# Unit 4 Integration

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 4**
Length: **15 weeks**
Status: **Published**

## Brief Summary of Unit

Students will solve integrals for polynomial, rational, radical, exponential, logarithmic and trigonometric integrands. Students will work with both definite and indefinite integrals. The focus of the indefinite integrals will be initial value problems with connections to position (velocity) problems and economic (marginal) problems. The focus for definite integrals will be on the Fundamental Theorem of Calculus.

**Revised Date:** July 2025

## Standards

| | |
|---|---|
| 1 | Make sense of problems and persevere in solving them |
| 2 | Reason abstractly and quantitatively |
| 3 | Construct viable arguments and critique the reasoning of others |
| 4 | Model with mathematics |
| 9.2.12.CAP.5 | Assess and modify a personal plan to support current interests and post-secondary plans. |
| 5 | Use appropriate tools strategically |
| 9.2.12.CAP.6 | Identify transferable skills in career choices and design alternative career plans based on those skills. |
| 3.a | Identify complex, interdisciplinary, real-world problems that can be solved computationally. |
| 6 | Attend to precision |
| 3.b | Decompose complex real-world problems into manageable sub-problems that could integrate existing solutions or procedures. |
| 7 | Look for and make use of structure |
| 3.c | Evaluate whether it is appropriate and feasible to solve a problem computationally. |
| 8 | Look for and express regularity in repeated reasoning |
| 8.1.12.AP.5 | Decompose problems into smaller components through systematic analysis, using constructs such as procedures, modules, and/or objects. |
| LA.K-12.NJSLSA.L4 | Determine or clarify the meaning of unknown and multiple-meaning words and phrases by using context clues, analyzing meaningful word parts, and consulting general and specialized reference materials, as appropriate. |
| LA.K-12.NJSLSA.L5 | Demonstrate understanding of word relationships and nuances in word meanings. |
| MA.9-12.III.A.1 | Definite integral as a limit of Riemann sums |
| MA.9-12.III.A.2 | Definite integral of the rate of change of a quantity over an interval interpreted as the |

| | |
|---|---|
| | change of the quantity over the interval: the integral of f'(x)dx = f(b) - f(a) on the interval [a, b] |
| MA.9-12.III.A.3 | Basic properties of definite integrals (examples include additivity and linearity) |
| MA.9-12.III.B | Applications of integrals |
| MA.9-12.III.C.1 | Use of the Fundamental Theorem to evaluate definite integrals |
| MA.9-12.III.C.2 | Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined |
| MA.9-12.III.D.1 | Antiderivatives following directly from derivatives of basic functions |
| MA.9-12.III.D.2 | Antiderivatives by substitution of variables (including change of limits for definite integrals) |
| MA.9-12.III.E.1 | Finding specific antiderivatives using initial conditions, including applications to motion along a line |
| MA.9-12.III.E.2 | Solving separable differential equations and using them in modeling (including the study of the equation y' = ky and exponential growth) |
| MA.9-12.III.F | Numerical approximations to definite integrals |
| SCI.9-12.2.1 | The student can justify the selection of a mathematical routine to solve problems. |
| SCI.9-12.2.2 | The student can apply mathematical routines to quantities that describe natural phenomena. |
| SCI.9-12.2.3 | The student can estimate numerically quantities that describe natural phenomena. |

## Essential Questions

- How is an anti derivative (integral) related to limits and derivatives?
- What does an anti derivative represent?
- What is the result of the integration of a rate of change?

## Enduring Understandings

- How area under a curve can be estimated by the sums of the rectangles, where volume of a solid comes from rotating those rectangles about the x-axis to create circles.
- How can the graphing calculator assist in finding a definite integral?
- How can we use "u" substitution to integrate?
- How do the six trigonometric derivative rules correspond to integration rules?
- How do we apply the General Exponential Rule of integration?
- How do we apply the General Logarithmic Rule of Integration?
- How is integration used to find the area between graphs?
- How is integration used to find the volume of solids of revolution?
- What are the basic rules of integration?
- What is an integral (anti derivative) and how is it calculated?
- What is the difference between an indefinite and a definite integral?
- What is the Fundamental Theorem of Calculus?

## Students Will Know

- Basic rules of integration (constant rule, constant multiple rule, sum/difference rule, simple power rule.
- Chain rule integration ("u" substitution).
- Different functions can have the same derivative due to the constant value.
- Exponential and Logarithmic Integral Rules.
- For each trigonometric differentiation rule, there is a corresponding integration rule.
- How integrals are used to find the volume of a solid of revolution including both the disc and washer method.
- How integration is used in marginal analysis.
- How integration is used to find the area bounded by two graphs.
- How the midpoint rule is used to approximate an integral.
- How to evaluate a definite integral.
- How to evaluate an indefinite integral.
- Proper notation for integration.
- The derivative and integral are inverse operations.
- The Fundamental Rule of Calculus allows to find the area under a curve over a closed interval.
- The process to solve an initial value problem.
- Why the value of "C" is necessary in initial value problems.

## Students Will Be Skilled At

- Applying integration to deriving position functions and economic functions.
- Applying the chain rule ("u" substitution) in integration.
- Evaluating a definite integral as the limit of a sum using the midpoint rule.
- Evaluating definite integrals algebraically and with graphing calculator.
- Evaluating definite integrals involving trigonometric functions.
- Evaluating indefinite integrals involving trigonometric functions.
- Evaluating indefinite integrals.
- Finding the area of a region bounded by two graphs through integration.
- Integrating exponential functions.
- Integrating logarithmic functions.
- Use the Disc and Washer Method to find the volume of a solid.
- Using the Fundamental Theorem of Calculus to find the area under a curve.

## Assessment

Assessments

- Formative: Daily assessments using examples from class notes, observation of student work on

problems in class, student group work
- Summative: Teacher-created assessments
- Alternative Assessments: Differentiated learning tasks in Khan Academy Edia, and/or DeltaMath

- Answer essential questions
- Class discussion of daily topic
- Classwork and homework that assess the essential questions
- Graded Do Now assessments that reflect student understanding of class notes and homework.
- Provide alternative means of assessments for certain students
- Teacher Observation
- Tests and quizzes that assess the essential questions
- Written assignments that assess the essential questions that involves providing explanations

## Learning Plan

A graphing calculator/Desmos will be utilized throughout the unit. Students will use these resources to confirm their answers to a variety of problems, and to help solve various problems in the unit.

15 weeks will be about 60 classes on the rotate drop schedule. Fifty classes are accounted for below with some time left for weeks that only have 3 class meetings as well as some extra days for larger assessments.

- Antiderivatives and indefinite integrals **(10 classes)**
- Chain rule integration **(3 classes)**
- Integration of exponential functions **(2 classes)**
- Integration of logarithmic functions **(2 classes)**
- Definite integrals **(3 classes)**
- Fundamental Theorem of Calculus **(5 classes)**
- Applications of integrals **(5 classes)**
- Area bounded by two graphs **(5 classes)**
- Volumes of solids of revolution **(5 classes)**
- Integration of trigonometric functions **(10 classes)**

## Materials

Core instructional material: Core Book List including Calculus Larson 12E

Supplemental instructional materials: Khan Academy, Edia, and DeltaMath

- District approved textbook
- Khan Academy
- Teacher created activiites
- Teacher created notes

## Integrated Accommodation & Modifications

Possible accommodations/modification for CP Calculus