4 - Abstract Reasoning

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 2**
Length: **5 weeks**
Status: **Published**

Summary of Abstract Reasoning

As a capstone to the previous three units, students are presented with the concept of abstract reasoning, and this includes both creative and analytic skills. Abstract reasoning is an ability to consider concepts beyond what is physically observed, it is a cognitive mechanism for reaching conclusions in the absence of concrete details. Ideally, students will now raise their ability to solve problems because they can identify patterns, work with logical systems, and think about ideas or concepts that are not immediately observable. With abstract thinking, an individual can understand complex and theoretical ideas and make informed decisions based on the possibilities that exist beyond what is presently known. In this unit, students will engage in sophisticated reasoning within a complex network of ideas. The mathematics content includes a meta-awareness of the four ways to represent a function; the chess content includes a study of the endgame. In chess, proper decision making in the endgame requires careful understanding of positional play in which every single move has significant implications. This unit will show students how mathematical modeling can connect any real-life scenario to the world of mathematics, and how chess moves require analysis, creativity, and the ability to imagine near-infinite possibilities.

Revision Date: July 2023

Essential Questions for Abstract Reasoning

- What are the differences between inductive, deductive, and abstract reasoning?
- How do first principles influence abstract reasoning?
- What are the four ways to represent a mathematical function?
- What are key steps in mathematical modeling?
- How does one make decisions during the endgame in chess?

Enduring Understandings for Abstract Reasoning

- Mathematical functions may be represented four ways.
- Mathematical modeling is a useful process, helpful in solving real-life problems.
- The endgame of chess requires sophisticated thinking, an understanding of which will influence a player's decision making in earlier stages of the game.

Objectives for Abstract Reasoning

Students Will Know

- The four ways to represent a mathematical function.
- How to create mathematical models of real-life situations.
- Checkmate combinations that arise from common endgame scenarios.

Objectives for Abstract Reasoning

Students Will Be Skilled At

- Deconstructing given word problems and creating mathematical models for real-life scenarios.
- Understanding when mathematical relationships are functions of independent and dependent variables and expressing the functions as equations, in tables, as graphs, and in common everyday language.
- Identifying and arranging checkmate combinations.
- Solving endgame puzzles in chess.

Learning Plan for Abstract Reasoning

To begin this unit, the instructor will review fundamental concepts about the axiomatic method, inductive and deductive reasoning processes. The instructor will use class time to present the notion of abstract reasoning and compare it to a student's meta-awareness of the four ways to represent a function. The relationship between independent and dependent variables is an essential topic in secondary mathematics, an understanding of which provides an analogy to comprehend the difference between uninformed and informed simplicity. In this unit, the instructor aims to clarify students' understanding of fundamental concepts, and to provide many classroom opportunities to strengthen their mathematical skills. Recognizing patterns and utilizing valid logical structures will help students better succeed with the content in their other math classes, as well as prepare them to make sound decisions throughout a game of chess. Class periods during the final weeks of this course will alternate among mathematical modeling with word problems, chess puzzles with endgame positions, and the experience of playing complete chess matches against their peers, followed by post-game analysis reports.

Evidence/Performance Tasks for Abstract Reasoning

Formative assessments will include classroom activities and discussions, with immediate feedback from the instructor. Short written quizzes will be administered and reviewed during class time to identify any challenges that students may have with their comprehension of these topics. Additionally, students may have the option to verbally articulate their understanding of key concepts or elaborate on their written work. Students will be assessed on their ability to create mathematical models for situations described in word problems, and their ability to successfully express function relationships in four different ways – algebraically, in tables, as a graph, and in words. For the chess component, students will be given endgame puzzles to be solved by identifying the optimal sequence of moves to follow. The growth of a student's chess knowledge will be measured objectively through their performances against a computer program at designated intervals during the course. The online resource *www.chess.com* provides a tool that evaluates a player's strength in any given match; students will have multiple matches recorded against the computer in order to compile the multiple data points needed to chart the growth of an individual player over time. The student's growth, along with their written explanations of their reasoning processes will be used to assess the knowledge they have

gained in this course.

Materials for Abstract Reasoning

Lecture notes and classroom activities designed by instructor
Internet resources, including YouTube instructional videos and teacher-recommended chess education websites.
Book: “Mathematical Reasoning Through Chess” by Dr. Gary Wenger (in progress)

Standards for Abstract Reasoning

Applying skills through chess provides an inclusive learning environment for all students. It promotes teamwork, mutual respect, and learning about each other's points of view. These activities incorporate the following elements:

New Jersey Diversity and Inclusion Law: In accordance with New Jersey’s Chapter 32 Diversity and Inclusion Law, this unit includes instructional materials that highlight and promote diversity, including: economic diversity, equity, inclusion, tolerance, and belonging in connection with gender and sexual orientation, race and ethnicity, disabilities, and religious tolerance.

LA.K-12.NJSLSA.L4	Determine or clarify the meaning of unknown and multiple-meaning words and phrases by using context clues, analyzing meaningful word parts, and consulting general and specialized reference materials, as appropriate.
LA.K-12.NJSLSA.L5	Demonstrate understanding of word relationships and nuances in word meanings.
LA.K-12.NJSLSA.L6	Acquire and use accurately a range of general academic and domain-specific words and phrases sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when encountering an unknown term important to comprehension or expression.
MA.F-BF	Building Functions
MA.F-BF.A	Build a function that models a relationship between two quantities
MA.F-BF.A.1	Write a function that describes a relationship between two quantities.
MA.F-BF.A.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
MA.F-IF.A	Understand the concept of a function and use function notation
MA.F-IF.B	Interpret functions that arise in applications in terms of the context
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.F-IF.C	Analyze functions using different representations
MA.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

MA.F-LE.A	Construct and compare linear and exponential models and solve problems
MA.F-LE.A.1	Distinguish between situations that can be modeled with linear functions and with exponential functions.
MA.F-LE.B	Interpret expressions for functions in terms of the situation they model
TECH.9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
TECH.9.4.12.CI.2	Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).
TECH.9.4.12.CT	Critical Thinking and Problem-solving
TECH.9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
TECH.K-12.P.4	Demonstrate creativity and innovation.
TECH.K-12.P.5	Utilize critical thinking to make sense of problems and persevere in solving them.
TECH.K-12.P.8	Use technology to enhance productivity increase collaboration and communicate effectively.

Suggested Accommodations and Modifications

Link to Google Doc with list of accommodations and modifications:

https://docs.google.com/spreadsheets/d/1jqF3mSHC48EXTGESYLOmnO3ZbM_R5_etPyYULfrQhwE/edit#gid=1426178898