such as tests and literature circles may need to be read aloud to the student.

Timers: The teacher will use timers as an instructional tool. The use of timers is beneficial for students who have trouble completing tasks. Timers can be helpful so the student is aware of how much time they have to complete an assignment.

Students with 504 Plans

*Always reference the students IEP for specific accommodations or modification per student need.

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Gifted & Talented Strategies

Extensions/Enrichments: Teachers will provide gifted and talented students with extension/enrichment projects. Students will be challenged to further their understanding, to apply acquired knowledge, and/or to produce something in reference to acquired knowledge.

Modify/Change Activities: Teachers will monitor and modify activities to accommodate those students who need to be challenged further. Additional reading, problem-solving, writing, or project work is necessary for those students who are ready to move on at a rate more accelerated than their peers. In this way, G & T students are provided the same opportunity for support as special needs students.

Students at Risk of School Failure

*Reach out to the Student Support team for assistance.

Directions or Instructions: Make sure directions and/or instructions are given in limited numbers. Give directions/instructions verbally and in simple written format. Ask students to repeat the instructions or directions to ensure understanding occurs. Check back with the student to ensure he/she hasn't forgotten.

Peer Support: Peers can help build confidence in other students by assisting in peer learning. Many teachers use the 'ask 3 before me' approach. This is fine, however, a student at risk may have to have a specific student or two to ask. Set this up for the student so he/she knows who to ask for clarification before going to you.

Alternate or Modified Assignments: Always ask yourself, "How can I modify this assignment to ensure the students at risk are able to complete it?" Sometimes you'll simplify the task, reduce the length of the assignment or allow for a different mode of delivery. For instance, many students may hand something in, the at-risk student may jot notes and give you the information verbally. Or, it just may be that you will need to assign an alternate assignment.

Increase One to One Time: When other students are working, always touch base with your students at risk and find out if they're on track or needing some additional support. A few minutes here and there will go a long way to intervene as the need presents itself.

Contracts: It helps to have a working contract between you and your students at risk. This helps prioritize the tasks that need to be done and ensure completion happens. Each day write down what needs to be completed, as the tasks are done, provide a checkmark or happy face. The goal of using contracts is to eventually have the student come to you for completion sign-offs.

Hands On: As much as possible, think in concrete terms and provide hands-on tasks. This means a child doing math may require a calculator or counters. The child may need to tape record

comprehension activities instead of writing them. A child may have to listen to a story being read instead of reading it him/herself.

Tests/Assessments: Tests can be done orally if need be. Break tests down in smaller increments by having a portion of the test in the morning, another portion after lunch and the final part the next day.

Seating: Seat students near a helping peer or with quick access to the teacher. Those with hearing or sight issues need to be close to the instruction which often means near the front.

Standards

New Jersey Student Learning Standards: Content Area

MATH.K-12.1	Make sense of problems and persevere in solving them
MATH.K-12.2	Reason abstractly and quantitatively
MATH.K-12.3	Construct viable arguments and critique the reasoning of others
MATH.K-12.5	Use appropriate tools strategically
MATH.K-12.6	Attend to precision
MATH.K-12.7	Look for and make use of structure
MATH.K-12.8	Look for and express regularity in repeated reasoning
MATH.9-12.A.SSE.B.3.c	Use the properties of exponents to transform expressions for exponential functions.
MATH.9-12.A.SSE.B.4	Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media Literacy

Diversity, Equity, and Inclusion

NCTM: Access and Equity in Mathematics Education

A Pathway to Equitable Math Instruction

Provide students with opportunities to give feedback to teachers about the classroom and instruction.

• Verbal Example: Fist to five, How well do you understand what we talked about today? Fist to five, How well did I teach this today?

• Classroom Activity: Exit tickets or surveys that ask students to identify how well teachers taught, what helped them learn, what got in the way of their learning, etc.

Treat mathematics as a language that everyone is learning while authentically centering students home languages.

• Classroom Strategies: Color-coding ideas, learning vocabulary in student languages, visual and kinesthetic learning, representations of learning without words.

• Classroom Activity: Multilingual Frayer Models for
definitions or concepts

Incorporate true culturally relevant pedagogy, practice, and curriculum.

• Verbal Example: What are some of your family traditions that you are proud of? Would you be okay if we brought some of those into the classroom?

• Classroom Activity: Use Ankara fabric to teach mathematical concepts such as tessellations, fractions, area, percentages, etc.

Incorporate the history of mathematics into lessons.
• Verbal Example: Why do you think we call it Pythagorean's theorem, when it was used before he was even born? What should we call it instead?
• Classroom Activity: Learn about different bases and numerical ideas: Base 2, binary and connections to computer programming, how the Yoruba of Nigeria used base 20, and how the Mayans conceptualized the number 0 before the first recording of it

Solicit student ways of thinking and processing.
• Verbal Example: How might you all go about this? What do you notice?
• Classroom Activity: Incorporate explorations, where students interact with mathematics in a way that allows them to "discover" or experience mathematics.

Reorganize your classroom teaching around concepts, and teach them more like a web rather than discrete sets of knowledge.
• Verbal Example: How does this connect to what you've learned in the past? How can you use that knowledge today?
• Classroom Activity: Learning webs that connect content

Start with more complex math problems and scaffold as necessary.
• Verbal Example: If we wanted to build a rocket, what are all the things we might need to know before we get started? Along the way, we decided that we want the rocket to reach the moon. What do we need to consider now?
• Classroom Activity: When solving equations, start with the most complex problem, generate ideas for how to solve it, and use the simpler equations as examples to support those ideas.

Offer a variety of ways to demonstrate thinking and knowledge.
• Verbal Example: Show your thinking with words, pictures, symbols.

Ask other questions that will demonstrate learning when it is not clear to you how students know the answer.
• Verbal Example: If you were working with a fellow mathematician who was absent this day, what might you tell them to help them learn it?

Learn about, engage with, and incorporate ethnomathematics.
• Verbal Example: Reflect on your day so far. What math have you already used today?
• Classroom Activity: Community walks to engage with slope.

Co-construct knowledge in the classroom.
• Verbal Example: Let's get into partners and do a think pair-share. We will incorporate everyone's ideas and try to synthesize them.
• Classroom Activity: Have students create mathematical definitions in their own words in groups, and bring the groups together to co-construct mathematical definitions as a class

Choose problems that have complex, competing, or multiple answers.
• Verbal Example: Come up with at least two answers that might solve this problem.
• Classroom Activity: Challenge standardized test questions by getting the "right" answer, but justify other answers by unpacking the assumptions that are made in the problem.
• Classroom Activity: Deconstructed Multiple Choice
- given a set of multiple choice answers, students discuss why these answers may have been included (can also be used to highlight common mistakes).

Identify what is right about the thinking, and highlight the mistake in what is factually or procedurally accepted.
• Verbal Example: You recognized that you had to combine the constants 27 and 9, could you explain your thinking?
• Classroom Activity: Error Analysis worksheets that highlight what is the right idea behind the mistake.

Using thoughtful questioning to solicit mathematical thoughts rather than telling.

• Verbal Example: What would a mathematician who is confused ask about this question?

• Classroom Activity: After students demonstrate knowledge of a topic, have them play a game where they have to explain their topic to a fellow mathematician and a skeptic. Develop their own reflective questioning/explaining in all three roles.

Create multiple ways of participating that honor myriad ways of thinking and being.
• Verbal Example: For this section, feel free to work alone, in pairs, trios, or quads (let them choose).
• Classroom Activity: Community circles or storytelling circles, incorporating dance, music, song, call and response, and other cultural ways of communicating.

Climate Change

Math Climate Change Companion Guide

• S.ID.B.6a Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.
Climate Change Example: Students may use linear or exponential functions fitted to geoscience data to solve problems and analyze the results from global climate models to make an evidence-based forecast of the current rate of global climate change.

• F.IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
Climate Change Example: Students may use function notation to determine the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline), m, where c(m) is the number of molecules of carbon dioxide.
• F.IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
Climate Change Example: Students may relate the domain of a function c(m) representing the amount of carbon dioxide produced by burning m molecules of ethane (gasoline), to its graph in order to determine the appropriate domain for c(m).
• F.IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
Climate Change Example: Students may calculate the average rate of change of a function c(m) presented symbolically or as a table, where c(m) represents the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline).

• A.CED.A.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
Climate Change Example: Students may create equations and/or inequalities to represent the economic impact of climate change.
• A.CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
Climate Change Example: Students may represent constraints describing the economic impact of climate change by equations, inequalities, and/or by systems of inequalities, and interpret solutions as viable or nonviable options.
• A.CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law to highlight resistance R.
Climate Change Example: Students may rearrange formulas related to the economic impact of climate change to highlight a quantity of interest, using the same reasoning as in solving equations.

• N.Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose

and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
Climate Change Example: Students may use units to guide the solution of multi-step problems about how variations in the flow of energy into and out of the Earth's systems result in climate change. Note: Changes in climate are limited to changes in surface temperatures, precipitation patterns, glacial ice volumes, sea levels, and biosphere distribution.
• N.Q.A.2 Define appropriate quantities for the purpose of descriptive modeling.
Climate Change Example: Students may define appropriate quantities for a descriptive model of how variations in the flow of energy into and out of Earth's systems result in climate change. Note: changes in climate are limited to changes in surface temperatures, precipitation patterns, glacial ice volumes, sea levels, and biosphere distribution.

21st Century Life and Career

CRP.K-12.CRP1	Act as a responsible and contributing citizen and employee.
CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP3	Attend to personal health and financial well-being.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP5	Consider the environmental, social and economic impacts of decisions.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP10	Plan education and career paths aligned to personal goals.
CRP.K-12.CRP12	Work productively in teams while using cultural global competence.

Interdisciplinary Connections: NJSLS for ELA, Social Studies, Science and/or Math

SOC.K-12.1	Developing Questions and Planning Inquiry
LA.W.9-10.4	Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)
LA.SL.9-10	Speaking and Listening
SCI.9-12.5.1.12.A	Students understand core concepts and principles of science and use measurement and observation tools to assist in categorizing, representing, and interpreting the natural and designed world.

Integration of Career Readiness. Life Literacies and Key Skills

TECH.9.4.2.CI	Creativity and Innovation
TECH.9.4.2.CT	Critical Thinking and Problem-solving
TECH.9.4.2.IML	Information and Media Literacy

Integration of Computer Science and Design Thinking

CS.9-10.3A-AP-14	Use lists to simplify solutions, generalizing computational problems instead of repeatedly using simple variables.
CS.9-10.3A-AP-17	Decompose problems into smaller components through systematic analysis, using constructs such as procedures, modules, and/or objects.