Unit 3 Derivatives

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 2**
Length: **15 weeks**
Status: **Published**

Brief Summary of Unit

Students will learn how to find derivatives algebraically and to apply derivatives to real-world applications. Students will also apply their knowledge of derivatives in graphical analysis. Derivatives will be examined in the context of polynomial, square root, rational, exponential, logarithmic, and trigonometric functions.

Revised Date: July 2025

Standards

MATH.K-12.1	Make sense of problems and persevere in solving them
MATH.K-12.2	Reason abstractly and quantitatively
MATH.K-12.3	Construct viable arguments and critique the reasoning of others
MATH.K-12.4	Model with mathematics
MATH.K-12.5	Use appropriate tools strategically
MATH.K-12.6	Attend to precision
MATH.K-12.7	Look for and make use of structure
MATH.K-12.8	Look for and express regularity in repeated reasoning
MATH.9-12.F.IF.C.7.b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
MATH.9-12.F.IF.C.7.e	Graph exponential and logarithmic functions, showing intercepts and end behavior.
MATH.9-12.F.LE.A.1.b	Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
MATH.9-12.F.LE.A.1.c	Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
MATH.9-12.F.LE.A.4	Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
LA.K-12.NJSLSA.L4	Determine or clarify the meaning of unknown and multiple-meaning words and phrases by using context clues, analyzing meaningful word parts, and consulting general and specialized reference materials, as appropriate.
LA.K-12.NJSLSA.L5	Demonstrate understanding of word relationships and nuances in word meanings.
MATH.9-12.F.TF.A.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
MATH.9-12.F.TF.A.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric

	functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
MATH.9-12.F.TF.A.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
MATH.9-12.F.TF.A.4	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
CS.9-12.8.1.12.AP.5	Decompose problems into smaller components through systematic analysis, using constructs such as procedures, modules, and/or objects.
CS.K-12.3.a	Identify complex, interdisciplinary, real-world problems that can be solved computationally.
CS.K-12.3.b	Decompose complex real-world problems into manageable sub-problems that could integrate existing solutions or procedures.
CS.K-12.3.c	Evaluate whether it is appropriate and feasible to solve a problem computationally.
MA.9-12.II.A.1	Derivative presented graphically, numerically, and analytically
MA.9-12.II.A.2	Derivative interpreted as an instantaneous rate of change
MA.9-12.II.A.3	Derivative defined as the limit of the difference quotient
MA.9-12.II.A.4	Relationship between differentiability and continuity
MA.9-12.II.B.1	Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
MA.9-12.II.B.2	Tangent line to a curve at a point and local linear approximation
MA.9-12.II.B.3	Instantaneous rate of change as the limit of average rate of change
MA.9-12.II.C.1	Corresponding characteristics of graphs of f and f'
MA.9-12.II.C.2	Relationship between the increasing and decreasing behavior of f and the sign of f'
MA.9-12.II.C.4	Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.
MA.9-12.II.D.1	Corresponding characteristics of the graphs of f, f', and f''
MA.9-12.II.D.2	Relationship between the concavity of f and the sign of f''
MA.9-12.II.D.3	Points of inflection as places where concavity changes
MA.9-12.II.E.1	Analysis of curves, including the notions of monotonicity and concavity
MA.9-12.II.E.2	Optimization, both absolute (global) and relative (local) extrema
MA.9-12.II.E.3	Modeling rates of change, including related rates problems
MA.9-12.II.E.4	Use of implicit differentiation to find the derivative of an inverse function
MA.9-12.II.E.5	Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration
MA.9-12.II.F.1	Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions
MA.9-12.II.F.2	Derivative rules for sums, products, and quotients of functions
MA.9-12.II.F.3	Chain rule and implicit differentiation
SCI.9-12.2.1	The student can justify the selection of a mathematical routine to solve problems.
SCI.9-12.2.2	The student can apply mathematical routines to quantities that describe natural phenomena.
SCI.9-12.2.3	The student can estimate numerically quantities that describe natural phenomena.

WRK.9.2.12.CAP.5	Assess and modify a personal plan to support current interests and post-secondary plans.
WRK.9.2.12.CAP.6	Identify transferable skills in career choices and design alternative career plans based on those skills.

Essential Questions

- How is a derivative a limit?
- How is a derivative an instantaneous rate of change?
- How is the limit definition used to find a derivative?
- What algebraic procedures can be used to find a derivative?
- What conditions must be satisfied for a function to have a derivative?

Enduring Understandings

- How are calculus rules of power rule, product rule, quotient rule and chain rule used to find the derivative of a function? What determines which rule should be used?
- How are Related Rates utilized to find the instantaneous rate of change of a variable with time?
- How do we find higher order derivatives?
- How exponential functions are used to model growth and decay?
- How implicit differentiation can be used to develop the derivative of the natural logarithmic function?
- How is an average rate of change represented mathematically (secant lines)?
- How is an instantaneous rate of change represented mathematically (tangent lines)?
- How is the first derivative of a function related to the optimization of the function?
- How is the second derivative of a function related to the concavity of the function?
- How the natural logarithmic function is the inverse to the natural exponential function?
- What can the slope of a curve tell us about the graphical behavior of a function?
- What is the slope of a curve?
- When is implicit differentiation necessary to find a derivative?
- Why is the Chain Rule used to find the derivative of all six trigonometric functions?
- Why the irrational number e is defined by a limit?

Students Will Know

- How the first derivative test is used to find relative maximum/minimum for a function.
- How the second derivative of a function can determine the concavity of a graph at a point.
- How to apply the general chain rule to a function.
- How to find a derivative when a variable is changing with time.
- How to use and apply implicit differentiation.
- The derivative rules for exponential and logarithmic functions.
- The derivative rules for the six trigonometric functions.

- The derivatives of sums, differences, products and quotients of functions.
- The limit definition of a derivative.
- The process for finding the slope of a secant line to a curve.
- The process for finding the slope of the tangent to a curve at the point of tangency.
- The proper notation for a derivative.
- What determines if the graph of a function is increasing or decreasing.
- Why the first derivative of function can show optimization of a function.

Students Will Be Skilled At

- Finding higher order derivatives.
- Finding optimization points for a function.
- Finding related rates.
- Finding the derivative of exponential and logarithmic functions.
- Finding the derivative of the six trigonometric functions.
- Graphical and algebraic understanding of secant and tangent lines.
- The computation and application of chain rule and implicit differentiation.
- The formulas necessary to find derivatives algebraically (power rule, product rule, quotient rule, sum and difference).
- Understanding the relationship between demand, revenue and profit.
- Understanding the relationship between position, velocity and acceleration.
- Using derivatives to help graph functions through descriptions of increasing/decreasing and concavity.

Assessment

Assessments

- Formative: Daily assessments using examples from class notes, observation of student work on problems in class, student group work
- Summative: Teacher-created assessments
- Alternative Assessments: Differentiated learning tasks in Khan Academy Edia, and/or DeltaMath

- Answer essential questions
- Class discussion of daily topic
- Classwork and homework that assess the essential questions
- Graded do now assessments that reflect student understanding of class notes and homework
- Provide alternative means of assessments for certain students
- Teacher observation
- Tests and quizzes that assess the essential questions
- Written assignments that assess the essential questions that involves providing explanations

Learning Plan

A graphing calculator/Desmos will be utilized throughout the unit. Students will use these resources to confirm their answers to a variety of problems, and to help solve various problems in the unit.

15 weeks will be about 60 classes on the rotate drop schedule. About 50 classes are accounted for below with some time left for weeks that only have 3 class meetings as well as some extra days for larger assessments.

- Limit definition of a derivative **(2-3 classes)**
  - Various animations exist online to show how the slop of a secant line becomes the slope of the tangent line as the change in x approaches zero. Use these to help students visualize the meaning of the limit definition of a derivative.
- Power Rule **(2-3 classes)**
  - Show students how the power rule can be derived from the limit definition of a derivative. Students do not need to be assessed on this, but showing them the derivation helps them to understand that these rules are shortcuts we can use, but they do have a relationship to the original limit definition.
  - Use the power rule for integer and rational exponents. Answers involving rational exponents do not need to have irrational denominators rationalized.
- Applications **(2-3 classes)**
  - Velocity
  - Marginal cost
- Product rule **(2 classes)**
  - Include tips for how to recognize when it could be used.
- Quotient rule **(2 classes)**
  - Include tips for how to recognize when it could be used.
- Chain rule **(2-3 classes)**
  - Include tips for how to recognize when functions are "inside" of other functions.
  - Problems should require the use of the rules students have learned so far in varying combinations.
- Find point of tangency to a curve given the tangent line's slope. **(2-3 classes)**
  - Incorporate technology
- Find equations for secant lines and tangent lines **(2-3 classes)**
  - Incorporate technology
- Higher-order derivatives **(2-3 classes)**
  - Include acceleration
- Implicit differentiation **(3 classes)**
- Related rates problems **(3 classes)**
- Graph analysis of the first derivative to show increasing and decreasing intervals for a function **(2 classes)**
- Graph analysis to find relative extrema using the first derivative **(1-2 classes)**
- Graph analysis of the second derivative to discuss concavity and points of inflection **(1-2 classes)**
- Derivatives of exponential functions **(2 classes)**
- Derivatives of logarithmic functions **(2 classes)**
- Trigonometry review **(5 classes)**
- Derivatives of trigonometric functions **(4 classes)**
- L'Hopital's rule **(2 classes)**

Materials

Core Book List including Calculus Larson 12E

Supplemental materials: Khan Academy, Edia, and DeltaMath

- District approved textbook
- Khan Academy
- Teacher created activiites
- Teacher created notes

Integrated Accommodation & Modifications

Possible accommodations/modification for CP Calculus