Unit 05 Functions

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 2**
Length: **2 weeks**
Status: **Published**

Brief Summary of Unit

Students will spend this unit learning about functions. They will evaluate functions at specific values and expressions, evaluate piecewise functions and graph them, find the domain of a function without a graph, and find the domain and range (with unions) of the graph of a function. They will be able to define a function, the vertical line test, and determine where a function is increasing, decreasing, or constant. To be successful with their piecewise graphing, students will revisit their rules of transformations. We will complete the unit with combining functions, composition of functions, one-to-one functions, and finding their inverses (and proving inverses)

Revised Date: July 2025

Standards

ELA.K-12.1	Developing Responsibility for Learning: Cultivating independence, self-reflection, and responsibility for one's own learning.
MATH.9-12.F.BF	Building Functions
MATH.9-12.F.BF.A	Build a function that models a relationship between two quantities
MATH.9-12.F.BF.A.1	Write a function that describes a relationship between two quantities.
ELA.K-12.3	Valuing Evidence in Argumentation: Constructing viable claims and evaluating, defending, challenging, and qualifying the arguments of others.
MATH.9-12.F.BF.A.1.a	Determine an explicit expression, a recursive process, or steps for calculation from a context.
ELA.K-12.4	Building Knowledge: Building strong content knowledge and connecting ideas across disciplines using a variety of text resources and media.
MATH.9-12.F.BF.A.1.b	Combine standard function types using arithmetic operations.
ELA.K-12.5	Leveraging Technology: Employing technology and digital media thoughtfully, strategically and capably to enhance reading, writing, speaking, listening, and language use.
	For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
MATH.9-12.F.BF.A.1.c	Compose functions.
	For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.
MATH.9-12.F.BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\ f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k(both positive and negative); find the value of k given the graphs.

	Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MATH.9-12.F.BF.B.4	Find inverse functions.
MATH.9-12.F.BF.B.4.a	Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = (x + 1)/(x - 1)$ for $x \neq 1$.
MATH.9-12.F.BF.B.4.b	Verify by composition that one function is the inverse of another.
MATH.9-12.F.BF.B.4.c	Read values of an inverse function from a graph or a table, given that the function has an inverse.
MATH.9-12.F.BF.B.4.d	Produce an invertible function from a non-invertible function by restricting the domain.
MATH.9-12.F.IF	Interpreting Functions
MATH.9-12.F.IF.A	Understand the concept of a function and use function notation
MATH.9-12.F.IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
MATH.9-12.F.IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MATH.9-12.F.IF.A.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ for $n \geq 1$.
MATH.9-12.F.IF.B	Interpret functions that arise in applications in terms of the context
MATH.9-12.F.IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
MATH.9-12.F.IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MATH.9-12.F.IF.C	Analyze functions using different representations
MATH.9-12.F.IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
MATH.9-12.A.REI.D.11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
MATH.9-12.F.IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Essential Questions

- Can a function exist if two x-values are paired to one y-value? What is one x-value is paired to two y-values?

- How are we able to prove two functions are inverses of each other?
- How can a graph help us interpret a function and its values?
- What is the purpose of the vertical line test?

Enduring Understandings

- A function will only have an inverse if it passes the horizontal line test
- Functions are the baseline for a large part of our course and students will need to learn the proper language and terminology
- Later in the course, we will put restrictions on the values of x to ensure that it can pass as a function, a one-to-one, and therefore have an inverse

Students Will Know

- How to determine a function exists
- How to evaluate functions
- How to find an inverse
- How to graph piecewise functions
- That a function must pass the vertical line test
- That a one-to-one must pass the horizontal line test
- That an inverse can be proven when f(f-1(x)) = x
- The domain of a function is equivalent to the range of the inverse of the same function

Students Will Be Skilled At

- Adding / subtracting / multiplying / dividing functions
- Evaluating a function at a specific value
- Evaluating compositions such as f(g(-3))
- Finding an inverse
- Finding domain and range of functions
- Graphing piecewise functions
- Interpreting data of a function from a graph
- Proving two functions are inverses of each other

Evidence/Performance Tasks

Assessments

- Formative: Daily assessments using examples from class notes, NJSLA test bank problems, and/or

Albert/AP Classroom assessments

- Summative: Teacher-created assessments, NJSLA test bank problems, Big Ideas Math online platform problems, Albert/AP Classroom
- Benchmark: I teacher created diagnostic assessments in addition to unit assessments
- Alternative Assessments: Student-centered activities such as scavenger hunts, various projects involving real world applications, and differentiated learning tasks in Khan Academy, and DeltaMath
- Quick quizzes
- Practice sheets
- Accurate homework completion
- Review sheets completed and checked

- Answer essential questions
- Class discussion of daily topic
- Classwork and homework that assess the essential questions
- Provide alternative means of assessments for certain students
- Teacher Observation
- Tests and quizzes that assess the essential questions
- Written assignments that assess the essential questions that involves providing explanations

Learning Plan

Functions

Day 1

- Define a function
- Create a mapping diagram of the domain to the range and have students decide from the mapping diagram if it is a function or not
- Have students examine various graphs and decide if the graph represents a function based on the vertical line test
- Have students evaluate a function
    - Evaluate with integers, rational numbers
    - Evaluate with variables
    - Evaluate with expressions
- Have students find the domain of a function just from an equation and not a graph
    - Define the 4 rules of domain
        - If x is not in a denominator or radical - the domain is all real numbers
        - If x is in a radical then set the value under the radical to be greater than or equal to zero and solve for x
        - If x is in a denominator then solve for x in the denominator and make that the excluded value
        - If x is in both a radical and a denominator then set what it is in the radical in the denominator to just be greater than zero

        - Have students express the domain in interval notation
- Have students evaluate a piecewise function from just the equation(s) and restricted values

Day 2

- Practice problems
- Quick quiz

Day 3

- Refresh the parent functions and their graphs
- Graph piecewise functions
    - Graph the entire component and then erase the part that is not needed from the restriction
    - State the domain and range of the piecewise functions
    - Evaluate the piecewise function at various values using the equation or the graph
- Give students a graph of a piecewise function and see if they can create an equation for the piecewise function that satisfies all conditions
- Make sure students are using open and closed circles properly

Day 4

- Continue graphing piecewise functions and evaluating them
- Review

Day 5

- Quiz

Day 6

- Students will examine graphs of various functions and define where the graphs are:
    - increasing
    - decreasing
    - constant
    - have a local max
    - have a local min
    - define where $f(x) = g(x)$
    - define where $f(x) < g(x)$ and so on

Day 7

- Combining functions
    - find (f + g)(x)
    - find (f - g)(x)
    - find (fg)(x)
    - find (f/g)(x)
    - find f(g(x))
    - find f(f(x))
    - find f(g(3)) and so on
- Give the students a graph (you do not need to a have a defined equation for it) and have them evaluate functions from a graph such as f(g(g(4)))
- Complete practice worksheets

Day 8

- Continue with practice on combining functions and interpreting graphs
- Quick Quiz
- Begin one-to-ones and inverses
    - define a one-to-one
    - give a mapping diagram with the domain and range and have students decide if it is a one-to-one
    - give students a graph and have them examine if it is a one-to-one with the horizontal line test
- Begin to find an inverse
    - basic problems of switch x and y and solving for y
    - problems where there will end up having y in the problem twice and how to solve for it
    - Have students realize that if f(x) has points (-3, 4) on the graph, then the inverse has (4, -3) on the graph and they can use a graph to find the inverse
    - Show that a function and the inverse have symmetry over the line y = x
    - Show that f(x) and f^-1(x) are inverses by proving that f(f^-1(x)) = x and f^-1(f(x)) = x
    - Give the graph of a function (no equation) and have students graph the inverse by finding (x, y) on f(x) and graphing (y, x) for f^-1(x)

Day 9

- Practice inverses
- Practice all work from Day 8

Day 10

- Review

Day 11 - Assessment (quiz or test)

Materials

Core instructional materials:Core Book List including Algebra & Trigonometry 4E by Stewart

Supplemental materials: Khan Academy, Edia, and DeltaMath

- District approved textbook and ancillary materials.
- Graphing utility (calculator or desmos.com)
- Online materials such a as Khan Academy, Delta Math, Edia, Ed Puzzle
- Teacher created activiites
- Teacher created notes

Integrated Accommodation & Modifications

Integrated Accommodation & Modifications for Algebra 2/Intro to Trig Honors