

Unit 1: Principles of Probability

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 1**
Length: **5 weeks**
Status: **Published**

Summary of the Principles of Probability

Principles of probability form a foundation for the study of statistics. Hence, the first unit of study in this course addresses the development of key concepts in probability and provides a framework for understanding the elements of uncertainty that are inherent in statistical studies. Probability explores the world of uncertainty. The study of probability is a mathematical way to investigate, calculate, and describe the chances of something happening. More precisely, a probability is the likelihood of an outcome of a random process. Random means without pattern, and a random process is one in which each possible outcome has an equal chance of being selected. Simple probability examines the chances of something happening, in relation to the total number of all possibilities that could happen. With simple probability, we look at one event in isolation. This unit also explores compound probability with multiple events, and conditional probability in which the likelihood of an outcome depends on initial conditions and related events. Students will learn about permutations and combinations, independent and dependent events, mutually exclusive and non-mutually exclusive events, and the use of visual aids to organize compound relationships, including Venn Diagrams, systematic lists, and tree diagrams.

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Analyzing various sets of data will allow students to explore studies about people from different backgrounds. Statistical studies and analysis provides students an opportunity to read about historical statistics about people's cultures. Embracing the diversity within society incorporates the following:

Amistad Commission

This unit also reflects the goals of the Department of Education and the Amistad Commission including the infusion of the history of Africans and African-Americans into the curriculum in order to provide an accurate, complete, and inclusive history regarding the importance of of African-Americans to the growth and development of American society in a global context.

Asian American and Pacific Islander History Law

This unit includes instructional materials that highlight the history and contributions of Asian Americans and Pacific Islanders in accordance with the New Jersey Student Learning Standards in Social Studies.

New Jersey Diversity and Inclusion Law

In accordance with New Jersey’s Chapter 32 Diversity and Inclusion Law, this unit includes instructional materials that highlight and promote diversity, including:

economic diversity, equity, inclusion, tolerance, and belonging in connection with gender and sexual orientation, race and ethnicity, disabilities, and religious tolerance.

Commission on Holocaust Education

This unit further reflects the goals of the Holocaust Education mandate where students are able to identify and analyze applicable theories concerning human nature and behavior; understand that genocide is a consequence of prejudice and discrimination; understand that issues of moral dilemma and conscience have a profound impact on life; and understand the personal responsibility that each citizen bears to fight racism and hatred whenever and wherever it happens.

CS.K-12.3.a	Identify complex, interdisciplinary, real-world problems that can be solved computationally.
CS.K-12.3.b	Decompose complex real-world problems into manageable sub-problems that could integrate existing solutions or procedures.
LA.K-12.NJSLSA.W4	Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.
LA.K-12.NJSLSA.W5	Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach.
MA.S-CP.A.1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).
MA.S-CP.A.2	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
MA.S-CP.A.3	Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A , and the conditional probability of B given A is the same as the probability of B .
MA.S-CP.A.5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
MA.S-CP.B.6	Find the conditional probability of A given B as the fraction of B ’s outcomes that also belong to A , and interpret the answer in terms of the model.
MA.S-CP.B.7	Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.
MA.S-CP.B.8	Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = [P(A)] \times [P(B A)] = [P(B)] \times [P(A B)]$, and interpret the answer in terms of the model.

MA.S-CP.B.9	Use permutations and combinations to compute probabilities of compound events and solve problems.
WRK.K-12.P.5	Utilize critical thinking to make sense of problems and persevere in solving them.
TECH.9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
TECH.9.4.12.CT	Critical Thinking and Problem-solving
TECH.9.4.12.IML.3	Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8).

Essential Questions / Enduring Understandings

- What is the relationship between probability and statistics?
- How does an understanding of probability provide a foundation for studying statistics?
- What are the essential components of simple probability?
- How is the simple probability formula supported by the counting rules, including permutations and combinations?
- How is probability used to make informed decisions about uncertain events?
- How are Venn diagrams used with compound probability?
- How are the words and/or used in determining compound probabilities?
- How does one know whether to use conditional probability with dependent events or compound probability with independent events?
- The principles of probability form a foundation for statistical inference.
- Permutations and combinations are understood through fundamental counting rules and are often necessary for determining the values within probability formulas.
- A probability distribution shows the corresponding probabilities for all values in the sample space, and may be used to answer a variety of probability questions.
- Venn diagrams are useful visual aids for understanding associated values among independent events in compound probability.
- Compound events may be dependent or independent, mutually exclusive or non-mutually exclusive, and these distinctions determine how to proceed forward when calculating compound probability values.

Objectives for the Principles of Probability

Students Will Know

- Applications of the counting rules to determine the total number of possibilities for given situations of simple and compound probability.
- How to compute the simple probability for a single event using the simple probability formula.
- The difference between theoretical and empirical probability.
- How to design and simulate probability experiments.
- How to identify the sample space for all possible outcomes of a random phenomenon.
- The significance of the words “and” & “or” in determining the appropriate mathematical operations for calculating compound probabilities.

- How to construct and interpret a two-way contingency table for conditional probabilities.
- How to identify the difference between independent and dependent events, in order to distinguish between principles of compound and conditional probability.

Objectives for the Principles of Probability

Students Will Be Skilled At

- Constructing Venn diagrams to model independent events within compound probability.
- Using survey results to complete a Venn diagram and distinguishing between the union and intersection of sets.
- Determining the simple probability for a single event.
- Listing the probability distribution to account for all possible probabilities of the sample space for a given random phenomenon.
- Using a probability distribution to construct a histogram.
- Calculating the compound probability of multiple independent events.
- Using two-contingency tables to organize joint and marginal probability distributions for dependent events within the context of conditional probability.

Learning Plan for the Principles of Probability

The study of probability is facilitated with the use of hands-on learning activities and classroom manipulatives that allow for students to experiment with scenarios of simple and compound probability. The instructor will allocate class time to simulate games and situations of chance in order to engage students and create opportunities for them to directly experience random phenomena, and to compare their expectations with their observations. Key vocabulary and essential formulas may be presented via whole group instruction, and students are expected to read through corresponding sections of the textbook, the teacher's guided note-taking handouts, and to explore online resources. Throughout this unit, the instructor will explain how principles of probability appear in real-life applications. Results in probability are often counterintuitive, accordingly, students will be consistently encouraged to reflect on their observations as they relate to their expectations with the classroom activities. Finally, the instructor will facilitate both small group and whole group discussions allowing for students to make personal connections to the essential questions and objectives for enduring understandings with this unit.

Evidence/Performance Tasks for the Principles of Probability

Given that many students have not had significant prior experience with the principles of probability, topics and solutions to exercises in this unit are often both unfamiliar and counterintuitive. As such, the instructor will need to carefully scaffold concepts into concrete components, making extensive use of classroom manipulatives with hands-on learning activities. These same classroom activities will serve as formative

assessments, with the instructor providing immediate feedback – both at the small group level and through whole group discussions. Additionally, short written quizzes in the form of entrance and exit tickets will be frequently administered and reviewed during class time in order to identify students’ misconceptions and/or struggles with comprehension. Through the teacher’s observations and reflections on homework assignments, students will consistently receive guidance as they explore the principles of probability. The unit will conclude with a summative written assessment that addresses concepts in simple, compound, and conditional probability. For the summative assessment, students will have the opportunity to verbally articulate their understanding of key concepts or elaborate on their written responses.

Assessments

- Formative: Daily assessments using examples from class notes, NJSLA test bank problems, and/or Albert/AP Classroom assessments
- Summative: Teacher-created assessments, NJSLA test bank problems, Big Ideas Math online platform problems, Albert/AP Classroom and/or Big Ideas Math unit assessments
- Benchmark: IXL or teacher created diagnostic assessments in addition to unit assessments
- Alternative Assessments: Student-centered activities such as scavenger hunts, various projects involving real world applications, and differentiated learning tasks in Khan Academy, DeltaMath, and IXL

Materials for the Principles of Probability

Core instructional materials: [Core Book List](#) including Mathematical Ideas 14E, Miller, Heeren, & Hornsby, Savvas

Supplemental materials:

Lecture notes and classroom activities designed by instructor

Internet resources, including Khan Academy, YouTube instructional videos and teacher-recommended math education websites