

Unit 3: Trigonometric Equations and Graphs

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 2**
Length: **10 weeks**
Status: **Published**

Brief Summary of Unit

Students will begin by exploring radian and degree measure, and finding co-terminal angles. Conversion between the two measures will be discussed, and special emphasis will be placed on common angles (30° , 45° , 60° , 90° , and their multiples up to 360°). The six basic trigonometric functions will be introduced and discussed in the context of a right triangle and the unit circle. Students will use Pythagorean identities to evaluate trigonometric expressions. Trigonometric functions will then be extended to quadrants II – IV using reference angles. Students will then explore sketching transformations of the graphs of the six basic trigonometric functions building off of techniques introduced in Unit 1, and using their properties; domain, range, amplitude, period, phase shift, and symmetry. Inverse trigonometric functions and their properties will be discussed, and students will sketch the inverse sine, cosine, and tangent functions. Problems involving composition of trigonometric functions will be examined, and students will explore real world problems involving right triangles, bearing, angle of elevation/depression, the Law of Sines, and the Law of Cosines. Fundamental Identities and their role in evaluating trigonometric functions and simplifying trigonometric expressions will be discussed, in preparation for verifying trigonometric identities and solving trigonometric equations. After solving trigonometric equations, the sum, difference, and double angle formulae will be introduced and applied.

Standards

Students will analyze geometric designs which connect to various cultures. Embracing the diversity within society incorporates the following:

Amistad Commission

This unit also reflects the goals of the Department of Education and the Amistad Commission including the infusion of the history of Africans and African-Americans into the curriculum in order to provide an accurate, complete, and inclusive history regarding the importance of African-Americans to the growth and development of American society in a global context.

Asian American and Pacific Islander History Law

This unit includes instructional materials that highlight the history and contributions of Asian Americans and Pacific Islanders in accordance with the New Jersey Student Learning Standards in Social Studies.

New Jersey Diversity and Inclusion Law

In accordance with New Jersey's Chapter 32 Diversity and Inclusion Law, this unit includes instructional materials that highlight and promote diversity, including:

economic diversity, equity, inclusion, tolerance, and belonging in connection with gender and sexual orientation, race and ethnicity, disabilities, and religious tolerance.

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.F-IF.C.7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
MA.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
LA.K-12.NJSLSA.L4	Determine or clarify the meaning of unknown and multiple-meaning words and phrases by using context clues, analyzing meaningful word parts, and consulting general and specialized reference materials, as appropriate.
MA.F-TF.A	Extend the domain of trigonometric functions using the unit circle
LA.K-12.NJSLSA.L5	Demonstrate understanding of word relationships and nuances in word meanings.
MA.F-TF.A.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
MA.F-TF.A.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
MA.F-TF.A.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x , where x is any real number.
MA.F-TF.A.4	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
MA.F-TF.B.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
MA.F-TF.B.6	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
MA.F-TF.B.7	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
MA.F-TF.C.8	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
MA.F-TF.C.9	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to

solve problems.

Mathematical and computational thinking in 9–12 builds on K–8 experiences and progresses to using algebraic thinking and analysis, a range of linear and nonlinear functions including trigonometric functions, exponentials and logarithms, and computational tools for statistical analysis to analyze, represent, and model data. Simple computational simulations are created and used based on mathematical models of basic assumptions.

TEC.K-12.8.1	All students will use computer applications to gather and organize information and to solve problems.
TEC.K-12.8.2	All students will develop an understanding of the nature and impact of technology, engineering, technological design, and the designed world as they relate to the individual society, and the environment.
WORK.K-12.9.1	All students will develop career awareness and planning, employability skills and foundational knowledge necessary for success in the workplace.
WORK.K-12.9.2	All students will develop career awareness and planning, employability skills and foundational knowledge necessary for success in the workplace.

Transfer

- Students will differentiate and integrate trig functions and their inverses in calculus.
- Students will use trigonometry in calculus, evaluating trigonometry-based limits

Essential Questions

- Can any sine function be written as a cosine function and vice versa? Why or why not? What is the graphical explanation? What is the algebraic explanation?
- How can knowledge of a trigonometric function's graph assist in determining the number of solutions to the corresponding equation in a given interval?
- How does knowledge of a trigonometric function's period assist in sketching the function's graph?
- How does knowledge of simplifying trigonometric equations using different identities assist solving those equations?
- How many answers do trig equations have over the real line?
- When applying the Law of Sines/Cosines to a real world situation, how can one tell which to use?
- When solving a triangle using Law of Sines, why are there ambiguous cases for SSA?

Essential Understandings

- A smaller period means more solutions in a given interval, while a larger period means few solutions. For example, in a given interval, if a sine function's period is halved, there will be double the answers as compared to the original sine function.
- In a triangle where two adjacent sides and one of the non-included angles are known, there is more than one triangle that the information could describe.
- One cycle of a trig function's graph should have a length equal to its period.
- Simplifying trig equations using the identities makes it easier to solve the original trig equation.

- Sine functions can be written as cosine functions and vice versa. One is just a horizontal shift of the other. This can be seen on a graph, or proved via cofunction identities.
- The use of the law of sines requires that at least three pieces of information are known which include an angle and its opposite side. If this information is not given or cannot be found, the law of cosines must be used.
- Trig equations have an infinite number of solutions over the real line, due to the cyclic nature of trig functions. To find a finite number of answers, an interval in which to work must be given.

Students Will Know

- Amplitude, period, phase shift, and reflection affect the shape and location of the trigonometric graphs.
- The angle for a given trigonometric function indicates the quadrant in which the angle lies, and therefore the sign of the trigonometric function for that angle.
- The domain and range of the trig functions and their inverses are important when evaluating composite trig functions.
- The graphing calculator, when used carefully, can be useful in double-checking the validity of a trigonometric identity.
- There is no one right way to simplify trigonometric expressions and verify trigonometric identities.
- Using an inverse trig function to assist in solving a trigonometric equation may not yield all solutions to that equation in the given interval.

Students Will Be Skilled At

- Evaluating any of the six trig functions for angles in any quadrant.
- Evaluating composite trig functions by knowing about the domains and ranges of the six trig functions and their inverses.
- How to properly use a graphing calculator to verify trig identities
- Simplifying trig identities and verifying trig identities.
- Sketching the six trig functions.
- Solving trig equations through proper use of inverse trigonometry.

Evidence/Performance Tasks

Assessments

- Formative: Daily assessments using examples from class notes, NJSLA test bank problems, and/or Albert/AP Classroom assessments
- Summative: Teacher-created assessments, NJSLA test bank problems, Big Ideas Math online platform problems, Albert/AP Classroom and/or Big Ideas Math unit assessments
- Benchmark: IXL or teacher created diagnostic assessments in addition to unit assessments from Big

Ideas Math

- **Alternative Assessments:** Student-centered activities such as scavenger hunts, various projects involving real world applications, and differentiated learning tasks in Khan Academy, DeltaMath, and IXL

- Answer essential questions
- Class discussion of daily topic
- Classwork and homework that assess the essential questions
- Provide alternative means of assessments for certain students
- Teacher Observation
- Tests and quizzes that assess the essential questions
- Written assignments that assess the essential questions that involves providing explanations

Learning Plan

- Discuss degree measure versus radian measure and converting between the two.
- Evaluate and graph inverse trigonometric functions and composite trigonometric functions.
- Evaluate trigonometric functions in all quadrants.
- Introduce inverse sine, cosine, and tangent, emphasize domain and range and their relationship to the unit circle.
- Introduce trigonometric functions and their relationship to the right triangle and unit circle.
- Simplify trigonometric expressions using identities.
- Sketch the six trigonometric functions (sine, cosine, tangent, and their reciprocal functions) using amplitude, period, etc.
- Solve real world problems involving angle of elevation, angle of depression, bearing, Law of Cosines, and Law of Sines.
- Solve triangles using Law of Sines and Law of Cosines.
- Solve trigonometric equations over a given interval.
- Use graphing calculator as appropriate, discuss the importance of window settings.
- Use sum, difference, and double angle formulae to evaluate and simplify trigonometric equations, and to verify trigonometric identities.
- Verify trigonometric identities.

Materials

Core instructional materials: [Core Book List](#) including PreCalculus with Limits 5E, Larson & Battaglia, Cengage

Supplemental materials: Khan Academy, Edia, and DeltaMath

- Desmos
- District approved textbook
- Khan Academy
- Teacher created activiites
- Teacher created notes

Suggested Strategies for Modifications

[Possible accommodations/modification for PreCalc Honors](#)