Unit 4 - Relative and Absolute Extrema

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 2**
Length: **5 weeks**
Status: **Published**

Brief Summary of Unit

Students will apply derivatives to analyze curves and make connections between the graphs of f'(x) and f''(x). Students will utilize critical points and the Extreme Value Theorem to solve optimization problems. As for all of the topics in the curriculum, the content of this course follows the Advanced Placement Program Course Description for Calculus AB provided by The College Board. In addition, there are some Calculus BC topics that can be optionally covered and are denoted with an asterisk.

Standards

Analyzing various sets of data, through the lens of optimization in Calculus, will allow students to explore studies about people from different backgrounds. Statistical studies and analysis provides students an opportunity to read about historical statistics about people's cultures. Embracing the diversity within society incorporates the following:

Amistad Commission

This unit also reflects the goals of the Department of Education and the Amistad Commission including the infusion of the history of Africans and African-Americans into the curriculum in order to provide an accurate, complete, and inclusive history regarding the importance of of African-Americans to the growth and development of American society in a global context.

Asian American and Pacific Islander History Law

This unit includes instructional materials that highlight the history and contributions of Asian Americans and Pacific Islanders in accordance with the New Jersey Student Learning Standards in Social Studies.

New Jersey Diversity and Inclusion Law

In accordance with New Jersey's Chapter 32 Diversity and Inclusion Law, this unit includes instructional materials that highlight and promote diversity, including:

economic diversity, equity, inclusion, tolerance, and belonging in connection with gender and

sexual orientation, race and ethnicity, disabilities, and religious tolerance.

Commission on Holocaust Education

This unit further reflects the goals of the Holocaut Education mandate where students are able to identify and analyze applicable theories concerning human nature and behavior; understand that genocide is a consequence of prejudice and discrimination; understand that issues of moral dilemma and conscience have a profound impact on life; and understand the personal responsibility that each citizen bears to fight racism and hatred whenever and wherever it happens.

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
LA.K-12.NJSLSA.L5	Demonstrate understanding of word relationships and nuances in word meanings.
LA.K-12.NJSLSA.L6	Acquire and use accurately a range of general academic and domain-specific words and phrases sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when encountering an unknown term important to comprehension or expression.
	Analyzing data in 9–12 builds on K–8 experiences and progresses to introducing more detailed statistical analysis, the comparison of data sets for consistency, and the use of models to generate and analyze data.
MA.9-12.II.B.1	Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
MA.9-12.II.C.2	Relationship between the increasing and decreasing behavior of f and the sign of f'
MA.9-12.II.C.3	The Mean Value Theorem and its geometric interpretation
MA.9-12.II.C.4	Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.
MA.9-12.II.E.1	Analysis of curves, including the notions of monotonicity and concavity
MA.9-12.II.E.2	Optimization, both absolute (global) and relative (local) extrema
MA.9-12.II.E.9	L'Hospital's Rule, including its use in determining limits and convergence of improper integrals and series
TEC.K-12.8.1	All students will use computer applications to gather and organize information and to solve problems.

Transfer

- Students will connect position, velocity and acceleration to solve problems about motion.
- Students will see the connection between the graphs of f(x), f'(x), and f''(x).
- Students will use optimization techniques to solve problems involving relative and absolute extrema.

Essential Questions

- How are position, velocity and acceleration connected?
- How can a derivative be used to optimize?
- What are relative and absolute extrema and how are they found?
- What is the connection between the graphs of the original function, the first derivative and the second derivative?

Essential Understandings

- How can a slope field give you a clue about the solution to a differential equation?
- How can optimization be used to solve real-life problems?
- How is optimizing on an open interval different from optimizing on a closed interval?
- What are the corresponding characteristics of f, f ', and f ''?
- What does the second derivative reveal about the graph of f(x) and f '(x)?
- What information can be gleaned from the graph of f '(x)? How about the equation or table of values of f '(x)?
- What is the difference between relative and absolute extrema?
- What is the Extreme Value Theorem and how can it be used to find absolute extrema?
- What is the First and Second Derivative Test and when does it apply?
- What is the Mean Value Theorem and when does it apply?
- What is the relationship between position, velocity and acceleration?

Students Will Know

- *Derivatives of parametric, polar, and vector functions.
- How to find critical points and how they relate to the graphs of f(x) and f '(x).
- How to find the maximum and minimum values on an interval using the Extreme Value Theorem.
- How to geometrically interpret differential equations via slope fields and the relationship between slope fields and derivatives of implicitly defined functions.
- How to make sign charts for the first and second derivative and how to apply these charts to graph and analyze f(x).
- How to sketch the position of a particle's motion using the velocity function.

- Velocity is the derivative of position and acceleration is the second derivative of position.
- When to use and apply the Mean Value Theorem.

Students Will Be Skilled At

- *Computing derivatives of parametric, polar and vector functions.
- Applying the Extreme Value Theorem and Mean Value Theorem.
- Computing velocity and acceleration given the position.
- Solving optimization real-world problems.
- Understanding the importance of the hypothesis in the Mean Value Theorem.
- Utilizing f''(x) to determine concavity and points of inflection.
- Utilizing f'(x) to find intervals of increasing and decreasing, as well as the relative extrema.

Evidence/Performance Tasks

Assessments

- Formative: Daily assessments using examples from class notes, NJSLA test bank problems, and/or Albert/AP Classroom assessments
- Summative: Teacher-created assessments, NJSLA test bank problems, Big Ideas Math online platform problems, Albert/AP Classroom and/or Big Ideas Math unit assessments
- Benchmark: IXL or teacher created diagnostic assessments in addition to unit assessments from Big Ideas Math
- Alternative Assessments: Student-centered activities such as scavenger hunts, various projects involving real world applications, and differentiated learning tasks in Khan Academy, DeltaMath, and IXL

- Answer essential questions
- Class discussion of daily topic
- Students will create a reference sheet summarizing the properties of f'(x) and f''(x) and what they reveal about the graph of f(x).
- Students will take tests and quizzes and will work on weekly written projects that assess the essential questions and involve written paragraph proofs.
- Students will work on an optimization problem that involves constructing an underground pipeline of least cost that can be made from an offshore oil well to a refinery on the coast.
- Teacher Observation
- Utilizing a graphing calculator students will explore the graphs of f(x), f'(x) and f''(x) and make conjectures about the relationship.

Learning Plan

Graphing calculators will be utilized throughout the unit in both an exploratory context (like the assessment suggestions below) and as a check for the more complex derivatives.

- Find the absolute maximum and minimum of a function by graphing calculator analysis as well as the Extreme Value Theorem.
- Find the relative maximum and minimum of a function utilizing the first and second derivative tests.
- Graph f(x) utilizing its first and second derivatives both with and without a graphing calculator.
- Solve particle motion problems.
- Use and apply Rolle's Theorem and the Mean Value Theorem.

Materials

Core Book List including AP Calculus Larson 11E

Supplemental materials: Khan Academy, Edia, and DeltaMath

- Curriculum modules and practice problems from https://apcentral.collegeboard.org
- District approved textbook
- Khan Academy
- Teacher-created graphing calculator explorations

Suggested Strategies for Modifications

Possible accommodations/modification for AP Calculus