Unit 03 Quadratic Functions and Equations

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 1**
Length: **17 days**
Status: **Published**

Brief Summary of Unit

The major work of this chapter is understanding characteristics of quadratic functions, solving for x in a quadratic equation, determining the discriminant value, graphing quadratic equations that are in the vertex form and standard form. Students will solve by factoring, the quadratic formula, find imaginary solutions, simplify imaginary numbers, and rationalize a denominator that has the imaginary number in it. Furthermore, we will take a quadratic that is in standard form and complete the square to put it into the vertex form. We will write equations of parabolas, find the x- and y-intercepts, determine if the graph opens up or down based on the "a" value, and based on where the graph is located and the direction, we will determine if there are any real x-intercepts. Modeling with quadratic functions and graphing transformations of quadratic functions are also included in this chapter. The same transformations from the first chapter are reviewed with quadratic functions. The vertex of an absolute value function and the vertex of a quadratic function are key points that help with graphing. Where is the function increasing or decreasing? Where is the axis of symmetry? What is the maximum or minimum value of the function? There are three common forms in which quadratic functions are written, and each gives information about the graph and the behavior of the function. Understanding the connection between the characteristics of a quadratic function and its equation can help students apply their knowledge when working with real-life applications.

Revised Date: July 2025

Standards

ELA.K-12.1	Developing Responsibility for Learning: Cultivating independence, self-reflection, and responsibility for one's own learning.
MATH.9-12.N.RN	The Real Number System
ELA.K-12.2	Adapting Communication: Adapting communication in response to the varying demands of audience, task, purpose, and discipline.
ELA.K-12.3	Valuing Evidence in Argumentation: Constructing viable claims and evaluating, defending, challenging, and qualifying the arguments of others.
ELA.K-12.4	Building Knowledge: Building strong content knowledge and connecting ideas across disciplines using a variety of text resources and media.
MATH.9-12.A.APR.B	Understand the relationship between zeros and factors of polynomials
MATH.9-12.N.RN.A.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
MATH.9-12.N.RN.A.3	Simplify radicals, including algebraic radicals (e.g., $\sqrt[3]{54} = 3\sqrt[3]{2}$, simplify $\sqrt{32x^2}$).
MATH.9-12.S.ID.B.6.a	Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function

	suggested by the context. Emphasize linear and exponential models.
MATH.9-12.F.BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\ f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k(both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MATH.9-12.N.CN	The Complex Number System
MATH.9-12.N.CN.A	Perform arithmetic operations with complex numbers
MATH.9-12.N.CN.A.1	Know there is a complex number i such that $i^2 = –1$, and every complex number has the form $a + bi$ with a and b real.
MATH.9-12.N.CN.A.2	Use the relation $i^2 = –1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
MATH.9-12.S.ID.C.8	Compute (using technology) and interpret the correlation coefficient of a linear fit.
MATH.9-12.N.CN.A.3	Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
MATH.9-12.A.CED.A	Create equations that describe numbers or relationships
MATH.9-12.A.CED.A.1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
MATH.9-12.N.CN.B.4	Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
MATH.9-12.A.CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MATH.9-12.N.CN.B.5	Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.
MATH.9-12.N.CN.C.7	Solve quadratic equations with real coefficients that have complex solutions.
MATH.9-12.N.CN.C.8	Extend polynomial identities to the complex numbers.
MATH.9-12.A.REI.A	Understand solving equations as a process of reasoning and explain the reasoning
MATH.9-12.A.REI.A.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
MATH.9-12.N.CN.C.9	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
MATH.9-12.A.REI.A.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
MATH.9-12.A.REI.B	Solve equations and inequalities in one variable
MATH.9-12.F.IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MATH.9-12.A.REI.B.4	Solve quadratic equations in one variable.
MATH.9-12.A.REI.B.4.a	Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
MATH.9-12.A.REI.B.4.b	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

MATH.9-12.F.IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
MATH.9-12.F.IF.C.7.c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
MATH.9-12.F.IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MATH.9-12.A.SSE.A.2	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
MATH.9-12.A.SSE.B	Write expressions in equivalent forms to solve problems
MATH.9-12.A.SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
MATH.9-12.A.SSE.B.3.a	Factor a quadratic expression to reveal the zeros of the function it defines.
MATH.9-12.A.SSE.B.3.b	Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

Essential Questions

- How can characteristics of quadratic functions help analyze the end behavior and intercepts of graphs?
- How can we distinguish between the various forms of quadratic graphs?
- How does the domain and range describe the behavior of a quadratic function?
- What are important characteristics of the graph of a quadratic function?
- Why is the vertex very important when analyzing the behavior of a quadratic function in real life settings?

Enduring Understandings

- Add, subtract, multiply, and divide complex numbers.
- Define the imaginary unit "I" and use it to rewrite the square root of a negative.
- Find and interpret the discriminant of an equation.
- Find complex solutions of quadratic equations and complex zeros of quadratic equations.
- Quadratic functions are graphed by a symmetric curve with a highest or lowest point (vertex) corresponding to an absolute maximum or minimum value.
- Solve quadratic equations by completing the square.
- Solve quadratic equations graphically.
- Students will apply the 1, 3, 5 rule from previous graphing in Unit 1 for the parabolas unless the "a" value alters this pattern.
- Students will realize that if there are zero x-intercepts, that the discriminant value must equal a negative value.
- The family of quadratic functions models certain situations where the rate of change is NOT CONSTANT.

Students Will Know

- Students will know how to identify characteristics of the graphs of quadratic functions.
- Students will know that they can model real-world situations with quadratic functions.
- Students will write different forms of equations of parabolas.
- The quadratic formula by either memorizing it or singing the song.
- The quadratic formula is only used with solving x for where the highest power is 2. Redefine the word quadratic vs linear vs cubic to comprehend.
- They can always solve for x by using the quadratic formula.

Students Will Be Skilled At

- Adding, subtracting, multiplying, and dividing with complex numbers.
- Finding and interpreting the discriminant of an equation.
- Knowing that the degree of the polynomial/ of x tells you how many solutions there are to a polynomial.
- Knowing that the square root of -1 is the same as 'i'.
- Simplifying a negative square root with the value of 'i'.
- Solving a quadratic equation by completing the square, the quadratic formula, and using square roots.
- Students will be skilled at applying quadratics to word problems and interpreting data of the word problem.
- Students will be skilled at describing transformations of quadratic functions.
- Students will be skilled at graphing transformations of quadratic functions.
- Students will be skilled at identifying characteristics of quadratic functions and their graphs.
- Students will be skilled at using characteristics of quadratic functions to solve real life problems.
- Students will be skilled at using technology to find a quadratic model for a set of data.
- Students will be skilled at using three forms of quadratic equations (vertex, intercept, standard) to graph quadratic equations.
- Students will be skilled at writing equations of quadratic functions using vertices, points, and x-intercepts.
- Students will be skilled at writing functions that represent transformations of quadratic functions.
- Students will be skilled at writing quadratic equations to model data sets.
- Students will know how to find an x- and y-intercept of a graph.
- Understanding why there is a +/- in front of our solutions, when that is needed and why.

Evidence/Performance Tasks

Assessments

- Formative: Daily assessments using examples from class notes, NJSLA test bank problems, and/or Albert/AP Classroom assessments

- Summative: Teacher-created assessments, NJSLA test bank problems, Big Ideas Math online platform problems, Albert/AP Classroom and/or Big Ideas Math unit assessments
- Benchmark: Quick quizzes teacher created diagnostic assessments in addition to unit assessments from teacher produced assessments
- Alternative Assessments: as needed per IEP/504

- Class discussion of daily topic
- Classwork and homework that assess the essential questions
- Provide alternative means of assessments for certain students
- Quick quizzes
- Teacher Observation
- Tests and quizzes that assess the essential questions
- Written assignments that assess the essential questions that involves providing explanations

Learning Plan

Unit 2 Graphing Quadratic Functions (3 weeks)

2.1 Students will define the vocabulary of classifying a polynomial by the degree and the number of terms (1/2 day)

- Students will FOIL/expand to classify polynomials - students will create a box chart to multiply anything higher than a 2x2.
- Students will also write in factored form when possible.
- Students will complete a chart (check boxes) with the vocabulary to classify (constant, monomial, binomial, trinomial, polynomial, linear, quadratic, cubic, quartic, quintic, etc).

2.2 Intro to solving quadratics (1 day)

- Quick quiz on classifying
- Students will solve for x by factoring
    - Review the zero product property.
- Students will solve for x using the quadratic formula and properly identify the a, b, and c values
    - Why are there two solutions?
    - Review how to simplify a radical.
    - Sing the song to help.
    - Review how to simplify your answer from the quadratic formula. Make sure students simplify the radical first, then reduce the entire fraction last.
    - Show the students how this answer represents two solutions. Have them use their calculator to find the decimal value(s).
    - If the quadratic formula created an answer with integer solutions, show them how it could also have been solved by factoring
- Students will solve for x by completing the square

- Students will complete the square when the "a" value is 1 and again when it is not one.
- Students will balance their equation by adding the value properly to both sides.

2.2 Day 2 (1 day)

- Students will practice solving for x with the above methods.
  - Students will be given one equation (that is factorable). They will be asked to solve that one equation by factoring, then again by the quadratic formula, then again by completing the square. They will see how all 3 methods yield the same solution and the matter of solving may be a preference.
- Students will be able to find the discriminant value and describe the solutions of x.
- Students will find the discriminant and then also solve for x with the quadratic formula to show the values line up and how it relates to the solution.
- Students will make sure they simplify all radicals and properly reduce (many students reduce first and simplify the radical last which is incorrect). Students will use their calculator to convert their quadratic answers into decimals for understanding.

2.2 Day 3 (1 day)

- Quick quiz on solving
- Review solving - quiz to follow on the next day.
- Students will begin to graph quadratics that are not in vertex form by finding the vertex. The method will be x = -b/2a, and then plug in x to solve for y. Students will plot the vertex. They will find the y-intercept and realize that it is always just the "c" value. Students will graph the axis of symmetry from the vertex and reflect the y-intercept over the axis of symmetry. Students will find the x-intercepts by either factoring the original problem or using the quadratic formula. Remind students that the x-intercept is the same as the solution to the equation. It is also called the root, zero, solution, x-intercept. Use the vocabulary throughout the year.
- Students will begin to solve word problems. They will find the maximum or minimum value (how can you quickly tell if there will be a max or min?). Students will apply information to the vertex. For example if 1 x unit is equivalent to $500 then they will multiply their x-value by 500.
  - Students will graph a sketch of the problem to understand what is being asked of them. They will show the axis of symmetry to help with calculations.

2.2 Day 4 (1 day)

- Quick on solving and graphing quadratics - full period

2.2 Day 4 (1 day)

- Students will complete word problems on max/mins.
- Students will take a quick quiz on max/mins.
- Students will begin to solve for equations that lack a "b" term by isolating the x and taking the square root of both sides. They will understand there are two solutions. We will plug in the solutions to show why it works and both solutions must be given. For example: $x^2 + 20 = 0$

- Students will solve with imaginary solutions as well and plug them in to confirm they work.

2.3 Day 5 and Day 6 (2 days)

- Students will be introduced to the idea of complex numbers. Students will write in a + bi form.
- Students will add and subtract with complex numbers.
- Students will learn about the powers of i and simplify.
  - Students will learn about the value of i^1, i^2, i^3, and i^4. We will use the divide by 4 method to simplify things like i^30, i^21
- Students will multiply with powers of i.
- Students will divide with a monomial and then a binomial value of I. They will realize that they need to use the conjugate as i represents a radical and you cannot leave a radical in the denominator.
  - All fractions must be properly simplified
- Put together different types of problems: (2 + 3i)^2 multiplied by the fraction (1 + 8i/3-7i)
- Students will spend a period just practicing.

2.3 Day 7 (1 day)

- Students will simplify radicals with imaginary values in them.
- Students will FOIL with imaginary numbers but first simplify all radicals with imaginary numbers. We will go over the correct and incorrect way to multiply and show the different.
  - For example: (1 + square root of -2)(3 - square root of negative 12). Show how the answers are not the same when you just foil right away vs. simplify the radicals first with the imaginary value (which is the correct way).
- Students will use the quadratic formula to solve for x where there are imaginary solutions.

2.3 Day 8 and 9

- Review 1 day
- Quiz 1 day / begin the item below

2.4 Day 10 Transformations - refresh (1/2 day)

- Students will apply knowledge from the previous chapter to quadratic functions, using x^2 as the new parent function.
- Transformations will include: horizontal and vertical translations, reflections over both x and y axis, and vertical stretches.
- Students will graph a transformed equation.
- Students will write an equation, given a graph.
- Given a description of transformations, students will name the equation and graph the equation.

- Students will name the vertex as a characteristic of the graph and identify the values for (h, k) in the equation.

- Students will recognize y=a(x-h)2 +k as the vertex form of a quadratic equation.

Day 11 and Day 12: Other methods of solving

- Students will solve equations by other methods (which we have seen before but now we are putting it all together)
    - Solve by factoring
        - Refresh how to solve by grouping
    - Solve by multiplying out the LCD
        - check for extraneous solutions
    - Solve when there is a radical on one side of the equation
        - Square both sides
        - FOIL as needed
        - Factor to solve
        - Check for extraneous solutions
    - Solve when there is a radical on both sides on no other constants
    - Solve when there is a radical on both sides but there is a constant
        - Show how you can either bring both radicals to one side, square each side, but then FOIL
        - Show how you can leave the problem as is, square both sides, and FOIL
        - Be sure to check your solution(s).
    - Solve 4th degree polynomials
        - Factor the problem if it has a GCF or is a trinomial
            - Break up the leading term of the trinomial into x^2
            - Continue to factor as needed
    - Solve quadratic types using "u."
        - Students will realize the "a" and "b" term both have a value in common which they will call "u."
            - They will substitute this value of "u" into the equation and solve for "u" by either factoring or the quadratic formula.
            - Students will now take the "u" value and replace it to solve for the x-value of the equation using any of the skills they have been taught.
        - Students may need to factor out a rational exponent like x^2/3 and then use their solving skills properly from this unit and past units.
    - Students will solve for problems when there is a square root inside of another square root. This may involve squaring twice. Some students may recognize using a power of 4 instead if possible.

Day 13 - Practice and quick quiz if possible

Day 14 - Unit review

Day 15 - Unit test or wrap it up with a quiz on the topics that were not assessed at this point. The timeline

may be broken to have a test due to holiday breaks.

Frequent assessments, with next-day results, should be given throughout this chapter as there is a lot of new material that should be memorized.

Each section, in and of itself, could have it's own big assessment.

Real life examples could be used to review for a large chapter test.

Materials

Core instructional materials:Core Book List including Algebra & Trigonometry 4E by Stewart

Supplemental materials: Khan Academy, Edia, and DeltaMath

- District approved textbook and ancillary materials.
- Online resources: Delta Math, Edia, Ed Puzzle, Khan Academy
- Teacher created activiites
- Teacher created notes

Integrated Accommodation & Modifications

Integrated Accommodation & Modifications for Algebra 2/Intro to Trig Honors