Unit 02: Properties and Graphs of Functions

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 1**
Length: **4 weeks**
Status: **Published**

Brief Summary of Unit

The students will investigate properties of functions: domain, range, symmetry, inverses, composite functions, adding/subtracting/multiplying/dividing functions, vertical and horizontal shrinking and stretching, and vertical and horizontal translations. There will be an emphasis on the different representations of functions including table of values, graphs and equations and students will explore these properties through discovery-based activities with the graphing calculator. Piecewise functions will be graphed, knowledge of parent functions, function notation, and word problems will be completed as well.

Standards

LA.K-12.NJSLSA.L4	Determine or clarify the meaning of unknown and multiple-meaning words and phrases by using context clues, analyzing meaningful word parts, and consulting general and specialized reference materials, as appropriate.
LA.K-12.NJSLSA.L5	Demonstrate understanding of word relationships and nuances in word meanings.
MA.F-BF.A.1	Write a function that describes a relationship between two quantities.
MA.F-BF.A.1a	Determine an explicit expression, a recursive process, or steps for calculation from a context.
MA.F-BF.A.1b	Combine standard function types using arithmetic operations.
MA.F-BF.A.1c	Compose functions.
MA.F-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MA.F-BF.B.4	Find inverse functions.
MA.F-BF.B.4a	Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.
MA.F-BF.B.4b	Verify by composition that one function is the inverse of another.
MA.F-BF.B.4c	Read values of an inverse function from a graph or a table, given that the function has an inverse.
MA.F-BF.B.4d	Produce an invertible function from a non-invertible function by restricting the domain.
MA.F-IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MA.F-IF.C.7a	Graph linear and quadratic functions and show intercepts, maxima, and minima.
MA.F-IF.C.7b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
MA.F-IF.C.7c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
MA.F-IF.C.7d	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
MA.F-IF.C.7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
MA.A-REI.B.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
MA.A-REI.B.4	Solve quadratic equations in one variable.
MA.A-REI.B.4a	Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
MA.A-REI.B.4b	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
TEC.K-12.8.1	All students will use computer applications to gather and organize information and to solve problems.
TEC.K-12.8.2	All students will develop an understanding of the nature and impact of technology, engineering, technological design, and the designed world as they relate to the individual society, and the environment.
WORK.K-12.9.1	All students will develop career awareness and planning, employability skills and foundational knowledge necessary for success in the workplace.
WORK.K-12.9.2	All students will develop career awareness and planning, employability skills and foundational knowledge necessary for success in the workplace.
	Mathematical and computational thinking in 9–12 builds on K–8 experiences and progresses to using algebraic thinking and analysis, a range of linear and nonlinear functions including trigonometric functions, exponentials and logarithms, and computational tools for statistical analysis to analyze, represent, and model data. Simple computational simulations are created and used based on mathematical models of basic assumptions.

Transfer

- Apply qualities of linear functions involved in real world slope-intercept equations.

- Apply qualities of quadratic functions involved in throwing/falling objects.
- Apply qualities of rational functions involved in economic situations.

Essential Questions

- How can these functions and their graphs be used as mathematical models to solve real-world problems?
- If a graph has symmetry, what does that mean for the domain and range?
- If functions are mathematically inverses, what does that mean for their graphs?
- What are the characteristics of linear, quadratic and rational functions?

Essential Understandings

- Any function that is monotonic will be symmetrical with respect to the line y = x with its inverse function.
- Graphing of piecewise functions consists of making tables of values and noting domain restrictions.
- If a function has origin symmetry, then it is an odd function.
- If a function has y-axis symmetry, then it is an even function.
- Indicators that the domain is not the set of real numbers include square roots and denominators with variables.
- The domain is the set of values that 'x' can take on and the range is the set of values, f(x), that are the answers when x is substituted into the rule.
- The horizontal line test is used to determine if an inverse exists. The vertical line test is used to determine if a graph is a function.
- There are simple formulas to find composition of functions as well as how to simplify using the four basic mathematical operations.
- To find the inverse of a function, interchange the x's and the y's and solve the equation for y.

Students Will Know

- A function is a rule that assigns every element in set D, the domain, to one and only one element in set R, the range.
- How to determine based on the equation if a function is vertically/horizontally stretched or shrunk.
- How to represent a function by equations, graphs and tables of values.
- How to simplify functions through the use of the four basic mathematical operations as well as composite functions.
- How to test for symmetry with respect to the x-axis, y-axis, the line y = x and the origin.
- How to utilize a graphing calculator to graph a function and solve an equation.
- The effects of the different constants in the general form for an equation: a●f(x+b) + c
- The methods for simplifying rational expressions and solving rational equations.
- The process for finding an inverse and when one does not exist.
- The relationship between these symmetries and if a function is odd, even, an inverse, or none of

these.

- When to apply the formula for the vertex of a parabola.
- Why y = x^2 is not the inverse of y = sqrt(x).

Students Will Be Skilled At

- Defining a function using terms like domain and range.
- Exploiting symmetry to define a function as odd, even, neither, or an inverse.
- Expressing a function on the graphing calculator by equation, graph, and table.
- Expressing in verbal words and by equation how a function can be transformed in any direction.
- Finding an inverse, or determining why there is none.
- Simplifying function operations, including composite.
- Understanding the symmetry of a function, and how the vertex of a quadratic function is involved in it.

Evidence/Performance Tasks

Assessments

- Formative: Daily assessments using examples from class notes, NJSLA test bank problems, and/or Albert/AP Classroom assessments
- Summative: Teacher-created assessments, NJSLA test bank problems, Big Ideas Math online platform problems, Albert/AP Classroom and/or Big Ideas Math unit assessments
- Benchmark: IXL or teacher created diagnostic assessments in addition to unit assessments from Big Ideas Math
- Alternative Assessments: Student-centered activities such as scavenger hunts, various projects involving real world applications, and differentiated learning tasks in Khan Academy, DeltaMath, and IXL

- Answer essential questions
- Class discussion of daily topic
- Classwork and homework that assess the essential questions
- Discover the domain/range of functions with and without a graphing calculator.
- Evaluate expressions involving operations on functions and composition of functions.
- Explore and recognize linear and quadratic functions, including their shifts, stretches and shape using graphing calculator exercises.
- Find the inverse of a function and describe why one exists.
- Graph and analyze linear functions.
- Graph and analyze quadratic and rational functions.
- Identify asymptotic behavior.

- Provide alternative means of assessments for certain students
- Recognize parent functions and their translations/reflections/stretches.
- Solve a variety of equations, including linear, quadratic, imaginary, rational and radical.
- Students can graph several functions in a graphing calculator and use the table feature to determine the domain and range. They can then make conjectures about the domains and ranges of rational functions, semicircles, etc.
- Students will graph several functions in a graphing calculator with different coefficients to make conjectures about vertical stretches and shrinks. They will then do the same with different numbers added and subtracted to make general rules for a function of the form a●f(x+b) + c.
- Take tests and quizzes that will assess the essential questions, including pre-assessments.
- Teacher Observation
- Tests and quizzes that assess the essential questions
- Use a graphing utility to solve equations.
- Use and apply function notation to write functions of one variable (such as V(x) = x^3 and x = 3b gives V(b) = 27b^3).
- Utilize the tests for symmetry to help graph more advanced functions
- Written assignments that assess the essential questions that involves providing explanations

Learning Plan

- Determine the general form a●f(x + b) + c and the effects of each of these constants both with and without a graphing calculator.
- Perform the four operations on a function as well as composition.
- Apply symmetry and reflections.
- As explained in the performance tasks above, students will use the graphing calculator as an exploratory device to make conjectures about the graphs of functions.
- Define function, domain and range.
- Explore inverse functions.
- Preview the essential questions and connect to learning throughout the unit.

Materials

Core instructional materials: Core Book List including PreCalculus Enhanced with Graphing Utilities, Sullivan, Savvas

Supplemental materials: Khan Academy, Edia, and DeltaMath

- District approved textbook
- Khan Academy
- Teacher created activiites
- Teacher created notes
- Whiteboard tables

Suggested Strategies for Modifications

Possible accommodations/modification for CP PreCalc & Trig