3 - Deductive Reasoning

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 2**
Length: **6 weeks**
Status: **Published**

Summary of Deductive Reasoning

The unit on deductive reasoning is the heart of the course. In this unit, students will build upon what they have learned about the axiomatic method and inductive reasoning, and they will be introduced to the use of truth tables to examine the validity of a logical argument. Students will learn about direct and transitive arguments, *modus ponens* and *modus tollens*, and the types of logical fallacies associated with deductive reasoning. Deductive reasoning and transitive arguments are used to construct theorems; students will see the application of logical structures and two-column proofs for familiar theorems from algebra and geometry. An exploration into deductive reasoning aligns with the study of the middlegame in chess. In this unit, students will examine middlegame tactics – including pins, forks, and skewers, through the lens of deductive reasoning. Numerous truth tables will be given to students for analysis of logical arguments, as well as countless puzzles with positional chess formations for students to decipher. An understanding of logically valid structures, along with training in how to navigate the complexities of the middlegame in chess, will give students a deeper appreciation for the mathematical reasoning processes inherent in both domains.

Revision Date: July 2023

Essential Questions for Deductive Reasoning

- What is deductive reasoning?
- What are logically valid structures, what are logically invalid structures?
- What are logical fallacies, as they relate to deductive reasoning?
- How can truth tables be used to understand valid logic?
- How is a two-column proof comparable to a transitive argument?
- What are middlegame tactics used in chess?
- How can deductive reasoning be used to better understand the chess middlegame?

Enduring Understandings for Deductive Reasoning

- Deductive reasoning is a logical process in which known general facts are applied to new specific cases.
- If the premises of an argument are true, and they are combined in a logically valid structure, then the conclusion must be true.
- Two or more direct arguments may be combined to form a transitive argument.
- Modus ponens and modus tollens are both logically valid structures.
- Inverse and converse arguments are logically invalid.
- Middlegame tactics include pins, forks, and skewers, and they comprise critical components of a chess player's arsenal.
- Chess tactics can be constructed as a sequence of moves and understood through the lens of deductive

reasoning.

Objectives for Deductive Reasoning

Students Will Know

- How to set up and interpret truth tables.
- How to select appropriate premises for a logical argument.
- How to identify logically valid structures.
- Two-column proofs are a logically valid form of deductive reasoning.
- How to use deductive reasoning to set up middlegame tactics in chess.

Objectives for Deductive Reasoning

Students Will Be Skilled At

- Constructing direct and transitive arguments to reach valid conclusions.
- Identifying invalid structures and finding counterexamples for invalid conclusions.
- Applying deductive reasoning to create pins, forks, and skewers as effective middlegame tactics in chess.
- Studying complex chess positions, solving middlegame puzzles, and explaining the rationale behind move sequences.

Learning Plan for Deductive Reasoning

The concept of logical validity will be presented in the context of real-life examples, and students will engage in guided note-taking activities to practice learning the mechanics of truth tables. In this part of the course, students will be given a lot of repetition in order to gain familiarity and comfort with logically valid structures. During instructional class time, the teacher will present content and clarify concepts, and respond to students' questions. The students will be expected to read the assigned material, as well as the supplementary resources found online. Classroom time will be used for students to collaborate on constructing and interpreting truth tables; working with peers will assist in developing and reinforcing their understanding of deductive reasoning and the ways in which it manifests in mathematical proofs and positions on the chess board. Many class periods will include complicated puzzles requiring students to engage in deductive reasoning to find solutions for optimal move sequences. Both small group and whole group discussions will be centered on developing the skills needed to articulate the rationale behind move sequences, the explanations of which contain the language of deductive reasoning.

Evidence/Performance Tasks for Deductive Reasoning

Assessments in this unit consist of many concrete questions with specific answers. Students will be given truth tables to complete, arguments to dissect and analyze for logical validity, and lots of chess puzzles

depicting middlegame scenarios in which students will need to ascertain optimal move sequences. Students will be tested on their ability to recognize and employ middlegame tactics founded upon deductive reasoning. To develop the desired skills in this unit, students will need to repetitively practice the task of deductive reasoning as it relates to math, logic, and chess. Formative assessments will include written tasks which require students to use logic in defense of their answers. The teacher may assign these formative assessments as either entrance or exit tickets, and then utilize the class time that follows to provide immediate feedback about the answers. In addition, daily classroom activities and discussions will serve as formative assessments with the teacher sharing feedback and commentary at the individual and whole group levels. Students will also have opportunities to exchange input with their peers during small group exercises which provide opportunities for them to practice working with the content. As a summative assessment, each student will present thorough explanations of their solutions to complex chess positions while making extensive use of the language of deductive reasoning.

Materials for Deductive Reasoning

Lecture notes and classroom activities designed by instructor
Internet resources, including YouTube instructional videos and teacher-recommended chess education websites.
Book: "Mathematical Reasoning Through Chess" by Dr. Gary Wenger (in progress)

Standards for Deductive Reasoning

Applying skills through chess provides an inclusive learning environment for all students. It promotes teamwork, mutual respect, and learning about each other's points of view. These activities incorporate the following elements:

New Jersey Diversity and Inclusion Law: In accordance with New Jersey's Chapter 32 Diversity and Inclusion Law, this unit includes instructional materials that highlight and promote diversity, including: economic diversity, equity, inclusion, tolerance, and belonging in connection with gender and sexual orientation, race and ethnicity, disabilities, and religious tolerance.

MA.G-CO.C	Prove geometric theorems
MA.G-CO.C.10	Prove theorems about triangles.
MA.G-SRT.B.4	Prove theorems about triangles.
LA.K-12.NJSLSA.L4	Determine or clarify the meaning of unknown and multiple-meaning words and phrases by using context clues, analyzing meaningful word parts, and consulting general and specialized reference materials, as appropriate.
LA.K-12.NJSLSA.L5	Demonstrate understanding of word relationships and nuances in word meanings.
MA.G-GPE.B.4	Use coordinates to prove simple geometric theorems algebraically.
LA.K-12.NJSLSA.L6	Acquire and use accurately a range of general academic and domain-specific words and phrases sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when encountering an unknown term important to comprehension or expression.

TECH.9.4.12.CI.2	Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).
TECH.9.4.12.CI.3	Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1).
TECH.9.4.12.CT	Critical Thinking and Problem-solving
TECH.K-12.P.4	Demonstrate creativity and innovation.
TECH.K-12.P.5	Utilize critical thinking to make sense of problems and persevere in solving them.

Suggested Accommodations and Modifications

Link to Google Doc with list of accommodations and modification:

https://docs.google.com/spreadsheets/d/1jqF3mSHC48EXTGESYLOmnO3ZbM_R5_etPyYULfrQhwE/edit#gid=1426178898