Unit 3 - Applications of Derivatives and Inverse Functions

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 2**
Length: **6 weeks**
Status: **Published**

Brief Summary of Unit

Students will learn about the existence of inverses, how to compute their derivatives and how to apply them to specific functions including logarithmic, exponential, and inverse trigonometric functions. Students will apply derivatives to related rates, L'Hopital's Rule and differentials. As for all of the topics in the curriculum, the content of this course follows the Advanced Placement Program Course Description for Calculus AB provided by The College Board.

Standards

Diversity and Inclusion: Students will focus on equity, inclusion, and tolerance when analyzing the comparison of various quantities regarding characteristics of people. Equality will also be highlighted which can be associated with both numerical representations through equations and the connection between people. This can be associated with treating people fairly and equally.

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
3.a	Identify complex, interdisciplinary, real-world problems that can be solved computationally.
3.b	Decompose complex real-world problems into manageable sub-problems that could integrate existing solutions or procedures.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
LA.K-12.NJSLSA.L5	Demonstrate understanding of word relationships and nuances in word meanings.
LA.K-12.NJSLSA.L6	Acquire and use accurately a range of general academic and domain-specific words and phrases sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when encountering an unknown term important to comprehension or expression.
	Stability and Change
MA.9-12.II.C	Derivative as a function
MA.9-12.II.C.1	Corresponding characteristics of graphs of f and f'

MA.9-12.II.C.2	Relationship between the increasing and decreasing behavior of f and the sign of f'
MA.9-12.II.C.3	The Mean Value Theorem and its geometric interpretation
MA.9-12.II.C.4	Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.
MA.9-12.II.D	Second derivatives
MA.9-12.II.D.1	Corresponding characteristics of the graphs of f, f', and f''
MA.9-12.II.D.2	Relationship between the concavity of f and the sign of f''
MA.9-12.II.D.3	Points of inflection as places where concavity changes
MA.9-12.II.E	Applications of derivatives
MA.9-12.II.E.1	Analysis of curves, including the notions of monotonicity and concavity
MA.9-12.II.E.2	Optimization, both absolute (global) and relative (local) extrema
MA.9-12.II.E.3	Modeling rates of change, including related rates problems
MA.9-12.II.E.4	Use of implicit differentiation to find the derivative of an inverse function
MA.9-12.II.E.5	Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration
MA.9-12.II.E.9	L'Hospital's Rule, including its use in determining limits and convergence of improper integrals and series
MA.9-12.II.F.1	Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions
MA.9-12.II.F.4	Derivatives of parametric, polar, and vector functions
TEC.K-12.8.1	All students will use computer applications to gather and organize information and to solve problems.

Transfer

- Students will apply related rates to solve problems involving motion, volume and area.
- Students will apply the idea of inverse functions to logarithm and inverse trig functions.
- Students will connect the ideas of L'Hopital's Rule to solve advanced limits.

Essential Questions

- How can L'Hopital's Rule be used to evaluate limits?
- How can related rates be used to model real-life problems about motion?
- What is an inverse function? How can the properties of inverses be related to exponential, logarithmic, and trigonometric functions?

Essential Understandings

- Can a function be its own inverse? What must be true of such a function?
- How are the derivatives of exponential and logarithmic functions found?
- How can one rate be related to another?

- How can we apply implicit differentiation to find the derivative of an inverse?
- How do we take the derivative of a^x?
- How is the domain of an inverse related to the range of the original function?
- In what context can L'Hopital's Rule be used?
- What are the domains, ranges and derivatives of the inverse trigonometric functions?
- What is a 1-to-1 function and how can a derivative be used to determine if a graph is 1-to-1?
- What is logarithmic differentiation and how is it used to take the derivative of more complex functions?
- What is the change of base formula and how is it used to find the derivative of logarithmic functions that are not of base e?
- When does an inverse exist?

Students Will Know

- How to compute the derivative of an inverse, both implicitly and, if possible, explicitly.
- How to use and apply the graphs of the logarithmic, exponential, and inverse trigonometric functions.
- If f(x) and g(x) are inverses, then f(g(x)) = g(f(x)) = x and the graphs of f(x) and g(x) are reflections across the line y = x.
- In order for an inverse to exist, a function must be one-to-one.
- L'Hopital's Rule can only be used in certain circumstances (0/0 or ∞/∞).
- The algebraic computation of limits using derivatives.
- The change of base formula for logarithms.
- The connection between e^x and a^x.
- The derivative formulas for the logarithmic, exponential, and inverse trigonometric functions.
- The domain and range of inverse trigonometric functions.
- The math modeling process of related rates.
- The properties of logarithms and their use in taking derivatives.

Students Will Be Skilled At

- Applying L'Hopital's Rule to evaluate complex limits.
- Applying the rules for finding inverses and their derivatives.
- Evaluating and differentiating inverse trigonometric functions.
- Evaluating and differentiating logarithmic and exponential functions.
- Using logarithmic differentiation and the change of base formulas to evaluate the derivatives of functions.
- Utilizing f'(x) to determine the existence of f^(-1)(x)
- Utilizing logarithmic and exponential properties to simplify derivatives.
- Utilizing the process of related rates to model real-world problems.

Evidence/Performance Tasks

- Formative: Daily assessments using examples from class notes, NJSLA test bank problems, and/or Albert/AP Classroom assessments
- Summative: Teacher-created assessments, NJSLA test bank problems, Big Ideas Math online platform problems, Albert/AP Classroom and/or Big Ideas Math unit assessments
- Benchmark: IXL or teacher created diagnostic assessments in addition to unit assessments from Big Ideas Math
- Alternative Assessments: Student-centered activities such as scavenger hunts, various projects involving real world applications, and differentiated learning tasks in Khan Academy, DeltaMath, and IXL

- Answer essential questions
- Class discussion of daily topic
- Students will begin with a function of the form y = f(x). Then, they will find the derivative implicitly of x = g(y) (where g is the inverse of f) and discover the formula for the derivative of an inverse. (This can be done for specific functions such as ln x or sin x instead of the general solution.)
- Students will discover e^x as an infinite series that contains factorials. They will be provided with the Maclaurin polynomial that represents e^x and show that its derivative is itself. Using the fact that the derivative of e^x is also itself, they will establish a connection between the two functions and make a conjecture. Using their graphing calculator, they will sketch both e^x and the series together and either prove or disprove their conjecture.
- Students will model real-world problems involving motion, volume, area, etc. through the use of related rates.
- Students will take tests and quizzes and will work on weekly written projects that assess the essential questions and involve written paragraph proofs.
- Teacher Observation

Learning Plan

A graphing calculator will continue to be used to find the derivative at a point and will be used for the discovery projects listed below in the assessment description.

- Apply L'Hopital's Rule to evaluate limits.
- Find the derivative of logarithmic and exponential functions, including the process of logarithmic differentiation.
- Review the properties of an inverse.
- Review the properties of logarithms and exponentials with and without the graphing calculator.
- Use and apply the derivatives of inverse trigonometric functions.
- Utilize calculus to prove an inverse's existence and discuss the method to find its derivative.
- Utilize related rates and graphing calculators to solve problems about motion.

Materials

Core Book List including AP Calculus Larson 11E

Supplemental materials: Khan Academy, Edia, and DeltaMath

- Curriculum modules and practice problems from https://apcentral.collegeboard.org
- District approved textbook
- Khan Academy
- Teacher-created graphing calculator explorations

Suggested Strategies for Modifications

Possible accommodations/modification for AP Calculus