Unit 6 - Applications of Integration

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 4**
Length: **6 weeks**
Status: **Published**

Brief Summary of Unit

Students will learn how to apply integrals to area under curves, area between curves, volumes of solids of revolution, average values of a function, lengths of plane curves, and *surface areas of solids of revolution. Students will learn other integration methods, including *Partial Fractions and *Integration By Parts. As for all of the topics in the curriculum, the content of this course follows the Advanced Placement Program Course Description for Calculus AB provided by The College Board. Any topic that is required for Calculus BC is noted with an asterisk and can be optionally covered in this class.

Standards

Students will analyze geometric designs which connects to various cultures. Embracing the diversity within society incorporates the following:

Amistad Commission

This unit also reflects the goals of the Department of Education and the Amistad Commission including the infusion of the history of Africans and African-Americans into the curriculum in order to provide an accurate, complete, and inclusive history regarding the importance of of African-Americans to the growth and development of American society in a global context.

Asian American and Pacific Islander History Law

This unit includes instructional materials that highlight the history and contributions of Asian Americans and Pacific Islanders in accordance with the New Jersey Student Learning Standards in Social Studies.

New Jersey Diversity and Inclusion Law

In accordance with New Jersey’s Chapter 32 Diversity and Inclusion Law, this unit includes instructional materials that highlight and promote diversity, including:

economic diversity, equity, inclusion, tolerance, and belonging in connection with gender and sexual orientation, race and ethnicity, disabilities, and religious tolerance.

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
3.a	Identify complex, interdisciplinary, real-world problems that can be solved computationally.
MA.K-12.6	Attend to precision.
3.b	Decompose complex real-world problems into manageable sub-problems that could integrate existing solutions or procedures.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
LA.K-12.NJSLSA.L4	Determine or clarify the meaning of unknown and multiple-meaning words and phrases by using context clues, analyzing meaningful word parts, and consulting general and specialized reference materials, as appropriate.
LA.K-12.NJSLSA.L5	Demonstrate understanding of word relationships and nuances in word meanings.
	Stability and Change
MA.9-12.II.E.2	Analysis of planar curves given in parametric form, polar form, and vector form, including velocity and acceleration
MA.9-12.III.A.2	Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval: the integral of f'(x)dx = f(b) - f(a) on the interval [a, b]
MA.9-12.III.B	Applications of integrals
MA.9-12.III.E.1	Finding specific antiderivatives using initial conditions, including applications to motion along a line
MA.9-12.III.E.2	Solving separable differential equations and using them in modeling (including the study of the equation y' = ky and exponential growth)
MA.9-12.III.F	Numerical approximations to definite integrals
MA.9-12.III.F	Numerical approximations to definite integrals
TEC.K-12.8.1	All students will use computer applications to gather and organize information and to solve problems.

Transfer

- Students will apply average value and average rate of change to real-world problems.
- Students will continue to apply integrals to data analysis through separable differential equations.
- Students will derive the geometric formulas for area and volume using integration.
- Students will discover which integration methods work best for any given integral.

Essential Questions

- How can an integral be used as an accumulator?
- How is volume and area computed using integration?
- What are the applications of an integral to the real world?
- What is a differential equation and how can it be solved by separation of variables?
- What possible methods are available for integration?

Essential Understandings

- *In what situations would integration by parts or partial fractions be used to solve an integral?
- How can integrals be used to solve problems of traffic flow, average value, and total distance traveled by a particle?
- How do you find the volume when an area is spun around a horizontal or vertical line?
- What are the different methods to find the volume of solids of revolution?
- What is the connection to area of a cross-section and the volume of the object?
- What is the difference between area under a curve and area between two curves?
- When is the area under a curve negative?

Students Will Know

- *How to find surface areas of solids of revolution.
- *How to integrate powers of sine, cosine, tangent, and secant
- *How to solve logistic differential equations utilizing Partial Fractions.
- *How to use and apply Euler's Method.
- *The procedures of integration by parts, trigonometric substitution, and partial fractions.
- An integral is an accumulation function.
- How to find volume using cross-sections, washer method, shell method and the disk method.
- How to set up integrals for area under a curve, area between two curves and volume of solids of revolution.
- The computation and applications of the average value and average rate of change of a function.
- The connection between volumes rotated about a line and the shifting of functions to rotate about the axis.
- The idea of displacement and how it can be used to find the final position of a particle.
- The process of finding total distance traveled by being given the information in equation, graphical, or table form.
- The process to find areas and volumes with piecewise functions.
- Volume is the integral of the area of a cross-section.

Students Will Be Skilled At

- *Finding surfaces of revolution and solving logistic differential equations.

- *Utilizing and applying Euler's Method.
- Applying definite integrals to solve real-world problems involving average value.
- Finding area and volume utilizing definite integrals.
- Finding total distance travelled and displacement of an object.
- Selecting the correct method to integrate a given quantity.
- Solving separable differential equations and applying initial value techniques to solve real-world problems.
- Understanding the difference between average value and average rate of change.

Evidence/Performance Tasks

Assessments

- Formative: Daily assessments using examples from class notes, NJSLA test bank problems, and/or Albert/AP Classroom assessments
- Summative: Teacher-created assessments, NJSLA test bank problems, Big Ideas Math online platform problems, Albert/AP Classroom and/or Big Ideas Math unit assessments
- Benchmark: IXL or teacher created diagnostic assessments in addition to unit assessments from Big Ideas Math
- Alternative Assessments: Student-centered activities such as scavenger hunts, various projects involving real world applications, and differentiated learning tasks in Khan Academy, DeltaMath, and IXL

- *Students will create a flow chart to help determine which methods of integration can be used for a given integral. These charts will be shared with the class.
- Answer essential questions
- Class discussion of daily topic
- Students will derive the geometric formulas for the volume of all 3-dimensional objects utilizing cross-sections and definite integrals. (For example, students will integrate the area of a semicircle of radius r from x = −r to x = r to find the formula for the volume of a sphere.)
- Students will explore the cost of heating a house using integrals and periodic functions and the cost savings when raising your thermostat in the summer.
- Students will explore the relationship between position, velocity and acceleration. These functions will be represented by graphs, tables of values and equations. The students will be asked to write a summary of the relationships that they discover.
- Students will take tests and quizzes and will work on weekly written projects that assess the essential questions and involve written paragraph proofs.
- Teacher Observation

Learning Plan

A graphing calculator will be utilized throughout the entire chapter. Students will use the integration feature on the graphing calculator to give numerical approximations to complex integrals.

- *Evaluate the surface area of a solid of revolution.
- *Show students how to select the best method for each integral.
- *Utilize and apply the integration methods (By Parts, Trigonometric Substitution, Tables and Partial fractions).
- *Utilize Euler's Method to estimate solutions to differential equations.
- Compute and apply the average value of a function and its average rate of change.
- Find area under a curve with and without a graphing calculator.
- Find the area between two curves, including areas that require more than one integral.
- Find the total distance that a particle traveled and relate it to rectilinear motion.
- Solve differential equations by separation of variables or estimation techniques.
- Use and apply the disk, shell, and washer methods.

Materials

Core Book List including AP Calculus Larson 11E

Supplemental materials: Khan Academy, Edia, and DeltaMath

- *a Calculus BC prep book
- Curriculum modules and practice problems from https://apcentral.collegeboard.org
- District approved textbook
- Khan Academy
- Teacher-created graphing calculator explorations

Suggested Strategies for Modifications

Possible accommodations/modification for AP Calculus