

# Unit 2: Polynomial and Rational Functions

Content Area: **Mathematics**  
Course(s):  
Time Period: **Marking Period 1**  
Length: **5 weeks**  
Status: **Published**

## Brief Summary of Unit

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Students will investigate properties of quadratics, polynomial functions of higher degree, polynomial and synthetic division, complex numbers, the zeros of polynomial functions, rational functions, and nonlinear inequalities. Nonlinear inequalities will use the functions of the unit as a basis.

## Standards

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MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.A-SSE.A.1	Interpret expressions that represent a quantity in terms of its context.
MA.F-IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$ . The graph of $f$ is the graph of the equation $y = f(x)$ .
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.A-SSE.A.1a	Interpret parts of an expression, such as terms, factors, and coefficients.
MA.A-SSE.A.1b	Interpret complicated expressions by viewing one or more of their parts as a single entity.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.A-SSE.B	Write expressions in equivalent forms to solve problems
MA.A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
MA.K-12.5	Use appropriate tools strategically.
MA.A-SSE.B.3a	Factor a quadratic expression to reveal the zeros of the function it defines.
MA.A-SSE.B.3b	Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.N-CN.A.1	Know there is a complex number $i$ such that $i^2 = -1$ , and every complex number has the form $a + bi$ with $a$ and $b$ real.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.N-CN.A.2	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to

	add, subtract, and multiply complex numbers.
MA.F-IF.C.7c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
MA.N-CN.A.3	Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
MA.F-IF.C.7d	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
MA.A-APR.B.2	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$ , the remainder on division by $x - a$ is $p(a)$ , so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$ .
MA.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
MA.F-IF.C.8a	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
MA.A-APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
MA.N-CN.C.7	Solve quadratic equations with real coefficients that have complex solutions.
MA.N-CN.C.8	Extend polynomial identities to the complex numbers.
MA.N-CN.C.9	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
MA.A-APR.D.6	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$ , where $a(x)$ , $b(x)$ , $q(x)$ , and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.
MA.F-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$ , $kf(x)$ , $f(kx)$ , and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MA.A-REI.B.4a	Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
MA.A-REI.B.4b	Solve quadratic equations by inspection (e.g., for $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$ .
MA.F-LE.B	Interpret expressions for functions in terms of the situation they model
LA.K-12.NJSLSA.L4	Determine or clarify the meaning of unknown and multiple-meaning words and phrases by using context clues, analyzing meaningful word parts, and consulting general and specialized reference materials, as appropriate.
LA.K-12.NJSLSA.L5	Demonstrate understanding of word relationships and nuances in word meanings.  Mathematical and computational thinking in 9–12 builds on K–8 experiences and progresses to using algebraic thinking and analysis, a range of linear and nonlinear functions including trigonometric functions, exponentials and logarithms, and computational tools for statistical analysis to analyze, represent, and model data. Simple computational simulations are created and used based on mathematical models of basic assumptions.
CS.K-12.3.a	Identify complex, interdisciplinary, real-world problems that can be solved computationally.
CS.K-12.3.b	Decompose complex real-world problems into manageable sub-problems that could integrate existing solutions or procedures.

TEC.K-12.8.1	All students will use computer applications to gather and organize information and to solve problems.
TEC.K-12.8.2	All students will develop an understanding of the nature and impact of technology, engineering, technological design, and the designed world as they relate to the individual society, and the environment.
WORK.K-12.9.1	All students will develop career awareness and planning, employability skills and foundational knowledge necessary for success in the workplace.
WORK.K-12.9.2	All students will develop career awareness and planning, employability skills and foundational knowledge necessary for success in the workplace.

## Transfer

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- Being able to algebraically manipulate the function in this unit will help students build their algebra skills which are necessary for calculus.
- Knowledge about the graphs of polynomial and rational functions is needed for analyzing those functions in calculus.

## Essential Questions

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- How does the discriminant reveal information about a quadratic function?
- How is it possible that a rational function can cross its horizontal asymptote (if it has one)?
- What is the rationale behind the rules for determining the end behavior of a polynomial function?
- What is the rationale behind the rules for determining the nature of a rational function's horizontal asymptote?
- What is the rationale behind the solution method for nonlinear inequalities?
- What is the relationship between the degree of a polynomial function and the number of zeros it has?
- What is the relationship between the multiplicity of a zero and the behavior of the graph at the x-intercept for that zero?
- What is the relationship between the zeros of a polynomial function, the x-intercepts of a polynomial graph, and the solutions to a polynomial equation?
- Where does the quadratic formula come from?

## Essential Understandings

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- At an x-intercept where the graph has a zero with even multiplicity, the graph will "bounce." At an x-intercept where the graph has a zero that appears once, the graph cuts through the x-axis. At an x-intercept where the graph has a zero with odd multiplicity higher than one, the graph will "wiggle" through the x-axis.
- For values of x that are increasing large or increasing small, the term with the leading term of the polynomial comes to dominate, outweighing contributions from other terms. Therefore, the graph's behavior as x increases and decreases can be determined solely from the leading term.
- Horizontal asymptotes are boundaries the graph approaches only as x approaches positive or negative infinity. Therefore, it is possible for a graph to cross a horizontal asymptote for values of x that are relatively small in absolute value.

- The degree of the polynomial is equal to the number of zeros the polynomial has, including repeated zeros.
- The discriminant of a quadratic function reveals the nature and type of its solutions, as well as whether or not the quadratic is factorable.
- The quadratic formula can be derived by completing the square on the standard form of a quadratic equation.
- The rationale is similar to that of the rationale for polynomial end behavior.
- The real zeros of a polynomial function correspond to the graph's x-intercepts, which are equal to the real solutions of the polynomial.
- We set the inequalities equal to zero first, because it makes them more easily solvable using known algebraic methods. We then find where the inequality is equal to zero using those known methods and identify any discontinuities. Since a function value can change sign only at its zeros or discontinuities, we choose arbitrary test values in the intervals created by these "critical numbers" and determine the sign of the answer.

## **Students Will Know**

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- A few key points of information are sufficient to sketch the graph of a quadratic function.
- A polynomial function can be sketched by knowing its zeros, intercepts, end behavior, and by picking some test points.
- A quadratic can be solved by taking square roots, factoring, completing the square, and the quadratic formula.
- Knowledge about asymptotes, intercepts, and some test points are sufficient to sketch rational functions.
- Knowledge about the zeros and any discontinuities of nonlinear inequalities can help to solve those inequalities.
- Polynomial division can help to solve polynomial equations, and to determine slant asymptotes for rational functions.

## **Students Will Be Skilled At**

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- Carrying out polynomial division.
- Sketching polynomial functions.
- Sketching quadratic functions.
- Sketching rational functions.
- Solving nonlinear inequalities.
- Solving quadratic equations.

## **Evidence/Performance Tasks**

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### Assessments

- Formative: Daily assessments using examples from class notes, NJSLA test bank problems, and/or

## Albert/AP Classroom assessments

- Summative: Teacher-created assessments, NJSLA test bank problems, Big Ideas Math online platform problems, Albert/AP Classroom and/or Big Ideas Math unit assessments
  - Benchmark: IXL or teacher created diagnostic assessments in addition to unit assessments from Big Ideas Math
  - Alternative Assessments: Student-centered activities such as scavenger hunts, various projects involving real world applications, and differentiated learning tasks in Khan Academy, DeltaMath, and IXL
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- Answer essential questions
  - Class discussion of daily topic
  - Classwork and homework that assess the essential questions
  - Graded Do Now assessments on homework and class notes.
  - Provide alternative means of assessments for certain students
  - Teacher Observation
  - Tests and quizzes that assess the essential questions
  - Written assignments (questions of the week) that assess the essential questions that involves providing explanations

## Learning Plan

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- Briefly review complex numbers.
- Discuss polynomial division
- Examine properties of polynomial functions needed to sketch
- Examine properties of rational functions needed to sketch
- Sketch quadratic functions
- Solve nonlinear inequalities based on polynomial and rational functions
- Solve polynomial equations by factoring
- Solve polynomial equations by using the rational root theorem
- Solve quadratic equations.

## Materials

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Core instructional materials: [Core Book List](#) including PreCalculus with Limits 5E, Larson & Battaglia, Cengage

Supplemental materials: Khan Academy, Edia, and DeltaMath

- Desmos
- District approved textbook
- Khan Academy
- Teacher created activities
- Teacher created notes

## **Suggested Strategies for Modifications**

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[Possible accommodations/modification for PreCalc Honors](#)