Unit 7 - Sequence and Series

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 4**
Length: **3 weeks**
Status: **Published**

Brief Summary of Unit

Students will learn about the concepts of sequences and series, with a major focus on series. They will discuss the many tests for convergence and use technology to explore the convergence and divergence of series. As for all of the topics in the curriculum, the content of this course follows the Advanced Placement Program Course Description. This entire unit is included only in a Calculus BC course.

Standards

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
	Patterns
	Stability and Change
MA.9-12.IV	Polynomial Approximations and Series
MA.9-12.IV.A	Concept of series
MA.9-12.IV.B	Series of constants
MA.9-12.IV.B.1	Motivating examples, including decimal expansion
MA.9-12.IV.B.2	Geometric series with applications
MA.9-12.IV.B.3	The harmonic series
MA.9-12.IV.B.4	Alternating series with error bound
MA.9-12.IV.B.5	Terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of p-series
MA.9-12.IV.B.6	The ratio test for convergence and divergence
MA.9-12.IV.B.7	Comparing series to test for convergence or divergence
MA.9-12.IV.C	Taylor series
MA.9-12.IV.C.1	Taylor polynomial approximation with graphical demonstration of convergence (for example, viewing graphs of various Taylor polynomials of the sine function approximating the sine curve)
MA.9-12.IV.C.2	Maclaurin series and the general Taylor series centered at x = a

MA.9-12.IV.C.3	Maclaurin series for the functions e to the x power, sin x, cos x, and 1/(1-x)
MA.9-12.IV.C.4	Formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, antidifferentiation, and the formation of new series from known series
MA.9-12.IV.C.5	Functions defined by power series
MA.9-12.IV.C.6	Radius and interval of convergence of power series
MA.9-12.IV.C.7	Lagrange error bound for Taylor polynomials
TEC.K-12.8.1	All students will use computer applications to gather and organize information and to solve problems.

Transfer

- *Students will apply series and sequences to model real-life phenomenon.
- *Students will connect improper integrals to series convergence.

Essential Questions

- *What are Taylor polynomials and how are their error bounds found?
- *What is the difference between a sequence and series and what are the applications of them?
- *What tests can be used to find the convergence of a series? How do you know which test to select?

Essential Understandings

- *How can an improper integral be used to determine the convergence of a series?
- *How can the Ratio Test, Limit Comparison Test and Root Test be used to determine the convergence of a series?
- *What are Taylor and Maclaurin polynomials?
- *What are the Maclaurin series for the functions e^x, sin x, and cos x?
- *What functions are defined by power series?
- *What is a sequence and how do you determine if it converges or diverges?
- *What is a series and how do you determine if it converges or diverges?
- *What is an improper integral and how is it evaluated?
- *What is the Lagrange error bound for Taylor polynomials?
- *What is the radius and interval of convergence of power series?
- *What properties determine the type of series (geometric, harmonic, p-series or alternating) and how does this lead to convergence or divergence?

Students Will Know

- *Alternating series can conditionally converge if they only converge based on the fact that they are alternating.

- *How to determine the accuracy of an approximation using a Taylor or Maclaurin polynomial.
- *How to utilize partial sums to estimate the value of a series.
- *The connection between improper integrals and series convergence.
- *The difference between series and sequence and how to determine the convergence of both of them.
- *The tests for series convergence.
- *The types of series: geometric, harmonic, alternating, power, Taylor, and Maclaurin.

Students Will Be Skilled At

- *Choosing the appropriate tests to determine the convergence and divergence of series.
- *Connecting the AB Calculus topics of linear and quadratic approximations to Taylor and Maclaurin polynomials.
- *Deciding if a sequence converges or diverges based on its limit.
- *Determining the convergence of alternating series.
- *Evaluating and applying Taylor and Maclaurin polynomials.
- *Finding the interval and radius of convergence for a given Power Series.
- *Recognizing the most common Taylor and Maclaurin polynomials, including sin x, cos x, e^x and using them to find sin(2x), cos(3x), e^(5x).
- *Relating the convergence of a sequence to the convergence of a series.

Evidence/Performance Tasks

Assessments

- Formative: Daily assessments using examples from class notes, NJSLA test bank problems, and/or Albert/AP Classroom assessments
- Summative: Teacher-created assessments, NJSLA test bank problems, Big Ideas Math online platform problems, Albert/AP Classroom and/or Big Ideas Math unit assessments
- Benchmark: IXL or teacher created diagnostic assessments in addition to unit assessments from Big Ideas Math
- Alternative Assessments: Student-centered activities such as scavenger hunts, various projects involving real world applications, and differentiated learning tasks in Khan Academy, DeltaMath, and IXL

- *Students will discuss how to find a Taylor polynomial to an actual polynomial and discuss the results. They will then compare it to finding a linear approximation to a linear function and prepare a presentation of why in each case the answer is the original function. They will then be asked to find a case where the answer is not the same and provide a reason for this phenomenon.
- Answer essential questions

- Class discussion of daily topic
- Students will take tests and quizzes and will work on weekly written projects that assess the essential questions and involve written paragraph proofs.
- Teacher Observation

Learning Plan

A graphing calculator will be utilized to explore the convergence of series.

- *Define sequence and its convergence.
- *Define series and its convergence, utilizing partial sums and graphing calculators.
- *Discuss alternating series and the idea of conditional convergence.
- *Discuss the Lagrange error bound for Taylor polynomials.
- *Discuss the Maclaurin and the general Taylor series centered at x = a.
- *Discuss the radius and interval of convergence of power series.
- *Discuss the relationship between series and improper integrals.
- *Students will determine what functions are defined by power series.
- *Use and apply the Limit Comparison Test, Root test and the Ratio Test.

Materials

Core Book List including AP Calculus Larson 11E

Supplemental materials: Khan Academy, Edia, and DeltaMath

- *a Calculus BC prep book
- Curriculum modules and practice problems from https://apcentral.collegeboard.org
- District approved textbook
- Khan Academy
- Teacher-created graphing calculator explorations

Suggested Strategies for Modifications

Possible accommodations/modification for AP Calculus