Unit 1: Properties and Graphs of Functions

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 1**
Length: **5 weeks**
Status: **Published**

Brief Summary of Unit

Students will investigate properties of functions; domain, range, symmetry, and operations on functions; addition, subtraction, multiplication, division, composition, and decomposition. Students will examine criteria for a function to be invertible, and how to invert a function. Students will explore the relationships between the graph of a function and an algebraic rule for a function through translation, reflection, and scaling. Students will use these functions as models for real-world problems. Students will also examine difference quotients including the average rate of change of a function, and they will extend the concepts of increasing/decreasing intervals to a myriad of functions. Given a multi-variable function, students will write the function in terms of a requested variable.

Standards

LA.K-12.NJSLSA.L4	Determine or clarify the meaning of unknown and multiple-meaning words and phrases by using context clues, analyzing meaningful word parts, and consulting general and specialized reference materials, as appropriate.
LA.K-12.NJSLSA.L5	Demonstrate understanding of word relationships and nuances in word meanings.
MA.F-BF.A.1b	Combine standard function types using arithmetic operations.
MA.F-BF.A.1c	Compose functions.
MA.F-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MA.F-BF.B.4a	Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.
MA.F-BF.B.4b	Verify by composition that one function is the inverse of another.
MA.F-BF.B.4c	Read values of an inverse function from a graph or a table, given that the function has an inverse.
MA.F-BF.B.4d	Produce an invertible function from a non-invertible function by restricting the domain.
MA.F-IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

MA.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MA.F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
MA.F-IF.C.7a	Graph linear and quadratic functions and show intercepts, maxima, and minima.
MA.F-IF.C.7b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
MA.F-IF.C.7c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
MA.F-IF.C.7d	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
MA.F-LE.B	Interpret expressions for functions in terms of the situation they model
MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.A-SSE.A.1	Interpret expressions that represent a quantity in terms of its context.
MA.A-SSE.A.1a	Interpret parts of an expression, such as terms, factors, and coefficients.
MA.A-SSE.A.1b	Interpret complicated expressions by viewing one or more of their parts as a single entity.
TEC.K-12.8.1	All students will use computer applications to gather and organize information and to solve problems.
TEC.K-12.8.2	All students will develop an understanding of the nature and impact of technology, engineering, technological design, and the designed world as they relate to the individual society, and the environment.
WORK.K-12.9.1	All students will develop career awareness and planning, employability skills and foundational knowledge necessary for success in the workplace.
WORK.K-12.9.2	All students will develop career awareness and planning, employability skills and foundational knowledge necessary for success in the workplace.
	Mathematical and computational thinking in 9–12 builds on K–8 experiences and progresses to using algebraic thinking and analysis, a range of linear and nonlinear functions including trigonometric functions, exponentials and logarithms, and computational tools for statistical analysis to analyze, represent, and model data. Simple computational simulations are created and used based on mathematical models of basic assumptions.

Transfer

- Being able to competently determine the domain of a function is important for Calculus topics such as continuity and differentiability.
- Difference quotients are used in calculus.
- Knowing about function inverses has applications in cryptography.
- When modelling real-world phenomena, it is important to consider the kinds of numbers that make sense in a given situation. Therefore, we must consider domain and range.

Essential Questions

- For function operations, how do the domains of the operand functions influence the domain of the result?
- How can the transformations on a particular parent graph be determined from the function for the graph?
- How does the horizontal line test assist in determining the invertibility of a function?
- What is the relationship between a function's average rate of change over a given interval, and how that function increases/decreases over that interval?

Essential Understandings

- Functions and their inverses are symmetric about the line y = x.
- If a function passes the horizontal line test, that function is called "one-to-one," and is invertible.
- One must consider the domains of the operand functions when performing function operations, since the results are influenced by those domains.
- The sign of a function's average rate of change over a given interval indicates how, on average, a function increased/decreased over that interval. For example, a positive rate of change means that, on average, the function increased over the interval. A positive rate of change does NOT mean that a function necessarily ONLY increased on the interval.
- the various parameters of a function can indicate how that function has been translated, dilated, and/or reflected.

Students Will Know

- Function composition allows us to verify that two functions are inverses of each other.
- Function operations are performed in much the same way that traditional arithmetic operations are, and that function composition involves plugging one function into another.
- Functions that are even are symmetric about the y-axis, and functions that are odd are symmetric about the origin.
- Interval Notation can be a concise way to represent a range of numbers.
- Inverting a function involves switching the x and y-coordinates.
- Piecewiese functions can be portions of graphs about which students have previously learned.
- Substitution can be used to write a multi-variable function in terms of a single variable.
- That the x and y intercepts of graphs have a y-coordinate and x-coordinate of zero, respectively.
- The answers to a difference quotient usually have variables in the answer.

- The domain of function is determined by various kinds of restrictions, like ensuring that there is no division by zero, and no negative radicands.
- The point-slope form of a line and when it is best-used
- The role that various parameters of a function play on transforming a function from its parent graph.
- The slope-intercept form of a line and when it is best-used.
- The x-values and y-values of points on a graph correspond to the input values and output values of the graph's function, respectively.
- There is a relationship between the real zeros of a function and the graph's x-intercepts.

Students Will Be Skilled At

- Determining the symmetry of a function.
- Evaluating a difference quotient.
- Finding the x and y-intercepts of a variety of functions.
- Graphing piecewise functions.
- How to carry out function operations.
- How to describe and sketch transformations of functions.
- How to find function inverses given various pieces of information.
- How to graph a function by substituting some values into its function.
- Identifying the domain and range of a function using interval notation.
- Identifying various restrictions on a function's domain.
- Verifying inverses using composition.
- Writing a multi-variable function in terms of a requested variable.
- Writing the equation of a line in point-slope form given a variety of information.
- Writing the equation of a line in slope-intercept form given a variety of information.

Evidence/Performance Tasks

Assessments

- Formative: Daily assessments using examples from class notes, NJSLA test bank problems, and/or Albert/AP Classroom assessments
- Summative: Teacher-created assessments, NJSLA test bank problems, Big Ideas Math online platform problems, Albert/AP Classroom and/or Big Ideas Math unit assessments
- Benchmark: IXL or teacher created diagnostic assessments in addition to unit assessments from Big Ideas Math
- Alternative Assessments: Student-centered activities such as scavenger hunts, various projects involving real world applications, and differentiated learning tasks in Khan Academy, DeltaMath, and IXL

- Answer essential questions
- Class discussion of daily topic
- Classwork and homework that assess the essential questions
- Graded Do Now assessments on homework and class notes.
- Provide alternative means of assessments for certain students
- Teacher Observation
- Tests and quizzes that assess the essential questions
- Written assignments (questions of the week) that assess the essential questions that involves providing explanations

Learning Plan

- Determine if a relation is a function
- Discuss function inverses
- Discuss intervals for increasing/decreasing
- Discuss relative extrema and how to find them by hand and with a graphing calculator.
- Discuss the average rate of change for a function over a given interval.
- Discuss the characteristics of the following functions: Constant, Identity, Squaring, Cubic, Square Root, Reciprical, Greatest Integer
- Discuss the symmetry of a function
- Evaluate and sketch piecewise functions
- Examine difference quotients
- Examine transformation of the aforementioned parent graphs
- Find the domain and range of a variety of functions/graphs
- Find the zeros of a variety of functions by hand and with a graphing calculator.
- Find x and y-intercepts for a variety of functions
- Graph linear and quadratic equations
- Review function notation with both numerical and algebraic arguments
- Write functions in terms of a requested variable.

Materials

Core instructional materials: Core Book List including PreCalculus with Limits 5E, Larson & Battaglia, Cengage

Supplemental materials: Khan Academy, Edia, and DeltaMath

- Desmos

- District approved textbook
- Khan Academy
- Teacher created activiites
- Teacher created notes

Suggested Strategies for Modifications

Possible accommodations/modification for PreCalc Honors