Unit_3 Derivatives of polynomial, square root, rational, exponential, logarithmic, and trigonometric functions

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 2**
Length: **15 weeks**
Status: **Published**

Brief Summary of Unit

Students will learn how to find derivatives algebraically and apply derivatives to real world applications. Students will also apply their knowledge of derivatives in graphical analysis.

Standards

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.F-IF.C.7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
MA.F-LE.A.1c	Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
MA.F-LE.A.4	Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to ab to the ct power $= d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
LA.K-12.NJSLSA.L4	Determine or clarify the meaning of unknown and multiple-meaning words and phrases by using context clues, analyzing meaningful word parts, and consulting general and specialized reference materials, as appropriate.
LA.K-12.NJSLSA.L5	Demonstrate understanding of word relationships and nuances in word meanings.
MA.F-TF.A.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
MA.F-TF.A.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
MA.F-TF.A.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

MA.F-TF.A.4	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
	Mathematical and computational thinking in 9–12 builds on K–8 experiences and progresses to using algebraic thinking and analysis, a range of linear and nonlinear functions including trigonometric functions, exponentials and logarithms, and computational tools for statistical analysis to analyze, represent, and model data. Simple computational simulations are created and used based on mathematical models of basic assumptions.
MA.9-12.II	Derivatives
MA.9-12.II.A	Concept of the derivative
MA.9-12.II.A.1	Derivative presented graphically, numerically, and analytically
MA.9-12.II.A.2	Derivative interpreted as an instantaneous rate of change
MA.9-12.II.A.3	Derivative defined as the limit of the difference quotient
MA.9-12.II.A.4	Relationship between differentiability and continuity
MA.9-12.II.B	Derivative at a point
MA.9-12.II.B.1	Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
MA.9-12.II.B.2	Tangent line to a curve at a point and local linear approximation
MA.9-12.II.B.3	Instantaneous rate of change as the limit of average rate of change
MA.9-12.II.C	Derivative as a function
MA.9-12.II.C.2	Relationship between the increasing and decreasing behavior of f and the sign of f'
MA.9-12.II.C.4	Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.
MA.9-12.II.D.1	Corresponding characteristics of the graphs of f, f', and f"
MA.9-12.II.D.2	Relationship between the concavity of f and the sign of f"
MA.9-12.II.D.3	Points of inflection as places where concavity changes
MA.9-12.II.E.1	Analysis of curves, including the notions of monotonicity and concavity
MA.9-12.II.E.2	Optimization, both absolute (global) and relative (local) extrema
MA.9-12.II.E.3	Modeling rates of change, including related rates problems
MA.9-12.II.E.4	Use of implicit differentiation to find the derivative of an inverse function
MA.9-12.II.E.5	Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration
MA.9-12.II.F.1	Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions
TEC.K-12.8.1	All students will use computer applications to gather and organize information and to solve problems.
TEC.K-12.8.2	All students will develop an understanding of the nature and impact of technology, engineering, technological design, and the designed world as they relate to the individual society, and the environment.
WORK.K-12.9.1	All students will develop career awareness and planning, employability skills and foundational knowledge necessary for success in the workplace.
WORK.K-12.9.2	All students will develop career awareness and planning, employability skills and foundational knowledge necessary for success in the workplace.

Transfer

- Students will apply derivatives to real life scenarios such as velocity, marginal profit, marginal revenue and marginal cost.
- Students will connect the concept of derivatives to data analysis.
- Students will connect the concept of derivatives to graphical analysis.

Essential Questions

- How is a derivative a limit?
- How is a derivative an instantaneous rate of change?
- What algebraic procedures can be used to find a derivative?

Essential Understandings

- Do all functions have a derivative?
- How are calculus rules of power rule, product rule, quotient rule and chain rule used to find the derivative of a function? What determines which rule should be used?
- How are Related Rates utilized to find the instantaneous rate of change of a variable with time?
- How do we find higher order derivatives?
- How exponential functions are used to model growth and decay?
- How implicit differentiation can be used to develop the derivative of the natural logarithmic function?
- How is an average rate of change represented mathematically (secant lines)?
- How is an instantaneous rate of change represented mathematically (tangent lines)?
- How is the first derivative of a function related to the optimization of the function?
- How is the limit definition used to find a derivative?
- How is the second derivative of a function related to the concavity of the function?
- How the natural logarithmic function is the inverse to the natural exponential function?
- What can the slope of a curve tell us about the graphical behavior of a function?
- What is the slope of a curve?
- When is implicit differentiation necessary to find a derivative?
- Why Chain Rule is used to find the derivative of all six trigonometric functions?
- Why the irrational number e is defined by a limit?

Students Will Know

- How the first derivative test is used to find relative maximum/minimum for a function.
- How the second derivative of a function can determine the concavity of a graph at a point.
- How to apply the general chain rule to a function
- How to find a derivative when a variable is changing with time.

- How to use and apply implicit differentiation.

- The derivative rules for exponential and logarithmic functions

- The derivative rules for the six trigonometric functions

- The derivatives of sums, differences, products and quotients of functions.

- The limit definition of a derivative.

- The process for finding the slope of a secant line to a curve.

- The process for finding the slope of the tangent to a curve at the point of tangency.

- The proper notation for a derivative.

- What determines if the graph of a function is increasing or decreasing.

- Why the first derivative of function can show optimization of a function.

Students Will Be Skilled At

- Finding higher order derivatives.

- Finding optimization points for a function.

- Finding related rates.

- Finding the derivative of exponential and logarithmic functions

- Finding the derivative of the six trigonometric functions

- Graphical and algebraic understanding of secant and tangent lines.

- The computation and application of chain rule and implicit differentiation.

- The formulas necessary to find derivatives algebraically (power rule, product rule, quotient rule, sum and difference).

- Understanding the relationship between demand, revenue and profit.

- Understanding the relationship between position, velocity and acceleration.

- Using derivatives to help graph functions through descriptions of increasing/decreasing and concavity.

Evidence/Performance Tasks

Assessments

- Formative: Daily assessments using examples from class notes, NJSLA test bank problems, and/or Albert/AP Classroom assessments

- Summative: Teacher-created assessments, NJSLA test bank problems, Big Ideas Math online platform problems, Albert/AP Classroom and/or Big Ideas Math unit assessments

- Benchmark: IXL or teacher created diagnostic assessments in addition to unit assessments from Big Ideas Math

- Alternative Assessments: Student-centered activities such as scavenger hunts, various projects involving real world applications, and differentiated learning tasks in Khan Academy, DeltaMath, and IXL

- Answer essential questions

- Class discussion of daily topic

- Classwork and homework that assess the essential questions

- Graded Do Now assessments that reflect student understanding of class notes and homework.

- Provide alternative means of assessments for certain students

- Teacher Observation

- Tests and quizzes that assess the essential questions

- Written assignments that assess the essential questions that involves providing explanations (problems of the week)

Learning Plan

- Chain Rule to find derivative.

- Derivative of exponential function

- Derivative of logarithmic function.

- Derivative of Trigonometric Functions

- Equation of secant line and tangent line.

- Find point of tangency given slope of tangent to a curve.

- Graph Analysis of second derivative to discuss concavity and points of inflection

- Graph Analysis of the first derivative to show increasing and decreasing functions

- Graph Analysis to find relative extrema from the first derivative

- Higher order derivatives of functions (acceleration)

- Implicit Differentiation to find the derivative.

- L'Hopitals Rule

- Limit Definition

- Power Rule to find derivative

- Product Rule to find derivative.

- Quotient Rule to find derivative.

- Related Rates to find the derivative.

- Trigonometry Review

- Velocity and Marginal applications to derivatives

Materials

Core Book List including AP Calculus Larson 12E

Supplemental materials: Khan Academy, Edia, and DeltaMath

- District approved textbook

- Khan Academy

- Teacher created activiites

- Teacher created notes

Suggested Strategies for Modifications

[Possible accommodations/modification for CP Calculus](#)