Unit 06 Polynomial Functions and Asymptotes

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 3**
Length: **3 weeks**
Status: **Published**

Brief Summary of Unit

This unit will focus on the end behavior of graphs of polynomials, long division, synthetic division, zeros of a polynomials, and multiplicities. Students will use the rational root theorem to find the zeros of a polynomial. In order to solve and graph polynomial equations, the Rational Root Theorem, Irrational Root Theorem, Complex Root Theorem, the Fundamental Theorem of Algebra and the Principle of Zero Products will be used. Students will recognize the relationship between the degree of a polynomial function and the number of zeros. Students will graph rational functions, find all intercepts, asymptotes, and find the domain and range of the polynomial.

Revised Date: July 2025

Standards

ELA.K-12.1	Developing Responsibility for Learning: Cultivating independence, self-reflection, and responsibility for one's own learning.
ELA.K-12.2	Adapting Communication: Adapting communication in response to the varying demands of audience, task, purpose, and discipline.
MATH.9-12.F.BF.A	Build a function that models a relationship between two quantities
ELA.K-12.3	Valuing Evidence in Argumentation: Constructing viable claims and evaluating, defending, challenging, and qualifying the arguments of others.
MATH.9-12.A.APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
ELA.K-12.4	Building Knowledge: Building strong content knowledge and connecting ideas across disciplines using a variety of text resources and media.
MATH.9-12.A.APR.B.2	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $x - a$ is a factor of $p(x)$.
ELA.K-12.5	Leveraging Technology: Employing technology and digital media thoughtfully, strategically and capably to enhance reading, writing, speaking, listening, and language use.
MATH.9-12.A.APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
MATH.9-12.A.APR.C.4	Prove polynomial identities and use them to describe numerical relationships.
MATH.9-12.F.BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\ f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k(both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

	Include recognizing even and odd functions from their graphs and algebraic expressions for them.
MATH.9-12.F.BF.B.4	Find inverse functions.
MATH.9-12.A.APR.D.6	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x)$ + $r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
MATH.9-12.F.BF.B.4.a	Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = (x + 1)/(x - 1)$ for $x \neq 1$.
MATH.9-12.F.BF.B.4.b	Verify by composition that one function is the inverse of another.
MATH.9-12.F.BF.B.4.c	Read values of an inverse function from a graph or a table, given that the function has an inverse.
MATH.9-12.F.BF.B.4.d	Produce an invertible function from a non-invertible function by restricting the domain.
MATH.9-12.F.IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
MATH.9-12.F.IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MATH.9-12.N.CN.C.7	Solve quadratic equations with real coefficients that have complex solutions.
MATH.9-12.N.CN.C.8	Extend polynomial identities to the complex numbers.
MATH.9-12.N.CN.C.9	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
MATH.9-12.F.IF.B	Interpret functions that arise in applications in terms of the context
MATH.9-12.F.IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
MATH.9-12.F.IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
MATH.9-12.F.IF.C	Analyze functions using different representations
MATH.9-12.F.IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MATH.9-12.F.IF.C.7.a	Graph linear and quadratic functions and show intercepts, maxima, and minima.
MATH.9-12.F.IF.C.7.c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
MATH.9-12.F.IF.C.7.d	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
MATH.9-12.A.SSE.A.2	Use the structure of an expression to identify ways to rewrite it.
CS.K-12.3.a	Identify complex, interdisciplinary, real-world problems that can be solved computationally.

CS.K-12.3.b	Decompose complex real-world problems into manageable sub-problems that could integrate existing solutions or procedures.

Essential Questions

- How are operations and properties of complex numbers related to those of real numbers?
- How do polynomial functions model real-world problems and their solutions?
- How do the characteristics of a polynomial equation appear in its graph?
- How do we determine the end behavior of a graph?
- What is the difference between a hole and an asymptote?
- What is the most efficient method for solving polynomial equations?
- What occurs in a function's equation for us to state that there is a vertical asymptote?

Enduring Understandings

- The characteristics of polynomial functions and their representations are useful in solving real-world problems.
- The domain and range of polynomial functions can be extended to include the set of complex numbers.

Students Will Know

- Students will know how factors, roots, zeros, and x-intercepts are related to each other and to the graph of a polynomial function.
- Students will know how polynomial functions can be used to model real-world applications.
- Students will know how to add, subtract, multiply, divide and factor polynomials.
- Students will know how to determine if a value is a factor of a polynomial.
- Students will know how to find the asymptotes, holes, and end behavior of a polynomial.
- Students will know how to graph a polynomial function.
- Students will know that all polynomials can be classified by specific characteristics.
- Students will know that every polynomial function will have defining features (relative minimum/maximum, domain, range, x-intercepts (may be complex), y-intercepts, and end behavior.)
- Students will know when and how to divide with long division vs synthetic division.

Students Will Be Skilled At

- The student will be skilled at adding and subtracting polynomials.
- The student will be skilled at completely factoring polynomials.
- The student will be skilled at describing the end behavior of polynomials.
- The student will be skilled at dividing polynomials by binomials of the form (x-k) using synthetic division.

- The student will be skilled at explaining how solutions of equations and zeros of functions are related.
- The student will be skilled at explaining the Remainder and Factor Theorems.
- The student will be skilled at finding all of the zeros of a polynomial function.
- The student will be skilled at graphing a rational function fully showing the asymptotes, holes, and possible domain and range.
- The student will be skilled at graphing polynomial functions.
- The student will be skilled at identifying and evaluating polynomial functions.
- The student will be skilled at identifying increasing and decreasing intervals.
- The student will be skilled at identifying the degree of a polynomial.
- The student will be skilled at multiplying polynomials and using special product patterns.
- The student will be skilled at solving polynomial equations.
- The student will be skilled at writing polynomial equations when given information about its zeros.
- The student will be skilled explaining the Fundamental Theorem of Algebra.
- The student will be skilled explaining the relationships among the degree of the polynomial function, real roots, and multiplicities.

Evidence/Performance Tasks

Assessments

- Formative: Daily assessments using examples from class notes, NJSLA test bank problems, and/or Albert/AP Classroom assessments
- Summative: Teacher-created assessments, NJSLA test bank problems, Albert/AP Classroom and/or Big Ideas Math unit assessments
- Benchmark: teacher created diagnostic assessments in addition to unit assessments / quick quizzes / homework
- Alternative Assessments: Student-centered activities such as scavenger hunts, various projects involving real world applications, and differentiated learning tasks in Khan Academy, and DeltaMath

- Answer essential questions
- Class discussion of daily topic
- Classwork and homework that assess the essential questions
- Provide alternative means of assessments for certain students
- Teacher Observation
- Tests and quizzes that assess the essential questions
- Written assignments that assess the essential questions that involves providing explanations

Learning Plan

Polynomials

Day 1

- Students will begin with the 4 basic rules of end behavior of a polynomial
    - If the leading term is positive with an even power --> the end behavior is up on both sides
    - If the leading term is negative with an even power --> the end behavior is down on both sides
    - If the leading term is positive with an odd power --> the end behavior is down on the left and up on the right
    - If the leading term is negative with an odd power --> the end behavior is up on the left and down on the right
- Students will find the zeros of the polynomial by factoring and graph the polynomial
    - If there is a multiplicity of 2 show a bounce
    - If there is a multiplicity of 3 show a flat/wiggle
    - Apply the end behavior

Day 2

- 1/2 of the period practice graphing polynomials from Day 1
- 1/4 of the period worksheet review
    - matching
- 1/4 of the period review how to do long division from elementary school so that the basic skills are covered
    - begin long division of a polynomial
    - show how to write the remainder in two ways
- Give students the graph of a polynomial and have them write the equation of the polynomial in factored form nothing any multiplicities

Day 3

- Continue with Long division. Students will use zero place holders for missing terms
- Students will learn synthetic division
    - Explain when synthetic division is allowed to be used and when only long division is to be used
- Use Synthetic Division to divide polynomials by binomials of the form (x-k)
    - make sure zero place holders are used as needed
    - show how to find the powers of the values when rewriting the answer
- Write remainder as the last term of the quotient over the binomial divisor
- Discuss the Remainder Theorem.
- Students should evaluate functions f(k) and divide by x-k to see that the remainder is the same as the output when evaluated
    - Show that when you divide by a value and get a remainder vs plug-in that same value to the

polynomial, the values are the same
- Apply the factor theorem
- Have students show that a value is a factor of a polynomial by showing that the remainder is zero

Day 4

- Have students find the original polynomial when you give them the answer of the division and what you divided by
- Practice
- Quick quiz

Day 5 - Quiz

Day 6

- Give students a solution to the polynomial and have them find the original polynomial (ie if x = 7 and -2 then f(x) = (x - 7(x + 2) and FOIL it out)
- Students will be given complex and radical solutions
    - They will realize that the conjugate must also be a solution (ie if x = 2i then x = -2i must also be a solution)
    - Have students multiply out the polynomial when given the roots to a polynomial when the roots are imaginary, complex, rational, radicals, or integers (Include roots that have multiplicities as well)
- Students will be given a root to a polynomial and have to find the remaining roots
    - They will recognize that the degree of the polynomial tells you the number of roots in the polynomial
    - They will use synthetic division for the root they were given to show that it is a root and the remainder is zero
    - They will use factoring or the quadratic formula to find the remaining roots

- Solve polynomials using factoring and the Zero-Product Property (Include repeated solutions and introduce the concept of multiplicities)

- Recognize that the terms zero, solution and root are synonymous

    - Also, that if k is a solution, then (x-k) is a factor

    - Students should understand that if they are given a factor of a polynomial, then they also know a solution of the polynomial and an x-intercept

Day 7

- Students will begin the Rational Root Theorem
    - Students will find factors of p
    - Students will find factors of q
    - Students will find factors of p/q
    - Students will use the graphing calculator to find which of the possible factors of p/q are actual solutions
        - Use the tables to check
        - Use the calculation key to check
    - Use the graphing calculator to find any other remaining roots from the p/q list
        - Use synthetic division of the correct roots to reduce the polynomial down to a quadratic
        - Use factoring or the quadratic formula to find the remaining roots

- Use Rational Root Theorem to name possible rational solutions

- Use Synthetic Division to test possible roots and find all solutions by factoring quotient

- Write polynomials in factored form after finding all solutions

- Discuss implications when the value of the lead coefficient changes from 1 to a different value

    - The total number of possible rational roots increases

    - Students can use technology to find rational roots, then use Synthetic Division to reduce the polynomial

- After using the Quadratic Formula to fully solve a polynomial, decide that irrational solutions are always found in pairs

- Discuss the Irrational Conjugates Theorem

Day 8

- Continue to practice with finding the roots of a polynomial with the help of a graphing calculator
    - Remind students who do not have a graphing calculator at home that they can use Desmos or an online graphing calculator
- Prep and review for quiz

Day 9

- Quiz
    - Give in two parts: Part 1 with a graphing calculator and Part 2 without a graphing calculator

Day 10

- Begin graphing rational functions

    - Define a rational function and define how to find a hole
    - Define a vertical asymptote and how it is found
        - set the denominator to zero and solve for x
    - Define a horizontal asymptote and how it is found
        - Go over the 3 rules
            - If the powers of x on the leading terms are the same, divide the coefficients
            - If the power of x is larger in the denominator, the HA is y = 0
            - If the power of x is larger in the numerator, the HA is none
                - There will be a slant asymptote (you can do this if there is enough time or skip slants)
- Use the graphing calculator to confirm your graph
- Continue practicing finding x- and y-intercepts

Day 11 - Review

- The degree of a polynomial in standard form is easily identified by the highest exponent
- The degree of a polynomial in factored form can be found by adding the multiplicities of the factors
- The Fundamental Theorem of Algebra states that the total number of solutions of a polynomial equals the degree, when the total number of solutions includes repeated solutions, and pairs of irrational and complex solutions
- Find all zeros of a polynomial by using the Rational Root Theorem and Factoring
- Students will discover the Complex Conjugates Theorem by solving the remaining quadratic
    - Complex solutions always come in +/- pairs like irrational solutions
- Students will write a polynomial in factored form and standard form given the zeros and at least one irrational or complex solution

Day 12 - Quiz or test

Materials

Core instructional materials:Core Book List including Algebra & Trigonometry 4E by Stewart

Supplemental materials: Khan Academy, Edia, and DeltaMath

- District approved textbook and ancillary materials.
- Online materials such as: Khan Academy, Delta Math, Ed Puzzle, Edia
- Teacher created activiites
- Teacher created notes

Integrated Accommodation & Modifications

Integrated Accommodation & Modifications for Algebra 2/Intro to Trig Honors