Unit 1 - Limits and Continuity

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 1**
Length: **3 weeks**
Status: **Published**

Brief Summary of Unit

The main focus of the unit is for students to understand different representations of functions (graph-based, equation-based, piece-wise and table-based) and about the properties of these functions. Students will learn the definitions of limits and continuity and will apply them to understand asymptotes, jumps and holes. With a focus on the consequences of continuity, students will explore the Intermediate Value Theorem. Students will also learn about asymptotic and unbounded behavior and will utilize graphing calculators to sketch complex functions in order to find their roots and properties. Students will also be introduced to mathematical writing and how to make complex arguments. As for all of the topics in the curriculum, the content of the course follows the Advanced Placement Program Course Description for Calculus AB and BC provided by The College Board.

Standards

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
LA.K-12.NJSLSA.L4	Determine or clarify the meaning of unknown and multiple-meaning words and phrases by using context clues, analyzing meaningful word parts, and consulting general and specialized reference materials, as appropriate.
LA.K-12.NJSLSA.L5	Demonstrate understanding of word relationships and nuances in word meanings.
MA.9-12.I	Functions, Graphs, and Limits
MA.9-12.I.A	Analysis of graphs
MA.9-12.I.B	Limits of functions (including one-sided limits)
MA.9-12.I.B.1	An intuitive understanding of the limiting process
MA.9-12.I.B.2	Calculating limits using algebra
MA.9-12.I.B.3	Estimating limits from graphs or tables of data
MA.9-12.I.C	Asymptotic and unbounded behavior
MA.9-12.I.C.1	Understanding asymptotes in terms of graphical behavior
MA.9-12.I.C.2	Describing asymptotic behavior in terms of limits involving infinity

MA.9-12.I.C.3	Comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth)
MA.9-12.I.D	Continuity as a property of functions
MA.9-12.I.D.1	An intuitive understanding of continuity. (The function values can be made as close as desired by taking sufficiently close values of the domain.)
MA.9-12.I.D.2	Understanding continuity in terms of limits
MA.9-12.I.D.3	Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem)
TEC.K-12.8.1	All students will use computer applications to gather and organize information and to solve problems.

Transfer

- Students will apply limits and continuity to model real life phenomenon that involve asymptotic behavior, such as terminal velocity and logistic differential equations.
- Students will connect the ideas of continuity to understand how to analyze a set of data utilizing the Intermediate Value Theorem.
- Students will fold a piece of paper several times and discuss the amount that is left after each fold. If this process was continued until the end of time, the students would have to figure out what amount of the paper would be left. This should introduce them to the concept of infinite limits and give them a visual representation of sequence convergence.

Essential Questions

- How are limits and continuity related to each other and how do they form the basis for all other calculus topics?
- How does continuity of a function effect data analysis?
- What algebraic methods are available to evaluate limits?
- What are the different representations of functions and how can we determine the properties of a function when given only one of these representations?

Essential Understandings

- How are limits calculated algebraically?
- How are limits estimated from graphs or tables of data?
- How are limits related to vertical and horizontal asymptotes?
- How can Intermediate Value theorem be used to prove the existence of a root on a given interval?
- How can the limit of f(x) as x approaches 2 exist without f(2) being defined?
- What are the basic properties of trigonometric and other non-polynomial functions?
- What are the graphical properties of a continuous function?
- What does it mean to approach a limit?
- What is continuity and how can a graph be right (or left) continuous?
- What is end behavior and how is it related to limits?

- What is the effect of an asymptote on the graphical behavior of a function?
- What is true if the left- and right-hand limits are the same? What if they are different?
- When given a graph of f(x) or a table of values for f(x), how can we find the difference quotient?
- Why does the function need to be continuous for Intermediate Value Theorem to apply?

Students Will Know

- A limit at a point has nothing to do with the function value at that point. It only is dependent on the values close to that point.
- The algebraic procedures to calculate limits including conjugates, factor/cancel, common denominator, L'Hopital's Rule and table methods.
- The connection between continuity and limits.
- The formal definition of a limit and its applications algebraically and graphically.
- The graphical properties of a continuous function.
- The importance of continuity to the Intermediate Value Theorem.
- The Intermediate Value Theorem and its applications.
- The intuitive meaning of a limit and how to explain it in non-math terminology.
- The intuitive meaning of continuity.
- The notation that applies to limits including sided limits.
- The properties of polynomials and non-polynomials.
- The types of discontinuity including jump, removable and infinite.

Students Will Be Skilled At

- Applying limits and continuity to other Calculus topics including derivatives and integrals.
- Applying the definitions of limits and continuity.
- Creating a mathematical argument in paragraph form.
- Evaluating limits from tables of data, algebraic processes, and graphs of functions.
- Understanding the properties of a continuous function.
- Using and applying the Intermediate Value Theorem.
- Utilizing different representations to model a situation.

Evidence/Performance Tasks

- Formative: Daily assessments using examples from class notes, NJSLA test bank problems, and/or Albert/AP Classroom assessments
- Summative: Teacher-created assessments, NJSLA test bank problems, Big Ideas Math online platform problems, Albert/AP Classroom and/or Big Ideas Math unit assessments
- Benchmark: IXL or teacher created diagnostic assessments in addition to unit assessments from Big Ideas Math

- Alternative Assessments: Student-centered activities such as scavenger hunts, various projects involving real world applications, and differentiated learning tasks in Khan Academy, DeltaMath, and IXL

- Answer essential questions
- Class discussion of daily topic
- Students are to find functions that meet certain conditions and write an explanation of their findings. (For example, write a function that has a domain of all real numbers, but does not have a limit anywhere except at x = 0.)
- Students will discuss how to construct a bridge, given a piecewise function that represents the road on each side. Using slope as a guide, they will determine if their bridge would be too steep. They must explain their solution in terms of limits.
- Students will explore asymptotic behavior and one-sided limits through a graphing calculator generated table of values and will make conjectures about limits and continuity.
- Students will take tests and quizzes and will work on weekly written projects that assess the essential questions and involve written paragraph proofs.
- Teacher Observation

Learning Plan

A graphing calculator will be utilized throughout the entire chapter. Students can use the table feature to estimate limits of a function that cannot be found algebraically. Students can discover asymptotic behavior of functions and create their own rules for infinite limits. Be sure to caution the students about the limitations of the graphing calculator, including misleading function values (with removable discontinuities) and inappropriate viewing windows.

- Discuss continuity intuitively and by definition. Explore continuous and discontinuous functions utilizing a graphing calculator.
- Discuss methods of calculating limits using algebra and graphing calculators.
- Discuss the limiting process given the graphs of specific functions and given tables of data related to a given function.
- Have students define an asymptote in terms of infinite limits.
- Introduce functions as mathematical models in many different contexts. Include one of each function type and how to utilize a graphing calculator to analyze each type.
- Introduce the notation for limits.
- State and apply the Intermediate Value Theorem. Students should know why the hypothesis must be true in order to use it.

Materials

Core Book List including AP Calculus Larson 11E

Supplemental materials: Khan Academy, Edia, and DeltaMath

- Curriculum modules and practice problems from https://apcentral.collegeboard.org
- https://www.khanacademy.org
- Teacher-created graphing calculator explorations
- Teacher-created summer packet that reviews algebraic processes, functions and trigonometry.
- Textbook