Unit 5: Vectors

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 3**
Length: **6 weeks**
Status: **Published**

Brief Summary of Unit

Vectors are used in calculus and physics to describe any quantity that has both a direction and a magnitude. In this unit students will represent vectors both algebraically and geometrically. Vectors will be used to find force, velocity, angles, area and equations of planes. Students will also use vectors to work with parametric equations to show movement along a vector according to time. Students will convert equations between parametric and Cartesian form. Also, students will use vectors to solve physics and navigation (bearing) problems. The unit of vectors will include applications in both two and three dimensions.

Standards

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.N-VM.B.4	Add and subtract vectors.
MA.N-VM.B.4a	Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
MA.N-VM.B.4b	Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
MA.N-VM.B.4c	Understand vector subtraction $\boldsymbol{v} - w$ as $\boldsymbol{v} + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
MA.N-VM.B.5a	Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(\boldsymbol{v}_x, \boldsymbol{v}$ subscript $y) = (c\boldsymbol{v}_x, c\boldsymbol{v}$ subscript $y)$.
MA.N-VM.B.5b	Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $\|\|c\boldsymbol{v}\|\| = \|c\|\boldsymbol{v}$. Compute the direction of $c\boldsymbol{v}$ knowing that when $\|c\|\boldsymbol{v} \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).
LA.K-12.NJSLSA.L4	Determine or clarify the meaning of unknown and multiple-meaning words and phrases by using context clues, analyzing meaningful word parts, and consulting general and specialized reference materials, as appropriate.

LA.K-12.NJSLSA.L5	Demonstrate understanding of word relationships and nuances in word meanings.
TEC.K-12.8.1	All students will use computer applications to gather and organize information and to solve problems.
TEC.K-12.8.2	All students will develop an understanding of the nature and impact of technology, engineering, technological design, and the designed world as they relate to the individual society, and the environment.
WORK.K-12.9.1	All students will develop career awareness and planning, employability skills and foundational knowledge necessary for success in the workplace.
WORK.K-12.9.2	All students will develop career awareness and planning, employability skills and foundational knowledge necessary for success in the workplace.

Transfer

- Denoting both direction and magnitude, vectors are utilized throughout science, especially physics.
- The concept of vectors is used throughout engineering in its applications.
- Vectors provide a speed (magnitude) and direction and as such are used in navigation.

Essential Questions

- How do you describe, compare, and solve problems involving vectors in three-dimensions?
- How do you describe, compare, and solve problems involving vectors in two-dimensions?
- How does a vector quantity compare to a scalar quantity?
- What mathematical operations are possible on vectors?

Essential Understandings

- A dot product of zero indicates that the two vectors are orthogonal.
- Parametric equations show how the x,y and possible z values of a curve change over time.
- Quantities such as force and velocity involve both magnitude and direction and cannot be characterized by a single number. Vectors allow you to show both the magnitude and direction of an object.
- Slope is used to show movement along a line in two dimensions. Vectors can show movement along a line in higher dimensions.
- The direction of an angle can be measured counter-clockwise from the positive x-axis.
- The dot product of two vectors yields a scalar value not a vector.
- The equation for a plane can be written knowing a point in the plane and a vector which is normal to the plane.
- Vector quantities can be added, subtracted and scaled by a multiplier k.
- Vectors can be written in component form (horizontal and vertical components) or as a linear combination.

Students Will Know

- How to apply vectors to solve real world applications (specifically travel applications of speed and direction)
- How to convert 3 dimensional vectors between parametric and rectangular form
- How to convert equations between parametric and rectangular form
- How to determine if 3 dimensional vectors are parallel or orthogonal
- How to determine when and where a vector intersects a plane; sphere; line; conic
- How to find the angle between two 3 dimensional vectors
- How to find the angle between two vectors
- How to find the area of a triangle from vectors
- How to find the cross product of vectors in 3 dimension
- How to find the direction (angle) of a vector
- How to find the dot product between two vectors
- How to find the dot product of a vector in 3 dimensions
- How to find the magnitude of a vector
- How to find the magnitude of a vector in 3 dimensions
- How to perform vector operations (addition, subtraction and scalar multiplication
- How to perform vector operations in 3 dimensions (add, subtract and scalar multiply)
- How to plot points in 3 dimensions
- How to sketch a vector diagram
- How to write 3 dimensional vectors in component form
- How to write a 3 dimensional vector equation
- How to write a vector equation from both its linear form and parametric form
- How to write a vector in component form
- How to write the equation of a plane
- How to write the equation of the sphere
- What determines parallel vectors
- What it means to be an orthogonal vector

Students Will Be Skilled At

- Answers real world scenarios involving vectors (ex. airplane and boat travel)
- Calculating the cross product of 2 three dimensional vectors
- Calculating the dot product for 3 dimensional vectors
- Completing vector operations in 3 dimension
- Converting 3 dimensional vectors between parametric and rectangular form
- Determine if 3 dimensional vectors are parallel or orthogonal or neither
- Determining if vectors are orthogonal or parallel
- Discovering the when and where a vector intersects a plane; sphere; conic
- Find the angle between 2 three dimensional vectors

- Find the direction of a vector
- Finding the angle between two vectors
- Finding the area of a triangle from vectors
- Finding the dot product of two vectors in component form
- Finding the horizontal and vertical components of a vector
- Finding the magnitude of a 3 dimensional vector
- Finding the magnitude of the vector
- Plotting points in 3 dimensions
- Sketch vectors using direction and magnitude
- Sketching a plane in 3 dimensions
- Vector conversions between rectangular and parametric form
- Vector operations which include adding, subtracting and scalar multiplication
- Write three dimensional vectors in component form
- Writing the equation of a plane
- Writing vector equations

Evidence/Performance Tasks

Assessments

- Formative: Daily assessments using examples from class notes, NJSLA test bank problems, and/or Albert/AP Classroom assessments
- Summative: Teacher-created assessments, NJSLA test bank problems, Big Ideas Math online platform problems, Albert/AP Classroom and/or Big Ideas Math unit assessments
- Benchmark: IXL or teacher created diagnostic assessments in addition to unit assessments from Big Ideas Math
- Alternative Assessments: Student-centered activities such as scavenger hunts, various projects involving real world applications, and differentiated learning tasks in Khan Academy, DeltaMath, and IXL

- Answer essential questions
- Class discussion of daily topic
- Classwork and homework that assess the essential questions
- Finding the dot product between two vectors
- Provide alternative means of assessments for certain students
- Sketching vectors with direction and magnitude
- Teacher Observation
- Tests and quizzes that assess the essential questions
- Writing vectors in component form

- Written assignments that assess the essential questions that involves providing explanations

Learning Plan

- 3 dimensional points
- Component form in 3 dimensions
- Component form of a vector
- Cross Product
- Direction of a vector
- Dot product
- Equation of a plane
- Magnitude (size) of vectors in 3 dimensions
- Magnitude of a vector
- Orthogonal vs. parallel vectors
- Parametric Equations
- Real world applications to vectors in 3 dimensions (boat and airplane problems)
- Sketch vectors in 2 dimension
- Vector Conversions
- Vector conversions between parametric and rectangular form
- Vector Equations
- Vector operations
- Vector operations in 3 dimensions
- When and Where questions for vectors in 2 dimensions
- When and where questions in 3 dimensions

Materials

Core instructional materials: Core Book List including PreCalculus with Limits 5E, Larson & Battaglia, Cengage

Supplemental materials: Khan Academy, Edia, and DeltaMath

- District approved textbook
- Khan Academy
- Teacher created activiites
- Teacher created notes

Suggested Strategies for Modifications

Possible accommodations/modification for PreCalc Honors