Unit 07 Exponential and Logarithmic Functions

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 4**
Length: **3 weeks**
Status: **Published**

Brief Summary of Unit

Most of the functions studied so far have involved a variable being raised to a power. The variable is the base and the exponent is constant. For exponential functions, the roles are reversed; the base is constant and the exponent varies. Inverses of exponential functions are logarithmic functions. Exponential growth and decay is often seen in real world examples. Students will also solve for compound interest problems.

Revised Date: July 2025

Standards

ELA.K-12.1	Developing Responsibility for Learning: Cultivating independence, self-reflection, and responsibility for one's own learning.
ELA.K-12.2	Adapting Communication: Adapting communication in response to the varying demands of audience, task, purpose, and discipline.
ELA.K-12.3	Valuing Evidence in Argumentation: Constructing viable claims and evaluating, defending, challenging, and qualifying the arguments of others.
ELA.K-12.4	Building Knowledge: Building strong content knowledge and connecting ideas across disciplines using a variety of text resources and media.
ELA.K-12.5	Leveraging Technology: Employing technology and digital media thoughtfully, strategically and capably to enhance reading, writing, speaking, listening, and language use.
MATH.9-12.F.BF.B.4	Find inverse functions.
MATH.9-12.F.BF.B.4.a	Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.
MATH.9-12.F.BF.B.5	Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.
MATH.9-12.F.IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
MATH.9-12.F.IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MATH.9-12.F.IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MATH.9-12.F.IF.C.7.e	Graph exponential and logarithmic functions, showing intercepts and end behavior.
MATH.9-12.F.IF.C.8.b	Use the properties of exponents to interpret expressions for exponential functions.

	For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.
MATH.9-12.F.IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MATH.9-12.F.LE	Linear and Exponential Models
MATH.9-12.F.LE.A	Construct and compare linear and exponential models and solve problems
MATH.9-12.F.LE.A.1	Distinguish between situations that can be modeled with linear functions and with exponential functions.
MATH.9-12.F.LE.A.1.a	Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
MATH.9-12.F.LE.A.1.b	Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
MATH.9-12.F.LE.A.1.c	Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
MATH.9-12.F.LE.A.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
MATH.9-12.F.LE.A.4	Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
MATH.9-12.F.LE.B.5	Interpret the parameters in a linear or exponential function in terms of a context.
	Scale, Proportion, and Quantity
SCI.HS.PS1.C	Nuclear Processes
	Mathematical and computational thinking in 9–12 builds on K–8 experiences and progresses to using algebraic thinking and analysis, a range of linear and nonlinear functions including trigonometric functions, exponentials and logarithms, and computational tools for statistical analysis to analyze, represent, and model data. Simple computational simulations are created and used based on mathematical models of basic assumptions.
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	Using Mathematics and Computational Thinking
CS.K-12.3.a	Identify complex, interdisciplinary, real-world problems that can be solved computationally.
CS.K-12.3.b	Decompose complex real-world problems into manageable sub-problems that could integrate existing solutions or procedures.

Essential Questions

- How can exponential and logarithmic equations be solved using multiple methods?
- How can exponential equations be rewritten in logarithmic form?
- How can we derive the value of e and why is it important when using exponential functions?

- What are the key features of the graphs of exponential functions and logarithmic functions?
- What is the relationship between compound interest and exponential/logarithmic graphs?
- What is the relationship between exponential and logarithmic functions?

Enduring Understandings

- Both exponential and logarithmic Functions have properties that are closely related. The properties are applied to solve logarithmic and exponential equations.
- Exponential functions are different from linear functions as exponential functions have a constant percent increase or decrease as opposed to Linear Functions that have a constant rate of change.
- The inverse of exponential functions are logarithmic functions.

Students Will Know

- Students will know how to apply logs to solving for time in compound interest.
- Students will know how to find the amount of money in an account after a period of time with the correct formula.
- Students will know how to solve exponential equations by using rules of exponents and equivalent bases, if possible.
- Students will know that the graphs of exponential and logarithmic functions have defining properties: domain, range, x-intercept, y-intercept and asymptote.
- Students will know that the properties of logarithms can be used to simplify logarithmic expressions.
- Students will know that the properties of logarithms can be used to solve logarithmic equations.
- Students will know the relationship between an asymptote and the function's domain and range for exponential and logarithmic functions.

Students Will Be Skilled At

- Students will be skilled at applying the change of base formula.
- Students will be skilled at calculating the interest and time for compound interest problems.
- Students will be skilled at evaluating exponential expressions and equations that model growth and decay in the real world.
- Students will be skilled at explaining the inverse relationship between exponential and logarithmic functions.
- Students will be skilled at graphing exponential and logarithmic functions using technology and algebraically.
- Students will be skilled at identifying key features of the graphs of logarithmic and exponential functions.
- Students will be skilled at rewriting exponential expressions using like bases if possible.
- Students will be skilled at using the properties of logs to simplify expressions and solve equations.
- Students will be skilled at writing equivalent forms of exponential and logarithmic equations, including base e and natural logs.

Evidence/Performance Tasks

Assessments

- Formative: Daily assessments using examples from class notes, NJSLA test bank problems, and/or Albert/AP Classroom assessments
- Summative: Teacher-created assessments, NJSLA test bank problems, Big Ideas Math online platform problems, Albert/AP Classroom
- Benchmark: teacher created diagnostic assessments in addition to unit assessments
- Alternative Assessments: Student-centered activities such as scavenger hunts, various projects involving real world applications, and differentiated learning tasks in Khan Academy, and DeltaMath

- Answer essential questions
- Class discussion of daily topic
- Classwork and homework that assess the essential questions
- Provide alternative means of assessments for certain students
- Teacher Observation
- Tests and quizzes that assess the essential questions
- Written assignments that assess the essential questions that involves providing explanations

Learning Plan

Day 1 -

- Define an exponential function and what it looks like. Give real life examples of when it would be seen
- Students will graph y = a^x and notice the direction of the graph
- Students will graph y = (1/a)^x and see the direction of the graph and how it differs from the previous statement
    - State the horizontal asymptote
- Students will compare y = 2^x with y = 5^x and share what they have in common (the point (0, 1) and the horizontal asymptote) and how they differ (steepness)
- Students will perform transformations of exponential graphs
    - y = 2^x
    - y = -2^x
    - y = 2^x + 3
    - y = 2^(x+3)
- Students will be given the graph of an exponential and will be asked to find the equation
    - Worksheet on matching equations to graphs
- Make sure students are always graphing the horizontal asymptote so that they are showing the translation up/down a specific set of values

Day 2 -

- Students will begin to find the value of "e"
- Students will evaluate (1 + 1/n)^n for increasing powers of "n"
    - Have students plug in:
        - 10
        - 1,000
        - 100,000
        - 1,000,000
        - 1,000,000,000
    - Students will begin to see that the decimal values is no longer changing by much
    - Have students go on their calculator and type in "e^1" and let them see how it is nearly identical to the last plug in value
- Begin compound interest
    - $A = P(1 + r/n)^{nt}$
    - Ask students if they notice that the value in the formula in parentheses reminds them of something? (it is "e")
    - Define annually, semiannually, quarterly, monthly, daily, hourly
    - Students will be reminded to write 9.5% interest as 0.095 on their calculator
    - Students will find the amount of money in an account after a specific amount of time and compounding period
    - Students will decide which is a better deal when comparing the same principal, time but different compounding periods and so on.
    - Students will find the initial deposit (P) when given the outcome of the money invested and work backwards.
        - Use the "ans" button on their calculator to be exact and not a low rounded answer
        - Show how rounding to just two decimal places on the backwards problems vs taking the entire decimal can yield two completely different answers and why we must use the entire value to be exact.
    - $A = Pe^{(rt)}$
        - Discuss what continuously compounded means
        - Compare how the outcome is different than just compounded
        - Discuss college loans, mortgages, credit cards, and which formulas are generally used for each one to calculate interest
        - Explain what is interest
        - Discuss car loans and how a $20,000 car loan paid back after 5 years is not at all $20,000
        - Discuss what a down payment means vs what you borrow
    - $n = ne^{(rt)}$
        - Discuss populations
        - Why are populations/living things increasing exponentially in most cases?
        - Where are populations decreasing?
    - Complete word problems

Day 3 -

- Review and practice!
- Have students solve for the interest rate
- Have students compare loan deals and find the better loan deal

Day 4 - Quiz

Day 5 -

- Students will graph the exponential function y = e^x
- Students will perform transformations to y = e^x
- Complete word problems that involve "e"

Day 6 and 7-

- Begin logarithms
- Students will practice writing from logarithmic form into exponential form
- Students will practice writing from exponential form into logarithmic form
- Students will be able to write into exponential form and solve / evaluate.
    - Example: $\log_3 9 = x$ --> then $3^x = 9$, thus $x = 2$
    - Solve where the answer could be a negative number or a fraction as well since a square root represents a 1/2 power
- Students will write problems into exponential form but make common bases to solve for x
    - Example: $\log_9 27 = x$ --> $9^x = 27$ --> thus $3^{2x} = 3^3$. Cancel the bases out and $2x = 3$ so $x = 3/2$.
- Students will be shown that the common log is in base 10
    - log 100 means log base 10 of 100
- Students will be shown the natural logarithm which is base e
    - ln means log base e
- Students will rewrite ln 5 = x into exponential form (e^x = 5)
- Students will use a calculator to evaluate logs in base 10 and round correctly
- Students will graph a logarithmic equation
    - $y = \log_3 x$
    - $y = \log_3 x + 2$
        - Students will notice the graph is similar yet different from the exponential graph
        - Students will notice that the vertical asymptote now changes and not the horizontal
        - Students will compare the graphs of $y = \log_3 x$ to y = 3^x and notice that they are inverses of each other
        - We will compare the points on each graph and notice that they are (x, y) on the first but then (y, x) on the other graph
        - We will also notice that the horizontal asymptote of the one graph is the vertical asymptote of the other graph

Day 8 -

- Practice problems independently
- Practice problems in pairs

- Quick quiz

Day 9

- Students will perform the Change of Base Formula and evaluate a log to a specific amount of decimal places
    - They will practice using the ANS key to plug it back in to verify that it is correct
- Students will learn the 3 Laws of Logarithms
    - log (AB) = log A + log B
    - log(A/B) = log A - log B
    - log A^C = C log A
    - Discuss how these rules are similar to the rules of exponents for example $x^6/x^2 = x^4$ because we subtract the powers on division just like in the log rules
- Students will expand logarithms
    - Radicals will be rewritten as fractional powers (square roots as 1/2)
- Students will condense logarithms from multiple logs into a single log
- Complete practice sheets

Day 10 -

- Full period practice on expanding and condensing, Change of Base, and basic evaluating
- Quick quiz

Day 11 - Solving log equations

- Solve equations like 3^(x + 1) = 9^(2x - 3) by making common bases of 3 and solving algebraically
- Solve equations where there is a log on each side of the equation (cancel the logs)
- Solve equations where there is a log on one side and a log and a constant on the other side (get the logs to one side, rewrite into exponential form and solve for x)
- Solve where an equation is simply in logarithmic form, rewrite into exponential form, and solve for
- Solve for problems that are in exponential form and require taking the log of both sides to solve them (Show how this is just like Change of Base Rule)
    - Example: $3^{x+2} = 7$
    - Example: $4^{3x+1} = 11^{x-2}$
        - Check your answers by plugging it in
- Solve equations that have "e" in them -- you should solve by taking the "ln" of both sides as the "ln of e" is just 1 and it will cancel easily
- Solve trinomial equations that have "e" in them by factoring with "e". Make sure you check your work as ln (-2) and ln (0) are not possible
- Solve log equations where you must condense logs together first using your laws of logs, and then cancel logs

Day 12 - Continue solving log equations of all types

Day 13 -

- More practice with solving log equations
- Quick quiz
- Begin to solve compound interest formulas again but this time solve the problem for time
- Make sure to take the log or ln of both sides when solving
- Make sure to use the proper interest equation
- Solve other styled word problems that would require logs

Day 14 - Quiz

Day 15

- If there is time, solve half-life /radioactive decay problems

Day 16 - Test or move on to the next unit

Materials

Core instructional materials:Core Book List including Algebra & Trigonometry 4E by Stewart

Supplemental materials: Khan Academy, Edia, and DeltaMath

- District approved textbook and ancillary materials.
- Graphing utility (online or calculator).
- Khan Academy, Delta Math, Edia, Ed Puzzle.
- Teacher created activiites
- Teacher created notes

Integrated Accommodation & Modifications

Integrated Accommodation & Modifications for Algebra 2/Intro to Trig Honors