Unit 07: Exponents and Logarithms

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 3**
Length: **2-3 weeks**
Status: **Published**

Brief Summary of Unit

The unit will begin with a review of the laws of exponents using positive integer exponents and the zero exponent. This will be followed by a study of negative integer exponents, rational number exponents, logarithms, and natural logs. Equations will be solved when the variable occurs in the exponent and when the variable is raised to a fractional power. Students will be introduced to the laws of logarithms that they will use to simplify and expand logarithmic expressions and solve logarithmic equations. Verbal problems involving compound interest rates, half-life, and annual growth will be discussed to demonstrate a link to real world problem solving.

Standards

MA.F-IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
MA.N-RN.A.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
MA.A-SSE.A.1	Interpret expressions that represent a quantity in terms of its context.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.A-SSE.A.1a	Interpret parts of an expression, such as terms, factors, and coefficients.
MA.N-RN.A.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
MA.A-SSE.A.1b	Interpret complicated expressions by viewing one or more of their parts as a single entity.
MA.A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
MA.A-SSE.B.3a	Factor a quadratic expression to reveal the zeros of the function it defines.
MA.A-SSE.B.3b	Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
MA.A-SSE.B.3c	Use the properties of exponents to transform expressions for exponential functions.
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.F-IF.C.7a	Graph linear and quadratic functions and show intercepts, maxima, and minima.
MA.F-IF.C.7b	Graph square root, cube root, and piecewise-defined functions, including step functions

	and absolute value functions.
MA.F-IF.C.7c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
MA.F-IF.C.7d	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
MA.F-IF.C.7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
MA.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
MA.F-IF.C.8a	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
MA.F-IF.C.8b	Use the properties of exponents to interpret expressions for exponential functions.
MA.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MA.A-REI.A.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
MA.F-LE.A.1	Distinguish between situations that can be modeled with linear functions and with exponential functions.
MA.F-LE.A.1a	Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
MA.F-LE.A.1b	Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
MA.F-LE.A.1c	Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
MA.F-LE.A.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
MA.F-LE.A.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
MA.F-LE.A.4	Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to ab to the ct power = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
MA.F-LE.B.5	Interpret the parameters in a linear or exponential function in terms of a context.
LA.K-12.NJSLSA.L4	Determine or clarify the meaning of unknown and multiple-meaning words and phrases by using context clues, analyzing meaningful word parts, and consulting general and specialized reference materials, as appropriate.
LA.K-12.NJSLSA.L5	Demonstrate understanding of word relationships and nuances in word meanings.
CS.K-12.3.a	Identify complex, interdisciplinary, real-world problems that can be solved computationally.
CS.K-12.3.b	Decompose complex real-world problems into manageable sub-problems that could integrate existing solutions or procedures.
TEC.K-12.8.1	All students will use computer applications to gather and organize information and to solve problems.
TEC.K-12.8.2	All students will develop an understanding of the nature and impact of technology, engineering, technological design, and the designed world as they relate to the individual

	society, and the environment.
WORK.K-12.9.1	All students will develop career awareness and planning, employability skills and foundational knowledge necessary for success in the workplace.
WORK.K-12.9.2	All students will develop career awareness and planning, employability skills and foundational knowledge necessary for success in the workplace.

Transfer

- Exponential functions can represent stock investments or debt payments.
- Exponents are most often used in scientific notation.
- Logarithmic functions are often used for bank interest or house payments.

Essential Questions

- How is an exponential function related to a logarithmic function?
- What are the properties of exponential functions and logarithmic functions? How can exponential functions and logarithmic functions be used to model real-life phenomena?

Essential Understandings

- Exponential growth is a phenomenon that occurs in the real world and can be solved using the following formulas: A=Ao(1+r)^t or y = y0e^(kt).
- If a negative fractional exponent is involved in a problem, one can deal with the negative exponent first followed by the fractional power or visa versa.
- If one was able to compound a sum of money continuously at rate r for a time period t, one would find that the total would approach 2.78^(rt). This irrational number rounded to 2.78 is known as e.
- It is advantageous to compound interest more frequently.
- One can use the laws of logs to simplify logs and solve equations involving logs.
- Products raised to powers are different than sums raised to powers. The laws of exponents can apply to the former, not the latter.
- Taking the ln of both sides is an approach that can be used when solving equations like A=Ao(1+r/n)^t for t.
- The bases must be the same in order to use the laws of exponents,
- The inverse of an exponential function y = b^x is a logarithm: x = logb(y). Thus, they are reflections across the line y = x
- To solve an equation when a variable is raised to a fractional power, each side of the equation should be raised to the reciprocal power.
- To solve an equation with the unknown in the exponent and ‘like bases’ cannot be found, one takes the ln of both sides and applies the laws of logs to solve for the unknown.
- To solve an equation with the unknown in the exponent, one tries to get ‘like bases’.

Students Will Know

- A logarithm is a way to express an exponent.
- A number raised to a fractional power a/b is equivalent to saying the bth root of the number raised to the ath power.
- A number raised to a negative exponent is equivalent to saying the reciprocal of that number has been raised to an exponent which is the additive inverse of the original exponent.
- Common logs have base 10.
- Natural logs have base e and are expressed ln.
- The equation y = y0e^(kt) can be used for all exponential growth and decay problems.
- The graphs of logarithmic and exponential functions.
- The properties of exponents.
- The properties of logarithms.

Students Will Be Skilled At

- Noticing the similar properties and graphs of exponents and logarithms.
- Rewriting a base with a negative exponent into its reciprocal form.
- Transitioning between exponent form and radical form if a base has a fraction exponent.
- Using exponents and logarithms to solve exponential growth and decay problems.
- Using the properties of exponents and logarithms.

Evidence/Performance Tasks

Assessments

- Formative: Daily assessments using examples from class notes, NJSLA test bank problems, and/or Albert/AP Classroom assessments
- Summative: Teacher-created assessments, NJSLA test bank problems, Big Ideas Math online platform problems, Albert/AP Classroom and/or Big Ideas Math unit assessments
- Benchmark: IXL or teacher created diagnostic assessments in addition to unit assessments from Big Ideas Math
- Alternative Assessments: Student-centered activities such as scavenger hunts, various projects involving real world applications, and differentiated learning tasks in Khan Academy, DeltaMath, and IXL

- Answer essential questions
- Apply the exponential growth/decay equations to a myriad of situations.
- Class discussion of daily topic
- Classwork and homework that assess the essential questions

- Discover when logs are needed to solve equations (instead of using like bases or rational exponents).
- Evaluate expressions involving exponents.
- Explain and understand the meaning of zero and negative exponents.
- Graph and apply exponential functions.
- Graph and apply logarithmic functions.
- Provide alternative means of assessments for certain students
- Students can prove what 5% compounded continuously is equal to if it was compounded yearly and research APR (Annual percentage rate). They can search online interest sites to compare the interest rate that appears on paper and the APR. They can see how the exponential function models this concept.
- Students will compare exponential functions to other known functions (linear, quadratic, etc.) They can discuss which prize they would rather have: $100 a day for 20 days or start with a penny and double each day for 20 days and other similar activities.
- Teacher Observation
- Tests and quizzes that assess the essential questions
- Use the properties of logarithms to solve equations and simplify expressions.
- Utilize rational exponents to simplify more advanced algebraic expressions including complex fractions.
- Written assignments that assess the essential questions that involves providing explanations

Learning Plan

- Rational number exponents.
- Review of the laws of exponents: positive integer exponents and the 0 exponent.
- Graphing calculator can be used throughout the unit.
- Logarithms and natural logs.
- Preview the essential questions and connect to learning throughout the unit.
- Solve equations when the variable occurs in the exponent and when a variable is raised to a fractional power.
- Solving equations when the variable occurs in the exponent and 'like bases' cannot be determined.
- Study of negative integer exponents.
- Verbal problems involving compound interest rates, half-life, and annual growth using knowledge of exponents and logarithms.

Materials

Core instructional materials: Core Book List including PreCalculus Enhanced with Graphing Utilities, Sullivan, Savvas

Supplemental materials: Khan Academy, Edia, and DeltaMath

- District approved textbook
- Khan Academy
- Teacher created activiites
- Teacher created notes
- Whiteboard tables

Suggested Strategies for Modifications

Possible accommodations/modification for CP PreCalc & Trig