Unit 4: Polar Coordinates and Complex Numbers

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 3**
Length: **4 weeks**
Status: **Published**

Brief Summary of Unit

The polar coordinate system is a set of rules for mapping a pair of numbers onto a point in the plane. Polar coordinates and bearings provide the means of describing a position in terms of distance and direction. In this unit we will study the relationship between Polar and Cartesian coordinates as well as their application to physics and engineering.

Standards

LA.K-12.NJSLSA.L4	Determine or clarify the meaning of unknown and multiple-meaning words and phrases by using context clues, analyzing meaningful word parts, and consulting general and specialized reference materials, as appropriate.
LA.K-12.NJSLSA.L5	Demonstrate understanding of word relationships and nuances in word meanings.
MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.N-CN.A	Perform arithmetic operations with complex numbers.
MA.N-CN.A.1	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
MA.N-CN.A.2	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
MA.N-CN.A.3	Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
MA.N-CN.B.4	Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
MA.N-CN.B.5	Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.
MA.N-CN.B.6	Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
TEC.K-12.8.1	All students will use computer applications to gather and organize information and to

	solve problems.
TEC.K-12.8.2	All students will develop an understanding of the nature and impact of technology, engineering, technological design, and the designed world as they relate to the individual society, and the environment.
WORK.K-12.9.1	All students will develop career awareness and planning, employability skills and foundational knowledge necessary for success in the workplace.
WORK.K-12.9.2	All students will develop career awareness and planning, employability skills and foundational knowledge necessary for success in the workplace.

Transfer

- Polar coordinates are used in Calculus when differentiating and integrating functions.
- Polar coordinates are useful in calculating equations of motion in physics.
- Polar coordinates help simplify equations for engineering applications to electric, magnetic and temperature fields.

Essential Questions

- How is trigonometry and the unit circle utilized when operating on complex numbers in polar form?
- What are the differences and similarities between the two-dimensional coordinate systems of polar and cartesian systems?

Essential Understandings

- Every rectangular (Cartesian) point can be represented by a polar point.
- The polar coordinate system is a two-dimensional coordinate system in which each point on a plane is determined by a distance and angle from a reference point.
- The trigonometric form of a complex number (polar form) allows us to multiply, divide, find roots and powers of numbers.
- Trigonometry and the unit circle are used in converting polar and rectangular points.

Students Will Know

- A negative r value relates to going through the pole (rotating 180 degrees) on a polar graph.
- A polar equation does not need to be a function.
- Equations can be converted between polar and rectangular form.
- How to perform mathematical operations such as multiplying, dividing, raise to higher powers and find roots of the trigonometric form of a complex number.
- In converting a polar point to rectangular point, the horizontal and vertical components are found by trigonometry.
- Pythagorean theorem is used to find the distance from the pole (r value) in a polar point.
- The pole (reference point) in the polar system is equivalent to the origin in the Cartesian plane.

- The roots of a complex number can be illustrated with a unit circle in the complex plane. The roots will be complex numbers in polar form having equal moduli and spaced evenly around the unit circle.
- Understanding the properties (domain, range, intercepts) of a rectangular form of a graph assists in graphing a polar equation.

Students Will Be Skilled At

- Converting equations between rectangular and polar form.
- Converting points between the rectangular (Cartesian) and polar form.
- Plotting polar points in the polar coordinate system.
- Using DeMoivre's Theorem to find powers of complex numbers.
- Using the graphing calculator to graph polar equations.
- Working DeMoivre's Theorem in reverse to find roots of complex numbers.

Evidence/Performance Tasks

Assessments

- Formative: Daily assessments using examples from class notes, NJSLA test bank problems, and/or Albert/AP Classroom assessments
- Summative: Teacher-created assessments, NJSLA test bank problems, Big Ideas Math online platform problems, Albert/AP Classroom and/or Big Ideas Math unit assessments
- Benchmark: IXL or teacher created diagnostic assessments in addition to unit assessments from Big Ideas Math
- Alternative Assessments: Student-centered activities such as scavenger hunts, various projects involving real world applications, and differentiated learning tasks in Khan Academy, DeltaMath, and IXL

- Answer essential questions
- Class discussion of daily topic
- Classwork and homework that assess the essential questions
- Graded Do Now assessments on homework and class notes.
- Provide alternative means of assessments for certain students
- Teacher Observation
- Tests and quizzes that assess the essential questions
- Written assignments (questions of the week) that assess the essential questions that involves providing explanations

Learning Plan

- Convert equations between rectangular and polar form.
- Covert coordinate points between polar and rectangular forms.
- Discuss the graphical implications of the roots of a complex number and the unit circle.
- Introduce and apply DeMoivre's Theorem to find powers of complex numbers.
- Introduce and apply the converted form of DeMoivre's Theorem to find the roots of a complex number.
- Plot polar points in the polar coordinate system.
- Review graphing of conic equations.
- Use the graphing calculator to graph the polar form of linear, polynomial, rational and conic equations.

Materials

Core instructional materials: Core Book List including PreCalculus with Limits 5E, Larson & Battaglia, Cengage

Supplemental materials: Khan Academy, Edia, and DeltaMath

- District approved textbook
- Khan Academy
- Teacher created activiites
- Teacher created notes

Suggested Strategies for Modifications

Possible accommodations/modification for PreCalc Honors