Unit 10 - Circles

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 4**
Length: **4-5 weeks**
Status: **Published**

Brief Summary of Unit

Students will recall general information about circles from middle school in order to extend into new knowledge. This information will be applied to angles and segments within and outside of a circle. Students will also explore how to find circles on the coordinate plane.

Revision Date: July 2024

Standards

MATH.9-12.G.C.A.1	Prove that all circles are similar.
MATH.9-12.G.C.A.2	Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
MATH.9-12.G.C.A.3	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
MATH.9-12.G.C.A.4	Construct a tangent line from a point outside a given circle to the circle.
ELA.L.KL.9–10.2.A	Acquire and use accurately general academic and domain-specific words and phrases, sufficient for reading, writing, speaking, and listening at the college and career readiness level.
MATH.9-12.G.CO.A.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
MATH.9-12.G.CO.D.13	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
MATH.9-12.G.GPE.A.1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
MATH.9-12.G.GPE.B.4	Use coordinates to prove simple geometric theorems algebraically.
ELA.SL.PE.11–12.1.A	Come to discussions prepared, having read and researched material under study; explicitly draw on that preparation by referring to evidence from texts and other research on the topic or issue to stimulate a thoughtful, well-reasoned exchange of ideas.
WRK.9.2.12.CAP.3	Investigate how continuing education contributes to one's career and personal growth.
WRK.9.2.12.CAP.6	Identify transferable skills in career choices and design alternative career plans based on those skills.

Essential Questions

- How can you use the Arc Addition Postulate to solve for unknown angles?
- How do you draw an inscribed polygon? Inscribed right angle? Inscribed quadrilateral?
- How do you find the center and radius of a circle's equation when it is written in polynomial form?
- How do you solve segment lengths when a secant and a tangent intersect?
- How do you solve segment lengths when two chords intersect?
- How do you solve segment lengths when two secants intersect?
- What are all the different parts of a circle? Are any of them related?
- What are congruent arcs and what important role do they hold when analyzing circles?
- What is the equation of a circle? What information do you need to write it?
- What is the relationship between a central angle and its arc?
- What is the relationship between an exterior angle and its arcs?
- What is the relationship between an inscribed angle and its arc?
- What is the relationship between an interior angle and its arcs?
- What makes tangents congruent when relating to the properties of circles?
- When can you use the Pythagorean Theorem in circles when solving various geometric problems?

Enduring Understandings

- Identify lines and segments that intersect circles and use them to solve problems.
- Understand and apply theorems about chords to solve for unknown measures.
- Understand angles formed by chords, secants, and tangents to solve for unknown measures.
- Understand arc measures and similar circles to solve for unknown measures.
- Understand equations of circles to analyze the radius, diameter, and center.
- Use properties of inscribed angles and inscribed polygons to solve for unknown measures.
- Use theorems about segments of chords, secants, and tangents to solve for unknown measures.

Students Will Know

- How to find angle and arc measures in circles.
- How to identify lines and segments that intersect circles.
- How to use circle relationships to solve problems.
- How to use circles to model and solve real-life problems.

Students Will Be Skilled At

- Constructing a square inscribed in a circle.
- Describing the relationship between a diameter and a chord perpendicular to a diameter.
- Drawing and identifying common tangents.

- Finding angle measures of inscribed polygons.
- Finding arc measures.
- Finding lengths of segments of chords.
- Finding measures of inscribed angles and intercepted arcs.
- Finding the center of a circle given three points on the circle.
- Graphing equations of circles and writing coordinate proofs involving circles.
- Identifying and find angle and arc measures determine by chords, secants, and tangents.
- Identifying and finding lengths of segments of secants and tangents.
- Identifying congruent arcs.
- Identifying special segments and lines that intersect circles.
- Proving that all circles are similar.
- Using chords of circles to find arc measures.
- Using chords of circles to find lengths.
- Using circumscribed angles to solve problems.
- Using properties of tangents to solve problems.
- Writing equations of circles by finding the center and radius of a circle.

Evidence/Performance Tasks

Assessments

- Formative: Daily assessments using examples from class notes, NJSLA test bank problems, and/or Albert/AP Classroom assessments
- Summative: Teacher-created assessments, NJSLA test bank problems, Big Ideas Math online platform problems, Albert/AP Classroom and/or Big Ideas Math unit assessments
- Benchmark: IXL or teacher created diagnostic assessments in addition to unit assessments from Big Ideas Math
- Alternative Assessments: Student-centered activities such as scavenger hunts, various projects involving real world applications, and differentiated learning tasks in Khan Academy, DeltaMath, and IXL

- Answer essential questions
- Class discussion of daily topic
- Classwork and homework that assess the essential questions
- Provide alternative means of assessments for certain students
- Teacher Observation
- Tests and quizzes that assess the essential questions
- Written assignments that assess the essential questions that involves providing explanations

Learning Plan

The following list is meant to create a day-to-day plan. Teachers are encouraged to slow down or condense days as appropriate to the student population in the class. Assessment(s) should be given when appropriate.

- Begin by having students identify as many vocabulary words regarding a circle as possible. Fill in any gaps of knowledge.
- Define the measure of a central angle and the relationship to its arc. Define the measure of a minor arc, major arc, and semicircle. Use adjacent arcs to set up the Arc Addition Postulate (and relate this back to Segment Addition Postulate and Angle Addition Postulate). Discuss how to determine congruent circles and similar circles. Use vertical angles to introduce congruent arcs.
- Introduce inscribed angles formed by two chords and their intercepted arc to determine each of their measures. Based on this information, discover how to solve for angles in an inscribed polygon. Extend this into theorem about inscribed right angles and inscribed quadrilaterals. Also include a discussion on inscribed angles formed by a tangent and chord.
- Students will likely need additional time to practice solving for these angles and their arc measures. Try to include algebraic equations that require solving quadratics and systems of equations to continue reinforcing students' Algebra 1 skills.
- This lesson should focus on solving for interior and exterior angles. Interior angles are formed by two chords, and students should be able to solve for either an angle or an arc, as well as the interior angle's linear pair. Exterior angle examples should include two secants, two tangents, and one secant and one tangent. Students should be able to solve for either the angle or one of the arcs. With the exterior angle formed by two tangents, discuss how the central angle formed by two radii drawn to the points of tangency is supplementary to the exterior angle.
- Discuss tangents in deeper context. Identify additional theorems regarding tangents and finding lengths of tangent segments. Spiral in the Pythagorean Theorem by using triangles formed by a tangent segment, a radius drawn to the point of tangency, and a secant segment drawn from the exterior point to the center. In these examples students should be able to solve for the tangent length, the secant length, or the radius length.
- Discover multiple theorems regarding chord lengths and the arc measures that they form. Also discuss the Perpendicular Chord Bisector Theorem, again using the Pythagorean Theorem when possible. This triangle solving method can also be applied to the Equidistant Chords Theorem.
- Students will likely need additional time to practice using tangents and chords. Again, continue to use quadratic solving methods and systems of equations, when possible.
- With multiple segments drawn, students should now focus solving the segment lengths when they intersect. Specifically for chord-chord, secant-secant, and secant-tangent relationships. Students should be able to solve for any piece of these segments.
- Introduce the standard equation of a circle, and how to plug in the center point and the radius length. Students should be able to find this equation whether they are given the center and radius, or given the center and a point on the circle. (You may need to review the distance formula.) If time allows, also extend to the given information of the diameter's endpoints. (You may need to review midpoint formula.) Students should absolutely be able to complete the square, and therefore solve for the center and radius when the circle equation is expanded into its polynomial form.

Materials

Core instructional materials: Core Book List including Big Ideas Math Common Core Geometry

Supplemental materials: Khan Academy, Edia, DeltaMath

- District approved textbook
- Khan Academy
- Teacher created activiites
- Teacher created notes
- Whiteboard tables

Suggested Strategies for Modifications

Possible accommodations/modification for Geometry