Unit 5 - Integration

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 3**
Length: **8 weeks**
Status: **Published**

Brief Summary of Unit

Students will study both definite and indefinite integrals. For indefinite integrals, there will be a focus on initial value problems with real-world applications. For definite integrals, there will be a focus on integrals as functions and the utilization of the First and Second Fundamental Theorems. They will also learn how to solve elementary differential equations and how to find the value of a definite integral with a graphing calculator. As for all of the topics in the curriculum, the content of this course follows the Advanced Placement Program Course Description for Calculus AB provided by The College Board. Any topic that is required for Calculus BC is noted with an asterisk and can be optionally covered in this class.

Standards

LA.K-12.NJSLSA.L5	Demonstrate understanding of word relationships and nuances in word meanings.
LA.K-12.NJSLSA.L6	Acquire and use accurately a range of general academic and domain-specific words and phrases sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when encountering an unknown term important to comprehension or expression.
MA.9-12.III.A	Interpretations and properties of definite integrals
MA.9-12.III.A.1	Definite integral as a limit of Riemann sums
MA.9-12.III.A.2	Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval: the integral of f'(x)dx = f(b) - f(a) on the interval [a, b]
MA.9-12.III.A.3	Basic properties of definite integrals (examples include additivity and linearity)
MA.9-12.III.B	Applications of integrals
MA.9-12.III.C	Fundamental Theorem of Calculus
MA.9-12.III.C.1	Use of the Fundamental Theorem to evaluate definite integrals
MA.9-12.III.C.2	Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined
MA.9-12.III.D	Techniques of antidifferentiation
MA.9-12.III.D.1	Antiderivatives following directly from derivatives of basic functions
MA.9-12.III.D.2	Antiderivatives by substitution of variables (including change of limits for definite integrals), parts, and simple partial fractions (nonrepeating linear factors only)
MA.9-12.III.D.2	Antiderivatives by substitution of variables (including change of limits for definite integrals)
MA.9-12.III.D.3	Improper integrals (as limits of definite integrals)
MA.9-12.III.E	Applications of antidifferentiation
MA.9-12.III.E.1	Finding specific antiderivatives using initial conditions, including applications to motion

	along a line
MA.9-12.III.E.3	Solving logistic differential equations and using them in modeling
MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
TEC.K-12.8.1	All students will use computer applications to gather and organize information and to solve problems.
	Stability and Change

Transfer

- Students will apply position, velocity and acceleration to solve problems about motion using integration.
- Students will connect the integral of a rate of change to the displacement and as an accumulator.
- Students will discover how integrals are used in data analysis (i.e. traffic flow, population growth, etc.).

Essential Questions

- How can a function be defined as an integral and what applications would this model?
- What does a Riemann Sum find and what are it applications to definite integrals?
- What is an anti-derivative (integral) and what is its relationship to the other major topics of Calculus (limits and derivatives).
- What is the result of the integration of a rate of change?

Essential Understandings

- *How is an improper integral evaluated?
- *What makes an integral improper?
- How can a graphing calculator evaluate a definite integral?
- How can an integral on one interval effect integrals on a different interval?
- How can integrals be used to solve particle motion problems?
- How can the idea of displacement be applied to finding function values when only the derivative is known?
- How do you find the derivative of a definite integral?
- How is sigma notation used to represent a sum?

- What are Riemann sums and how are they related to limits and integrals?
- What are the basic rules for evaluating an integral?
- What are the First and Second Fundamental Theorems of Calculus?
- What is an anti-derivative and how is it calculated?
- What is an initial value problem and how can it be applied to solve problems about displacement?
- What is substitution and how is it applied?
- What is the difference between a definite integral and an indefinite integral?
- Why are Riemann Sums needed when given a tabular function?

Students Will Know

- *How to evaluate improper integrals.
- *How to solve logistic differential equations and use them in modeling.
- *Other methods of solving first-order differential equations including integrating factors.
- Basic techniques of integration including approximation with Riemann sums and trapezoidal sums, u-substitution, and geometric evaluation.
- How to apply functions defined as integrals to solve problems about traffic flow, population growth, etc.
- How to apply the First and Second Fundamental theorems.
- How to evaluate definite and indefinite integrals.
- How to simplify differential equations of the form: $y' = ky$.
- If f(a) and f ’(x) are given, then you can find the function value at any given value of x by using displacement. (This works even if you don’t know the anti-derivative of f ‘(x).)
- Several functions have the same derivative. (For example, y = 2x has an infinite number of anti-derivatives and they can all be written in the form y = x^2 + c.)
- The derivative and integral are inverse operations.
- The evaluation of Riemann Sums using sigma notation and its significance to the value of the definite integral.
- The interpretation of a definite integral of the rate of change of a quantity over an interval as the change of the quantity over the interval.
- The process of separation of variables in order to solve a differential equation.
- The process to solve an initial value problem.
- The properties of definite integrals
- The significance of the value of c in initial value problems.
- The use of integrals with particle motion.
- To change the limits of integration when using substitution with definite integrals.

Students Will Be Skilled At

- Applying integral-defined functions to model growth and decay.
- Calculating Riemann Sums and recognizing them as an estimation of the real value of integrals. (If f(x)

is increasing, then the right sum is an overestimate and the left sum is an underestimate for the real value of the integral.) (If f(x) is concave up, then the trapezoidal rule is an overestimate and if f(x) is concave down, then the trapezoidal rule is an underestimate.)

- Demonstrating that an integral is an infinite limit of a summation, area under a curve and the anti-derivative of a function.
- Estimating definite integrals using Trapezoidal Rule.
- Evaluating integrals using multiplication, division and geometry formulas.
- Recognizing special integrals such as the ones for lnx and the inverse trigonometric functions.
- Solving and applying differential equations and using them to model different types of growth.
- Solving rate-in/rate-out problems involving sand on a beach, bees in a hive, etc.
- Using applying the First and Second Fundamental Theorems of Calculus.
- Utilizing slope fields and*Euler's Method to estimate the solution to a differential equation.
- Utilizing their graphing calculators to evaluate definite integrals.

Evidence/Performance Tasks

Assessments

- Formative: Daily assessments using examples from class notes, NJSLA test bank problems, and/or Albert/AP Classroom assessments
- Summative: Teacher-created assessments, NJSLA test bank problems, Big Ideas Math online platform problems, Albert/AP Classroom and/or Big Ideas Math unit assessments
- Benchmark: IXL or teacher created diagnostic assessments in addition to unit assessments from Big Ideas Math
- Alternative Assessments: Student-centered activities such as scavenger hunts, various projects involving real world applications, and differentiated learning tasks in Khan Academy, DeltaMath, and IXL

- Answer essential questions
- Class discussion of daily topic
- Students will explore integrals using a computer graphing program. They will see the definition of integration as the area under a curve and will discuss how to find the integral between the function and the x-axis using this definition. (Note: These values will only differ if the function goes below the x-axis.) This will be done at the end of the unit to prepare for the transition to the area and volume sections in the next unit.
- Students will explore problems about traffic flow, population growth, etc. Students will be provided with a rate of entry and rate of decay and will need to find the maximum and minimum values of the traffic flow, population growth, etc.
- Students will explore the functions y = 1/x and y = ln x. Using a graphing calculator, students will make conjectures about the two graphs (how one is the derivative of the other). They will then discuss why the power rule can't be used for the integral of 1/x. (You can't have (x^0)/0 since division by zero is

impossible.) Students will share their results with the class.

- Students will make conjectures about definite integrals on different intervals and create rules that explain the relationship known that explains why the sum of the parts equal the entire interval.
- Students will take tests and quizzes and will work on weekly written projects that assess the essential questions and involve written paragraph proofs.
- Teacher Observation

Learning Plan

A graphing calculator will be utilized throughout the entire chapter. Students will use the integration feature on the graphing calculator to give numerical approximations to complex integrals. This will be done with the purpose of finding function values when only the derivative and another function value are given. A computer graphing program should be used to see the area under a curve definition of integration.

- *Define and evaluate improper integrals.
- Define a definite integral.
- Define the indefinite integral and solve initial value problems.
- Demonstrate finding the value of a definite integral on the graphing calculator.
- Explain the interconnectivity of derivatives and integrals.
- Review sigma notation and its uses.
- Solve differential equations by separation of variables.
- Solve initial value problems and particle motion problems.
- Solve rate-in/rate-out problems and utilize a graphing utility to optimize.
- State and apply the First Fundamental Theorem and explain its relationship to area under a curve.
- State and apply the Second Fundamental Theorem and explain its relationship to derivative of an integral.
- State and derive the integration rules for exponential, logarithmic, and inverse trigonometric functions.
- Use and apply the sum, difference, constant multiple, power, and trigonometric rules for integration.
- Use geometry formulas and Riemann sums to evaluate integrals.
- Use substitution to solve definite and indefinite integrals.
- Use trapezoidal sums to estimate integrals.

Materials

Core Book List including AP Calculus Larson 11E

Supplemental materials: Khan Academy, Edia, and DeltaMath

- A Calculus BC prep book

- Curriculum modules and practice problems from https://apcentral.collegeboard.org
- District approved textbook
- Khan Academy
- Teacher-created graphing calculator explorations

Suggested Strategies for Modifications

Possible accommodations/modification for AP Calculus