Unit 02: Properties and Graphs of Functions

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 1**
Length: **4 weeks**
Status: **Published**

Brief Summary of Unit

The students will investigate properties of functions: domain, range, symmetry, inverses, composite functions, adding/subtracting/multiplying functions, factoring methods, vertical and horizontal shrinking and stretching, and vertical and horizontal translations. There will be an emphasis on the different representations of functions including table of values, graphs and equations, and students will explore these properties through discovery-based activities with the graphing calculator. Piecewise functions will be graphed, knowledge of parent functions, function notation, and word problems will be completed as well.

Revised Date: July 2025

Standards

6.1.12.CivicsPI.14.a

Use evidence to support a claim regarding how the decision to drop the atomic bomb enabled the United States to achieve its strategic goals and led to new power dynamics.

Connect by modeling exponential growth/decay of populations or radioactive decay, and examining ethical use of mathematics in military decision-making.

ELA.L.SS.11–12.1	Demonstrate command of the system and structure of the English language when writing or speaking.
MATH.9-12.F.BF.A.1	Write a function that describes a relationship between two quantities.
MATH.9-12.F.BF.A.1.a	Determine an explicit expression, a recursive process, or steps for calculation from a context.
MATH.9-12.F.BF.A.1.b	Combine standard function types using arithmetic operations.
MATH.9-12.F.BF.A.1.c	Compose functions.
ELA.L.VL.11–12.3	Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grades 11–12 reading and content, including technical meanings, choosing flexibly from a range of strategies.
MATH.9-12.F.BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\ f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k(both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MATH.9-12.F.BF.B.4	Find inverse functions.
MATH.9-12.F.BF.B.4.a	Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.
MATH.9-12.F.BF.B.4.b	Verify by composition that one function is the inverse of another.

MATH.9-12.F.BF.B.4.c	Read values of an inverse function from a graph or a table, given that the function has an inverse.
MATH.9-12.F.BF.B.4.d	Produce an invertible function from a non-invertible function by restricting the domain.
MATH.9-12.F.IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
MATH.9-12.F.IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MATH.9-12.F.IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MATH.9-12.A.REI.B.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
MATH.9-12.A.REI.B.4	Solve quadratic equations in one variable.
MATH.9-12.F.IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MATH.9-12.A.REI.B.4.a	Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
MATH.9-12.A.REI.B.4.b	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
MATH.9-12.F.IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MATH.9-12.F.IF.C.7.a	Graph linear and quadratic functions and show intercepts, maxima, and minima.
MATH.9-12.F.IF.C.7.b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
MATH.9-12.F.IF.C.7.c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
MATH.9-12.F.IF.C.7.d	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
MATH.9-12.F.IF.C.7.e	Graph exponential and logarithmic functions, showing intercepts and end behavior.
MATH.9-12.F.IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
WRK.9.2.12.CAP.5	Assess and modify a personal plan to support current interests and post-secondary plans.
WRK.9.2.12.CAP.13	Analyze how the economic, social, and political conditions of a time period can affect the labor market.
TECH.9.4.2.CI.2	Demonstrate originality and inventiveness in work (e.g., 1.3A.2CR1a).
TECH.9.4.2.CT.2	Identify possible approaches and resources to execute a plan (e.g., 1.2.2.CR1b, 8.2.2.ED.3).

Essential Questions

- How can these functions and their graphs be used as mathematical models to solve real-world problems?

- How do transformations effect a graphed function?
- If a graph has symmetry, what does that mean for the domain and range?
- If functions are mathematically inverses, what does that mean for their graphs?
- What are the characteristics of linear and quadratic functions?

Enduring Understandings

- Any function that is monotonic will be symmetrical with respect to the line y = x with its inverse function.
- Graphing of piecewise functions consists of making tables of values and noting domain restrictions.
- If a function has origin symmetry, then it is an odd function.
- If a function has y-axis symmetry, then it is an even function.
- Indicators that the domain is not the set of real numbers include square roots and denominators with variables.
- The domain is the set of values that 'x' can take on and the range is the set of values, f(x), that are the answers when x is substituted into the rule.
- The horizontal line test is used to determine if an inverse exists. The vertical line test is used to determine if a graph is a function.
- There are simple formulas to find composition of functions as well as how to simplify using the four basic mathematical operations.
- To find the inverse of a function, interchange the x's and the y's and solve the equation for y.

Students Will Know

- A function is a rule that assigns every element in set D, the domain, to one and only one element in set R, the range.
- How to determine based on the equation if a function is vertically/horizontally stretched or shrunk.
- How to represent a function by equations, graphs and tables of values.
- How to simplify functions through the use of the four basic mathematical operations as well as composite functions.
- How to test for symmetry with respect to the x-axis, y-axis, the line y = x and the origin.
- How to utilize a graphing calculator to graph a function and solve an equation.
- The effects of the different constants in the general form for an equation: a●f(x+b) + c
- The methods for simplifying rational expressions and solving rational equations.
- The process for finding an inverse and when one does not exist.
- The relationship between these symmetries and if a function is odd, even, an inverse, or none of these.
- When to apply the formula for the vertex of a parabola.
- Why y = x^2 is not the inverse of y = sqrt(x).

Students Will Be Skilled At

- Defining a function using terms like domain and range.
- Exploiting symmetry to define a function as odd, even, neither, or an inverse.
- Expressing a function on the graphing calculator by equation, graph, and table.
- Expressing in verbal words and by equation how a function can be transformed in any direction.
- Finding an inverse, or determining why there is none.
- Simplifying function operations, including composite.
- Understanding the symmetry of a function, and how the vertex of a quadratic function is involved in it.

Assessment

Assessments

- Formative: Daily assessments using examples from class notes, NJSLA test bank problems, and/or Albert/AP Classroom assessments
- Summative: Teacher-created assessments, NJSLA test bank problems, Big Ideas Math online platform problems, Albert/AP Classroom and/or Big Ideas Math unit assessments
- Benchmark: IXL or teacher created diagnostic assessments in addition to unit assessments from Big Ideas Math
- Alternative Assessments: Student-centered activities such as scavenger hunts, various projects involving real world applications, and differentiated learning tasks in Khan Academy, DeltaMath, and IXL

- Answer essential questions
- Class discussion of daily topic
- Classwork and homework that assess the essential questions
- Provide alternative means of assessments for certain students
- Take tests and quizzes that will assess the essential questions, including pre-assessments.
- Teacher Observation
- Use a graphing utility to solve equations.

Learning Plan

The following list is meant to create a day-to-day plan. Teachers are encouraged to slow down or condense days as appropriate to the student population in the class. Assessment(s) should be given when appropriate.

- Define domain and range in both inequality notation and interval notation. Students will also confirm functions versus relations using the Vertical Line Test.
- Review known transformations from Algebra 2, and fill in any gaps of knowledge. Reinforce that some reflections may look similar to original parent functions. Recall that the order of applied transformations matters.
- Identify transformations from a given graph. Write the equation of a transformed function based on either a description or a graph.

- Recall adding, subtracting, and multiplying polynomials. Review all factoring methods. Fill in gaps of knowledge as needed.
- Introduce the composite operation. Understand that composite operation can be used on the same function, or multiple functions.
- Extend the composite operation into a conversation about inverses. Examine the graphs of inverse functions, noting the symmetry about the line y=x. Prove two functions are inverses of each other.
- Define all symmetry cases, and explore with multiple types of functions.
- Model functions using the coordinate plane and real world examples (ie: cost and revenue).

Graphing calculators are encouraged to be used as an extension of these topics.

Materials

Core instructional materials: Core Book List including PreCalculus Enhanced with Graphing Utilities, Sullivan, Savvas

Supplemental materials: Khan Academy, Edia, and DeltaMath

- District approved textbook
- Khan Academy
- Teacher created activiites
- Teacher created notes
- Whiteboard tables

Integrated Accommodation & Modifications

Possible accommodations/modification for CP PreCalc & Trig