5 - Fair Division and Compensation Arrangements

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 2**
Length: **5 weeks**
Status: **Published**

Summary of Fair Division and Compensation Arrangements

This unit is an extension of the topics in social choice mathematics from the previous unit; students will continue to explore the role of mathematics in decision making and the subjective nature of knowledge. Apportionment is the act of dividing and assigning something according to a proportional distribution. Apportionment is most commonly associated with the process of determining the proportional number of members that each state has in the United States House of Representatives, based on population figures. However, the mathematical principles underlying apportionment may be used in a variety of situations that benefit from an understanding of proportional reasoning and serve to highlight the difference between equality and equity. In this unit, students will also be guided through the process of fairly dividing assets among entitled parties. First, students will examine how to manage the allocation of a single asset, in which one party receives the entirety of an object and must compensate others for their share. Following the study of fair shares, envy-free, and equitable arrangements, students will then learn how to make a fair division of contested items among exactly two sides.

Revision Date: July 2026

Essential Questions for Fair Division and Compensation Arrangements

- What is the best way to allocate resources and provide proportional representation across subsets of a population?
- How are ratios of representation calculated, and what insight do they offer towards understanding the impact of competing apportionment methods as they relate to managing the decimal remainders of quotas?
- What is the difference between an individual’s fair share and an equitable compensation arrangement?
- How can an estate consisting of objects with different perceived values be fairly divided among multiple participants who share entitlements?

Enduring Understandings for Fair Division and Compensation Arrangements

- Apportionment methods offer paths towards establishing proportional representation for subsets of a population and the proportional allocation of limited resources based upon selected factors.
- Equitable treatment is different than equal treatment.
- The perceived value of an object may differ among people.
- Individuals may assign monetary values, based on personal feelings, to objects that must be

shared among multiple parties so that compensation arrangements can be made to those who do not physically receive the objects to which they have entitlement.
- As it relates to the mathematics of fair division, students will learn to accept that an individual's feelings are not just relevant but in fact essential to the process, and as such, quantitative values must be deliberately attached to them.

Objectives for Fair Division and Compensation Arrangements

Students Will Know:

- Throughout history, the U.S. government has utilized several different apportionment methods to determine the number of representatives assigned to each state.
- Two particular methods of apportionment (Hamilton and Lowndes) highlight the necessary attention to detail and the fundamental challenge of managing decimal remainders within the process of proportional allocations, and may be applied in a variety of contexts.
- Among entitled parties, several mathematically-justifiable methods exist in which an impartial referee can select one individual to receive an object and make compensation payments to the other parties.
- For exactly two parties, each side can receive more than half of the perceived value of a basket of items by using the adjusted winner procedure, due to their differences among the perceived values of those items.

Objectives for Fair Division and Compensation Arrangements

Students Will Be Skilled At:

- Determining proportional distributions according Hamilton's and Lowndes' methods of apportionment.
- Calculating an individual's fair share within a compensation arrangement.
- Calculating an individual's envy-free payout within a compensation arrangement.
- Calculating the equitable payouts to all individuals within a compensation arrangement.
- Applying the adjusted winner procedure to allocate a collection of assets among two entitled parties.

Learning Plan for Fair Division and Compensation Arrangements

This unit addresses abstract concepts by exploring the differences between equality, equity, and fairness. Accordingly, the instructor needs to preview the essential questions and provide connections to the essential understandings throughout the unit. During instruction, the teacher will define and develop students' comprehension of relevant terms by consistently integrating models and real-life examples to illustrate the key concepts. Classroom time will consist of cooperative learning activities that emphasize the communication and presentation of mathematically-based decisions that demonstrate how to integrate individual emotions into calculations, in order to achieve a collectively acceptable balance among multiple parties. Whole group discussions should be employed frequently to clarify concepts, identify misunderstandings, and respond to

students' questions. Students are expected to read assigned material and utilize supplementary online resources.

These topics in social choice mathematics are highly compatible with cooperative learning and project-based assessments. During this unit of study, students will be assigned to work in small groups to simulate the real-life applications of apportionment and fair division. Instructional plans for the development and mastery of these concepts will employ the pedagogical practices of backward design and hands-on learning opportunities for students to collaborate with their peers. With these topics, simple problems are presented to students; the questions are easy to understand, however, the challenge and the value of their work lie with their efforts to navigate many possible paths to solution and ultimately agree on an optimal solution that best satisfies the individual and collective interests of all parties involved. Groups will be presented with real-life scenarios requiring applications of apportionment and fair division. After their initial efforts, the instructor will offer insight and guidance into the mechanics of apportionment and fair division so that students may strengthen the mathematical components of their solution plans. Throughout the unit, students will engage in note-taking and group discussions as they develop and reinforce their understanding of key principles.

Evidence/Performance Tasks for Fair Division and Compensation Arrangements

Formative assessments will include classroom activities and discussions, with immediate feedback from the instructor. Short written quizzes, in the form of entrance and exit tickets, will be frequently administered and reviewed during class time in order to identify students' misconceptions and/or struggles with comprehension. Evidence of student learning will be found by comparing students' initial work on a pre-test for the unit, serving as a benchmark, to similar tasks presented at the end of the unit. Prior to formally studying the topics in this unit, students are capable of understanding the questions posed about apportionment and fair division, as such, they can provide initial answers on the pre-test benchmark. After sufficient time working in small groups on collaborative projects for these topics, students will be given a written assessment and the opportunity to present their projects, both of which will serve as summative assessments of the growth in their learning. A significant amount of time in the course, inside and outside the classroom, is earmarked for project-based activities. The work students put into these projects allow the instructor to provide helpful feedback as formative assessments, and also to evaluate their overall understanding of curricular objectives as a summative assessment, upon the presentation of their final solutions.

Materials for Fair Division and Compensation Arrangements

Core instructional materials: Core Book List including district approved textbook: *Mathematical Ideas, 14th ed*. Miller, Heeran, Hornsby, and Heeran. Pearson (2020).

Supplemental instructional materials: Lecture notes and classroom activities designed by instructor. Internet resources, including instructional videos on YouTube, Khan Academy, and teacher-recommended math education websites.

Standards for Fair Division and Compensation Arrangements

Through collaborative learning about various discrete mathematics applications, students will embrace each other's differing points of view and logical reasoning. Through group and class discussions, students will reinforce their comfort with critiquing each other's thought process which incorporates elements of the following:

NJ Diversity and Inclusion Law:
In accordance with New Jersey's Chapter 32 Diversity and Inclusion Law, this unit includes instructional materials that highlight and promote diversity learning, including: economic diversity, equity, inclusion, tolerance, and belonging in connection with gender and sexual orientation, race and ethnicity, disabilities, and religious tolerance.

P.4	Demonstrate creativity and innovation.
	Innovative ideas or innovation can lead to career opportunities.
P.5	Utilize critical thinking to make sense of problems and persevere in solving them.
9.4.12.CI.2	Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).
9.4.12.CI.3	Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1).
9.4.12.CT	Critical Thinking and Problem-solving
MA.N-Q.A	Reason quantitatively and use units to solve problems.
P.9	Work productively in teams while using cultural/global competence.
9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
9.4.12.CT.3	Enlist input from a variety of stakeholders (e.g., community members, experts in the field) to design a service learning activity that addresses a local or global issue (e.g., environmental justice).
MA.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling.
MA.N-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
LA.K-12.NJSLSA.L4	Determine or clarify the meaning of unknown and multiple-meaning words and phrases by using context clues, analyzing meaningful word parts, and consulting general and specialized reference materials, as appropriate.
LA.K-12.NJSLSA.L6	Acquire and use accurately a range of general academic and domain-specific words and phrases sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when encountering an unknown term important to comprehension or expression.
6.2	World History: Global Studies
6.2.3	Age of Revolutions: Political and Industrial Revolutions, Imperialism, Reform and Global Impact (1750–1914)
	Democratic principles concerning universal human rights, concepts of equality, and the commitment to human freedom are commonly expressed in fundamental documents, values, laws, and practices.

Suggested Accommodations and Modifications

Link to Google Doc with list of accommodations and modifications:

https://docs.google.com/spreadsheets/d/1pRh--nhM8IFIomBxghCfN1PrwPPFT7n_y6BvOpAt6nQ/edit#gid=1426178898