Unit 07: Logarithms

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 3**
Length: **3 weeks**
Status: **Published**

Brief Summary of Unit

The unit will begin with a study of logarithms and natural logs. Equations will be solved when the variable occurs in the exponent and a common base cannot be found. Students will be introduced to the laws of logarithms that they will use to simplify and expand logarithmic expressions and solve logarithmic equations. Verbal problems involving compound interest rates will be discussed to demonstrate a link to real world problem solving.

Revised Date: July 2025

Standards

ELA.L.SS.11–12.1	Demonstrate command of the system and structure of the English language when writing or speaking.
MATH.9-12.N.RN.A.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
MATH.9-12.N.RN.A.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
ELA.L.VL.11–12.3	Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grades 11–12 reading and content, including technical meanings, choosing flexibly from a range of strategies.
MATH.9-12.F.IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
MATH.9-12.F.IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MATH.9-12.A.REI.A.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
MATH.9-12.F.IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MATH.9-12.F.IF.C.7.a	Graph linear and quadratic functions and show intercepts, maxima, and minima.
MATH.9-12.F.IF.C.7.b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
MATH.9-12.F.IF.C.7.c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

MATH.9-12.F.IF.C.7.d	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
MATH.9-12.F.IF.C.7.e	Graph exponential and logarithmic functions, showing intercepts and end behavior.
MATH.9-12.F.IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
MATH.9-12.F.IF.C.8.a	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
MATH.9-12.F.IF.C.8.b	Use the properties of exponents to interpret expressions for exponential functions.
MATH.9-12.A.SSE.A.1	Interpret expressions that represent a quantity in terms of its context.
MATH.9-12.A.SSE.A.1.a	Interpret parts of an expression, such as terms, factors, and coefficients.
MATH.9-12.A.SSE.A.1.b	Interpret complicated expressions by viewing one or more of their parts as a single entity.
MATH.9-12.F.LE.A.1	Distinguish between situations that can be modeled with linear functions and with exponential functions.
MATH.9-12.F.LE.A.1.a	Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
MATH.9-12.F.LE.A.1.b	Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
MATH.9-12.F.LE.A.1.c	Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
MATH.9-12.F.LE.A.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
MATH.9-12.A.SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
MATH.9-12.F.LE.A.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
MATH.9-12.A.SSE.B.3.a	Factor a quadratic expression to reveal the zeros of the function it defines.
MATH.9-12.F.LE.A.4	Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
MATH.9-12.A.SSE.B.3.b	Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
MATH.9-12.A.SSE.B.3.c	Use the properties of exponents to transform expressions for exponential functions.
MATH.9-12.F.LE.B.5	Interpret the parameters in a linear or exponential function in terms of a context.
WRK.9.2.12.CAP.5	Assess and modify a personal plan to support current interests and post-secondary plans.
WRK.9.2.12.CAP.13	Analyze how the economic, social, and political conditions of a time period can affect the labor market.
TECH.9.4.12.CI.2	Identify career pathways that highlight personal talents, skills, and abilities (e.g., 1.4.12prof.CR2b, 2.2.12.LF.8).
TECH.9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).

Essential Questions

- How is an exponential function related to a logarithmic function?
- What are the properties of logarithmic functions? How can logarithmic functions be used to model real-life phenomena?

Enduring Understandings

- If one was able to compound a sum of money continuously at rate r for a time period t, one would find that the total would approach 2.78^(rt). This irrational number rounded to 2.78 is known as e.
- It is advantageous to compound interest more frequently.
- One can use the laws of logs to simplify logs and solve equations involving logs.
- Taking the ln of both sides is an approach that can be used when solving equations like A=Ao(1+r/n)^t for t.
- The inverse of an exponential function y = b^x is a logarithm: x = logb(y). Thus, they are reflections across the line y = x
- To solve an equation with the unknown in the exponent and 'like bases' cannot be found, one takes the ln of both sides and applies the laws of logs to solve for the unknown.

Students Will Know

- A logarithm is a way to express an exponent.
- Common logs have base 10.
- Natural logs have base e and are expressed ln.
- The properties of logarithms.

Students Will Be Skilled At

- Noticing the similar properties and graphs of exponents and logarithms.
- Using logarithms to solve compounding interest problems.
- Using the properties of logarithms.

Assessment

Assessments

- Formative: Daily assessments using examples from class notes, NJSLA test bank problems, and/or Albert/AP Classroom assessments
- Summative: Teacher-created assessments, NJSLA test bank problems, Big Ideas Math online platform problems, Albert/AP Classroom and/or Big Ideas Math unit assessments
- Benchmark: IXL or teacher created diagnostic assessments in addition to unit assessments from Big

Ideas Math

- Alternative Assessments: Student-centered activities such as scavenger hunts, various projects involving real world applications, and differentiated learning tasks in Khan Academy, DeltaMath, and IXL

- Class discussion of daily topic
- Classwork and homework that assess the essential questions
- Provide alternative means of assessments for certain students
- Teacher Observation
- Tests and quizzes that assess the essential questions
- Written assignments that assess the essential questions that involves providing explanations

Learning Plan

The following list is meant to create a day-to-day plan. Teachers are encouraged to slow down or condense days as appropriate to the student population in the class. Assessment(s) should be given when appropriate.

- Define logarithm equations as another form of exponential equations. Show students how to translate from one form to the other.
- Introduce rules of logarithms. This should include key rules such as: the base must not be 1; the argument must be a positive number; if the base and the argument are equal, the result is 1; and if the argument can be rewritten as a power of the base, the result is the corresponding exponent. Have students discover these rules by example repetition and discovery.
- Introduce properties of logarithms. This should include expanding and condensing logarithm expressions.
- Solve expressions with base ten on the calculator. Explain the change of base formula for solving logarithms that do not have a base ten.
- Introduce natural logarithms and why it is special. Relate all logarithm rules and properties to natural logarithms, including the change of base formula.
- Begin solving various types of logarithms and natural logarithms. This should be split into multiple days, but can be broken up in anyway. The types of equations included should be: one logarithm on each side, only one logarithm on one side of the equation, multiple logarithms equal to one logarithm, multiple logarithms equal to multiple logarithms, and number bases with variable exponents but the bases cannot be manipulated to the same number.
- Use real world problems to demonstrate the use of these rules and properties. Include types of compound interest solving for any variable except n, half life, and growth or decay formulas.

Graphing calculators are encouraged to be used as an extension of these topics.

Materials

Core instructional materials: Core Book List including PreCalculus Enhanced with Graphing Utilities, Sullivan, Savvas

Supplemental materials: Khan Academy, Edia, and DeltaMath

- District approved textbook
- Khan Academy
- Teacher created activiites
- Teacher created notes
- Whiteboard tables

Integrated Accommodation & Modifications

Possible accommodations/modification for CP PreCalc & Trig