Unit 01 (Chpt P) Rational Exponents, Factoring, and Rational Expressions

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 1**
Length: **5-6 weeks**
Status: **Published**

Brief Summary of Unit

Students will be introduced to all of the rules of exponents. They will be able to simplify with basic powers, powers that have fractions, and learn to rewrite with rational exponents. Students will know when to add, subtract, multiply, reduce, and cancel as needed for their exponents. Students will simplify when the exponent is a number or a variable. Students will be able to simplify with negative exponents. Students will be able to simplify rational expressions, make common denominators with expressions, and simplify as needed. Students will solve rational equations and find the correct solutions (there may be more than one solution or an extraneous solution). Students will be able to simplify with radicals, they will simplify square roots, cubed roots, 4th roots, 5th roots. They will be able to rationalize a denominator when the denominator is any type of root. Students will simplify with variables in the radical and will decide when to use absolute value around the variable. Students will then review their rules of factoring. They will be able to factor a GCF first and always, basic trinomials, perfect square trinomials, trinomials with a leading term higher than 1, difference of square, difference of cubes, sum of cubes, and grouping. Students will rationalize a denominator with a binomial.

Revised Date: July 2025

Standards

This curriculum integrates algebra, geometry, statistics, and/or data analysis with diversity, equity, and inclusion by connecting mathematical concepts to cultural, historical, and social studies content. Students may explore statistical studies and data patterns tied to different cultures, analyze geometric designs rooted in global traditions, or use algebraic and quantitative reasoning to examine issues of equity and fairness. When relevant, units may incorporate elements such as the Amistad Commission mandate to highlight African and African-American history, the Asian American and Pacific Islander History Law to recognize contributions of AAPI communities, or New Jersey's Diversity and Inclusion Law to promote tolerance, belonging, and respect across gender, sexual orientation, race, ethnicity, disabilities, and religion. Instruction may also align with the Commission on Holocaust Education when appropriate, fostering awareness of human behavior, moral responsibility, and the importance of combating prejudice and discrimination. Units may also align to the English Language Development (ELD) standards for K–12, recognizing that multilingual learners develop content and language concurrently in English-medium classrooms; in particular, Standard 1 (Language for Social and Instructional Purposes) supports students in communicating effectively for both social and academic purposes. Across all domains of mathematics, students connect rigorous problem solving with real-world contexts that highlight the value of diversity in society.

ELA.K-12.1	Developing Responsibility for Learning: Cultivating independence, self-reflection, and responsibility for one's own learning.
MATH.9-12.N.RN.A	Extend the properties of exponents to rational exponents
MATH.9-12.N.RN.A.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
ELA.K-12.3	Valuing Evidence in Argumentation: Constructing viable claims and evaluating, defending, challenging, and qualifying the arguments of others.
ELA.K-12.4	Building Knowledge: Building strong content knowledge and connecting ideas across disciplines using a variety of text resources and media.
	For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.
MATH.9-12.F.BF.A.1.b	Combine standard function types using arithmetic operations.
MATH.9-12.N.RN.A.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
ELA.K-12.5	Leveraging Technology: Employing technology and digital media thoughtfully, strategically and capably to enhance reading, writing, speaking, listening, and language use.
MATH.9-12.N.RN.A.3	Simplify radicals, including algebraic radicals (e.g., $\sqrt[3]{54} = 3\sqrt[3]{2}$, simplify $\sqrt{32x^2}$).
MATH.9-12.F.BF.A.1.c	Compose functions.
MATH.9-12.A.APR.D	Rewrite rational expressions
MATH.9-12.A.APR.D.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
MATH.9-12.A.REI.A.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
MATH.9-12.F.IF.C.7.b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
MATH.9-12.A.SSE	Seeing Structure in Expressions
MATH.9-12.A.SSE.A.1	Interpret expressions that represent a quantity in terms of its context.
MATH.9-12.A.SSE.A.1.a	Interpret parts of an expression, such as terms, factors, and coefficients.
MATH.9-12.A.SSE.A.1.b	Interpret complicated expressions by viewing one or more of their parts as a single entity.
MATH.9-12.A.SSE.A.2	Use the structure of an expression to identify ways to rewrite it.
	For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
MATH.9-12.A.SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
	For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.
WRK.K-12.P.5	Utilize critical thinking to make sense of problems and persevere in solving them.
WRK.K-12.P.8	Use technology to enhance productivity increase collaboration and communicate effectively.
WRK.K-12.P.9	Work productively in teams while using cultural/global competence.
TECH.9.4.12.CT	Critical Thinking and Problem-solving

Essential Questions

- How are square root and cube root functions and their graphs similar to linear functions, quadratic functions, and polynomial functions? How are they different?
- How can solutions be determine extraneous and what do they reveal about functions?
- How can we compare and contrast the methods on simplifying a square root and cubed root?
- How can we determine when it is possible to take an nth root of a negative number?
- How can we use rational exponents to represent a power involving a radical?
- How can we use the properties of exponents to simplify products and quotients of radicals?
- How is factoring a difference of squares different from factoring a difference of cubes?
- Why do we have to simplify a radical in a denominator differently when it is a square root vs cubed root vs a variable?
- Why do we need absolute value around variables that are simpified from a radical?

Enduring Understandings

- Students will apply the song of cubes properly
- Students will be able to factor and then multiply their answers to the original polynomial.
- Students will be able to rewrite from a rational exponent into a radical and understand they represent the same value
- Students will comprehend the difference in simplifying exponents involving addition, multiplication, division, etc
- Students will foil (x + 2)(x + 2) vs (x + 2)(x - 2) and see that they are not the same answer and that x^2 + 4 is not the same as (x +2)(x+2)
- Students will know when to use factoring by grouping
- Students will understand how to apply exponents to solving equations
- Students will understand how to make a common denominator
- Students will understand how to rationalize a denominator
- Students will understand restrictions on x when solving rational equations
- Students will understand the relationship between rational exponents and radicals functions.
- Students will understand when a common denominator is needed and when one is not needed.

Students Will Know

- Students will know how to add, subtract, multiply, divide, and simplify compound fractions.
- Students will know how to check for extraneous solutions
- Students will know how to describe the properties of rational exponents and radicals.
- Students will know how to properly factor all types of polynomials
- Students will know how to rationalize a denominator
- Students will know how to represent roots using rational exponents.
- Students will know how to simplify with exponents

- Students will know how to solve rational equations

Students Will Be Skilled At

- Students will be skilled at being able to factor properly to simplify rational expressions involving multiplication and division (cross-cancel).
- Students will be skilled at being able to factor to make a common denominator to lead to adding or subtracting expressions.
- Students will be skilled at evaluating expressions with rational exponents.
- Students will be skilled at explaining the meaning of a rational exponent.
- Students will be skilled at explaining when radical expressions are in simplest form.
- Students will be skilled at factoring all polynomial types
- Students will be skilled at finding the arithmetic combinations of two functions.
- Students will be skilled at identifying extraneous solutions of rational equations.
- Students will be skilled at identifying rational equations.
- Students will be skilled at simplifying rational expressions with rational exponents.
- Students will be skilled at simplifying variable expressions containing rational exponents and radicals.
- Students will be skilled at solving equations using nth roots.
- Students will be skilled at solving radical equations.
- Students will be skilled at solving real life problems involving radical equations.
- Students will be skilled at writing functions that represent transformations of radical functions.

Evidence/Performance Tasks

Assessments

- Formative: Daily assessments using examples from class notes, NJSLA test bank problems, and/or Albert/AP Classroom assessments, homework check
- Summative: Teacher-created assessments, NJSLA test bank problems, Albert/AP Classroom assessments, quick quizzes
- Benchmark: teacher created diagnostic assessments, practice assignments
- Alternative Assessments: Student-centered activities

- Answer essential questions
- Class discussion of daily topic
- Classwork and homework that assess the essential questions
- Provide alternative means of assessments for certain students
- Teacher Observation

- Tests and quizzes that assess the essential questions
- Written assignments that assess the essential questions that involves providing explanations

Learning Plan

Unit P (front of book) - Exponents, factoring and rational equations (3 weeks)

This unit can be split into three mini units, with 3 assessments and a possible comprehensive unit assessment if time allows.

Prior to the start of the unit, rules for exponents should be reviewed. These are standards that are included in Algebra 1 and were included with the summer review.

Day 1: Exponents Review (3 days)

- Begin review of exponents: when to add, subtract, multiply
  - show how to simplify (2^2)(2^3) by keeping the same base. Students often multiply the bases
  - Make a chart/list of all of the powers: 2^2, 2^3, 2^4, etc so students can use it as needed in class
- Students will complete practice puzzles on exponents
- Students will complete practice sheets on exponents (work independently and in small groups)

Day 2: Exponents

- Review homework
- Practice sheet on exponents
- Quick quiz
- Begin rational exponents
  - Review how to add fractions and multiply fractions
- Have students simplify with negative exponents
  - When is it easier to flip the fraction?
  - Move negative exponents to make them positive
- Combine all exponent types

Day 3: Exponents

- Drill and review

Day 4: Quiz

Day 5: Simplifying radicals

- Students will make a list of the perfect squares, perfect cubes, perfect 4th roots, and 5th roots
- Students will simplify square roots, cued roots, 4th roots, and 5th roots where the root is a perfect root

or where the root has a perfect root contained in it and simplify

- Students will simplify with variables. They will decide if that variable needs absolute value around it
    - Rule: even root --> odd power in the answer needs absolute value
- Students will simplify things like cubed root of 8x^3y^6; cubed root of 27x^8y^2z and so on
- Students will begin to simplify with radicals in a denominator
    - simplify a square root denominator
        - rationalize the denominator by multiplying by the same square root (variables are different)
        - Show you are making a perfect square root
    - simplify a cubed root denominator
        - Show that you must make a perfect cube root in the denominator
            - simplify with variables as well - do you need absolute value?
    - Simplify a 4th root denominator

Day 6: Review day 5 and practice day 5

Day 7:

- Quick quiz
- Students will now rationalize a binomial denominator
    - Students may need to FOIL
    - Students may need to reduce their fractions
    - Students may need to simplify the radicals in their answer
- Practice how to multiply with radicals and simplify

Day 8: Review

Day 9: Quiz

Day 10: Factoring

- Review how to find a GCF properly
- Review factoring a basic trinomial
    - Factor with and without a GCF
- Review how to factor with a coefficient of "a" that is higher than 1
- Review how to factor a difference of squares
- New factoring: - show how to factor by grouping
    - Used when there is 4 terms
    - factor out the GCF of the two expressions
    - Check by foiling
- Practice

Day 11: Factoring day 2

- Review sheet of yesterday's material
- Scattered factoring instead of chunking the same types of problems - see if students can recognize each

type
- Begin factoring a sum and difference of two cubes
    - Sing the song to help
    - Discuss how the second quantity of the factorization (the trinomial) is never factorable any further and will solve only with the quadratic formula
- Factor with a GCF on the cubes
- Practice sheet

Day 12: Factoring day 3

- Review homework
- Quick quiz
- Review sheets for all factoring types
- Puzzle practice - group work or independent work

Day 13: Quiz on factoring

Day 14 - Rational expressions

- Students will begin to simplify a basic fraction by reducing
    - Complete a practice sheet on this topic
- Students will then factor a fraction and simplify (reduce the same factorization in a numerator and denominator to one)
    - Complete a practice sheet on this topic
    - Students will have problems with negative exponents
- Students will begin to simplify a a product of two fractions. Students will factor each numerator and denominator, then reduce where possible or "cross-cancel."
    - Students will complete practice sheets on this topic
- Students will begin division of two fractions
    - Step one: change the problem into a multiplication problem -- keep/change/flip or same/change/reciprocal are popular phrases to repeat
    - Step two: factor
    - Step three: cancel/reduce
    - Complete practice sheets on this topic

Day 15 - Spend 20 minutes reviewing yesterday just practice problems. Circulate and check their work for progress

- Students will begin to add and subtract fractions that have a monomial denominator with a variable in the numerator and/or denominator. They will make a common denominator
    - Prior to this review how to make a common denominator like 1/4 + 5/7
- Students will now factor each denominator and discuss how to make a common denominator when the denominator has a binomial expression.
    - When adding, make sure to distribute when making the common denominator and add like terms properly
    - When subtracting, make sure to distribute when making common denominators, but then properly distribute the subtraction sign to the ENTIRE expression after the subtraction sign. This is where the majority of errors will occur

- Increase the difficulty of the factoring to make common denominators
    - Complete problems where foiling will be necessary to simplify in the numerator for the common denominator

Day 16 - Practice, practice, practice!!

- Quick quiz
- After the quick quiz complete the practice puzzle in partners/small groups
- Begin compound fractions
    - Step 1: move negative exponents if present
    - Step 2: make one fraction in the numerator and one fraction in the denominator
        - Simplify each part
    - Step 3: change the problem to a multiplication problem (show them how the fraction bar means the division symbol)
    - Step 4: factor and reduce/simplify

Day 17: More practice on compound fractions -- increase the difficulty so that foiling will be required, add in negative exponents

- Review all rational expressions/prep for quiz
- Begin rational equations
    - Students will perform simple tasks like cross-multiplying
        - Students will distribute to solve
        - Students will need to FOIL then get all terms to one side and factor to solve
        - Students will check their solution to ensure that their answer is not an extraneous solution or excluded value. Discuss the meaning of this and how it causes a zero denominator.
    - Students will be able to find the least common denominator
    - Students will multiply the entire equation by the least common denominator. Show that this is over one and remind the students that we multiply fractions going across.
        - Cancel, there should no longer be a denominator in the problem
        - Solve for x (factoring could be involved)
        - Check your answer(s) for any extraneous solutions.
    - Have students start stating the excluded values of x before they start the problem

Day 18 - Quiz between the topics

Day 19 - Practice rational equations and consider having a quick quiz when done.

Day 20 - Students will solve power equations

- Students will solve for problems like $5x^{2/3} - 2 = 43$
    - Students will isolate the x

- Students will raise both sides to the reciprocal power (and recall how to rewrite the problem from the exponent format into the radical format)
- Students will ask if the original power is an even # over an odd # or not. If it is, then they will put +/- on the other side when solving.
- Have students check their solutions so that they can see why they may have needed two solutions. Discuss how taking an odd root but then raising it to an even power will result in the same solution whether or not the odd root had been negative or positive.
- Have students realize and discuss when there is no possible solution (for example x^1/2 + 2 = -6

Day 21 - Quick quiz

- Word problems -- students will define their variable for x and create an equation based on the word problems.
- Create a table using the words then compare to make the expression
- Complete practice problems

Day 22 - Unit review

Day 23 - Unit test

- It may be considerable to break this unit up into two smaller tests

Materials

Core instructional materials:Core Book List including Algebra & Trigonometry 4E by Stewart

Supplemental materials: Khan Academy, Edia, and DeltaMath

- District approved textbook and ancillary materials.
- Online programs: Khan Academy, Desmos, Edia, Ed Puzzle, Delta Math
- Teacher created activiites
- Teacher created notes

Integrated Accommodation & Modifications

Integrated Accommodation & Modifications for Algebra 2/Intro to Trig Honors