1 - Problem Solving

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 1**
Length: **4 weeks**
Status: **Published**

Summary of Problem Solving

The processes of mathematical reasoning may be simplified into two main types – inductive and deductive. Inductive reasoning is a natural human tendency in which we learn to recognize patterns and generalize conclusions based on our observations and experiences; in short, going "from local to global." With deductive reasoning, we employ known facts to new situations, we apply generalized understandings and truths to specific cases; in short, going "from global to local." In this unit, students will strengthen their pattern-recognition skills, learn the difference between inductive and deductive reasoning, and learn how to combine facts within a logical structure in order to reach reliable and valid conclusions. This unit presents students with many opportunities to develop problem-solving heuristics and work within Polya's four-step approach to solving problems. During this unit, students will engage in many cooperative learning activities and develop their mathematical reasoning and communication skills in small groups with their peers.

Revision Date: July 2023

Essential Questions for Problem Solving

- What is the difference between inductive and deductive reasoning?
- What are useful heuristics in problem solving?
- What is Polya's four-step approach to solving problems?

Enduring Understandings for Problem Solving

- Pattern recognition and the act of generalizing observations and experiences are natural human tendencies that lead us to reach conclusions, however, the process of inductive reasoning is based on limited information and such conclusions are not certain.
- Deductive reasoning applies established facts to new cases, and if the premises are true and the logical structure is valid, then a conclusion will be certain.
- It is important to first understand a problem, then devise and execute a plan, and finally, look back and reflect on the solution as it relates to the original question that was posed.
- Many problem-solving heuristics exist that offer paths to solutions; often more than one solution exists for a particular problem.

Objectives for Problem Solving

Students Will Know:

- Strategies for recognizing patterns.
- The difference between inductive and deductive reasoning processes.

Objectives for Problem Solving

Students Will Be Skilled At:

- Employing Polya's four step approach to solving problems.
- Appropriate use of problem-solving heuristics, including
    - Look for a pattern
    - Solve an easier, related problem
    - Work backwards
    - Make systematic lists
    - Deconstruct a problem into smaller pieces
    - Eliminate possibilities, and eliminate irrelevant information
    - Use an equation or formula
    - Represent the information in a different form
        - Draw a diagram
        - Make a table, chart, or graph
        - Consider physical representations, make a model

Learning Plan for Problem Solving

Class time will alternate between brief teacher lectures and longer group learning activities. It is important for the instructor to initially introduce key concepts, yet provide students with frequent opportunities to thoughtfully engage in problem-solving activities with their peers. This unit allows for many hands-on classroom exercises that develop classic heuristics which are transferrable across academic disciplines. This unit emphasizes students having the time to learn about and practice the application of problem-solving techniques within logically valid structures. Students will be guided to continuously work within George Polya's four-step framework for problem solving.

Evidence/Performance Tasks for Problem Solving

This unit will have multiple formative assessments; students will consistently be placed into small groups for cooperative problem-solving activities and they will be evaluated upon the strength of their solutions and the thoughtfulness of their efforts. The instructor will design and assign tasks that deliberately target the use of specific heuristics, and students will be encouraged to work together, communicate, and develop their problem-solving skills. Peer presentations will follow each major task, and students will immediately receive constructive feedback from each other and the instructor. At the conclusion of the unit, a summative written assessment will be administered, and students will be able to choose among a variety of problem-

solving heuristics to reach and defend their solutions. Additionally, students will have the opportunity to verbally elaborate on their written responses to demonstrate their understanding of key concepts.

Materials for Problem Solving

[Core Book List](#) including district approved textbook: *Mathematical Ideas, 14th ed*. Miller, Heeran, Hornsby, and Heeran. Pearson (2020).

Lecture notes and classroom activities designed by instructor.
Internet resources, including instructional videos on YouTube, Khan Academy, and teacher-recommended math education websites.

Standards for Problem Solving

Through collaborative learning about various discrete mathematics applications, students will embrace each other's differing points of view and logical reasoning. Through group and class discussions, students will reinforce their comfort with critiquing each other's thought process which incorporates elements of the following:

NJ Diversity and Inclusion Law:
In accordance with New Jersey's Chapter 32 Diversity and Inclusion Law, this unit includes instructional materials that highlight and promote diversity learning, including: economic diversity, equity, inclusion, tolerance, and belonging in connection with gender and sexual orientation, race and ethnicity, disabilities, and religious tolerance.

MA.F-IF.A	Understand the concept of a function and use function notation
MA.F-IF.B	Interpret functions that arise in applications in terms of the context
MA.F-BF.A	Build a function that models a relationship between two quantities
MA.F-BF.A.1	Write a function that describes a relationship between two quantities.
LA.K-12.NJSLSA.L4	Determine or clarify the meaning of unknown and multiple-meaning words and phrases by using context clues, analyzing meaningful word parts, and consulting general and specialized reference materials, as appropriate.
LA.K-12.NJSLSA.L5	Demonstrate understanding of word relationships and nuances in word meanings.
LA.K-12.NJSLSA.L6	Acquire and use accurately a range of general academic and domain-specific words and phrases sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when encountering an unknown term important to comprehension or expression.
CS.K-12.3.a	Identify complex, interdisciplinary, real-world problems that can be solved computationally.
CS.K-12.3.b	Decompose complex real-world problems into manageable sub-problems that could integrate existing solutions or procedures.
WRK.K-12.P.5	Utilize critical thinking to make sense of problems and persevere in solving them.
WRK.K-12.P.8	Use technology to enhance productivity increase collaboration and communicate effectively.

WRK.K-12.P.9	Work productively in teams while using cultural/global competence.
TECH.9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
TECH.9.4.12.CI.2	Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).
TECH.9.4.12.CI.3	Investigate new challenges and opportunities for personal growth, advancement, and transition (e.g., 2.1.12.PGD.1).
TECH.9.4.12.CT	Critical Thinking and Problem-solving
TECH.9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
TECH.9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
TECH.9.4.12.IML.3	Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8).
	Collaboration with individuals with diverse experiences can aid in the problem-solving process, particularly for global issues where diverse solutions are needed.

Suggested Accommodations and Modifications

Link to Google Doc with list of accommodations and modifications:

https://docs.google.com/spreadsheets/d/1pRh--nhM8IFIomBxghCfN1PrwPPFT7n_y6BvOpAt6nQ/edit#gid=1426178898