Unit 04 Polynomial Functions

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 2**
Length: **4-5 weeks**
Status: **Published**

Brief Summary of Unit

Students previously studied properties of the two simplest kinds of polynomial functions: linear and quadratic functions. Now they will examine polynomial functions in general. This unit reviews the basic operations involving polynomials, extending them to include polynomial functions, factoring by grouping and factoring sums and differences of cubes. In order to solve and graph polynomial equations, the Rational Root Theorem, Irrational Root Theorem, Compex Root Theorem, the Fundamental Theorem of Algebra and the Principle of Zero Products will be used. Students will recognize the relationship between the degree of a polynomial function and the number of zeros. The definitions of odd and even functions will be discussed.

Revised Date: June 2024

Standards

1	Developing Responsibility for Learning: Cultivating independence, self-reflection, and responsibility for one's own learning.
2	Adapting Communication: Adapting communication in response to the varying demands of audience, task, purpose, and discipline.
A.APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
3	Valuing Evidence in Argumentation: Constructing viable claims and evaluating, defending, challenging, and qualifying the arguments of others.
4	Building Knowledge: Building strong content knowledge and connecting ideas across disciplines using a variety of text resources and media.
A.APR.B.2	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $x - a$ is a factor of $p(x)$.
5	Leveraging Technology: Employing technology and digital media thoughtfully, strategically and capably to enhance reading, writing, speaking, listening, and language use.
A.APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
A.APR.C.4	Prove polynomial identities and use them to describe numerical relationships.
F.BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k(both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
3.a	Identify complex, interdisciplinary, real-world problems that can be solved computationally.

	Include recognizing even and odd functions from their graphs and algebraic expressions for them.
3.b	Decompose complex real-world problems into manageable sub-problems that could integrate existing solutions or procedures.
A.APR.D.6	Rewrite simple rational expressions in different forms; write □(□)/□(□) in the form □(□) + □(□)/□(□), where □(□), □(□), □(□), and □(□) are polynomials with the degree of □(□) less than the degree of □(□), using inspection, long division, or, for the more complicated examples, a computer algebra system.
N.CN.C.7	Solve quadratic equations with real coefficients that have complex solutions.
F.IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
N.CN.C.8	Extend polynomial identities to the complex numbers.
N.CN.C.9	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
F.IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
F.IF.C.7.c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
A.SSE.A.2	Use the structure of an expression to identify ways to rewrite it.

Essential Questions

- How are operations and properties of complex numbers related to those of real numbers?
- How do polynomial functions model real-world problems and their solutions?
- How do the characteristics of a polynomial equation appear in its graph?
- What is the most efficient method for solving polynomial equations?

Enduring Understandings

- The characteristics of polynomial functions and their representations are useful in solving real-world problems.
- The domain and range of polynomial functions can be extended to include the set of complex numbers.

Students Will Know

- Students will know how factors, roots, zeros, and x-intercepts are related to each other and to the graph of a polynomial function.
- Students will know how polynomial functions can be used to model real-world applications.
- Students will know how to add, subtract, multiply, divide and factor polynomials.
- Students will know that all polynomials can be classified by specific characteristics.

- Students will know that every polynomial function will have defining features (relative minimum/maximum, domain, range, x-intercepts (may be complex), y-intercepts, and end behavior.)

Students Will Be Skilled At

- The student will be skilled at adding and subtracting polynomials.
- The student will be skilled at completely factoring polynomials.
- The student will be skilled at describing the end behavior of polynomials.
- The student will be skilled at dividing polynomials by binomials of the form (x-k) using synthetic division.
- The student will be skilled at explaining how solutions of equations and zeros of functions are related.
- The student will be skilled at explaining the Remainder and Factor Theorems.
- The student will be skilled at finding all of the zeros of a polynomial function.
- The student will be skilled at graphing polynomial functions.
- The student will be skilled at identifying and evaluating polynomial functions.
- The student will be skilled at identifying increasing and decreasing intervals.
- The student will be skilled at identifying odd and even functions graphically and algebraically.
- The student will be skilled at identifying the degree of a polynomial.
- The student will be skilled at multiplying polynomials and using special product patterns.
- The student will be skilled at solving polynomial equations.
- The student will be skilled at writing polynomial equations when given information about its zeros.
- The student will be skilled explaining the Fundamental Theorem of Algebra.
- The student will be skilled explaining the relationships among the degree of the polynomial function, real roots, and multiplicities.

Evidence/Performance Tasks

Assessments

- Formative: Daily assessments using examples from class notes, NJSLA test bank problems, and/or Albert/AP Classroom assessments
- Summative: Teacher-created assessments, NJSLA test bank problems, Big Ideas Math online platform problems, Albert/AP Classroom and/or Big Ideas Math unit assessments
- Benchmark: IXL or teacher created diagnostic assessments in addition to unit assessments from Big Ideas Math
- Alternative Assessments: Student-centered activities such as scavenger hunts, various projects involving real world applications, and differentiated learning tasks in Khan Academy, DeltaMath, and IXL

- Answer essential questions

- Class discussion of daily topic
- Classwork and homework that assess the essential questions
- Provide alternative means of assessments for certain students
- Teacher Observation
- Tests and quizzes that assess the essential questions
- Written assignments that assess the essential questions that involves providing explanations

Learning Plan

Unit 4 Polynomials (16 days)

4.2 Adding, Subtracting, and Multiplying Polynomials (1 day)

- Name Polynomials according to the number of terms and degree (4.1).
- Adding, Subtracting, and Multiplying Polynomial Expressions.
- Remind students to distribute the negative sign when subtracting.
- Use both traditional distribution and box method to multiply.

4.3 Dividing Polynomials (2 days)

- Use Synthetic Division to divide polynomials by binomials of the form (x-k).
- Write remainder as the last term of the quotient over the binomial divisor.
- Discuss the Remainder Theorem. Students should evaluate functions f(k) and divide by x-k to see that the remainder is the same as the output when evaluated.

Quiz

4.4 Factoring Polynomials (2 days)

- Open lesson by multiplying polynomials where factoring patterns will be revealed later in lesson (ex: (2x)(3x-5)(3x+5) and (2x+1)(4x^2-2x+1)).
- Students will review all types of factoring, including GCF, Trinomials, Grouping, Difference of Squares. Students should factor completely where multiple types of factoring are required.
- Students will learn a new factoring pattern: sum and difference of cubes. Introduce, review from warm up, practice problems.
- Show students that the Factor Theorem is a specialized case of the Remainder Theorem; (x-k) is a

factor of f(x) if f(k)=0.

- Use the Factor Theorem to decide if a binomial is a factor of an unfactorable polynomial. Then use Synthetic Division to divide and continue to factor the polynomial.

4.5 Solving Polynomial Equations (2 days).

- Solve polynomials using factoring and the Zero-Product Property. Include repeated solutions and introduce the concept of multiplicities.
- Recognize that the terms zero, solution and root are synonymous. Also, that if k is a solution, then (x-k) is a factor. Students should understand that if they are given a factor of a polynomial, then they also know a solution of the polynomial and an x-intercept.
- Use Rational Root Theorem to name possible rational solutions.
- Use Synthetic Division to test possible roots and find all solutions by factoring quotient.
- Write polynomials in factored form after finding all solutions.
- Discuss implications when the value of the lead coefficient changes from 1 to a different value. The total number of possible rational roots increases. Students can use technology to find rational roots, then use Synthetic Division to reduce the polynomial.
- After using the Quadratic Formula to fully solve a polynomial, decide that irrational solutions are always found in pairs. Discuss the Irrational Conjugates Theorem.

Last big assessment before Midterms

SEMESTER BREAK

4.6 Fundamental Theorem of Algebra (2 days)

- The degree of a polynomial in standard form is easily identified by the highest exponent.
- The degree of a polynomial in factored form can be found by adding the multiplicities of the factors.
- The Fundamental Theorem of Algebra states that the total number of solutions of a polynomial equals the degree, when the total number of solutions includes repeated solutions, and pairs of irrational and complex solutions.
- Find all zeros of a polynomial by using the Rational Root Theorem and Factoring. Students will discover the Complex Conjugates Theorem by solving the remaining quadratic. Complex solutions always come in +/- pairs like irrational solutions.
- Students will write a polynomial in factored form and standard form given the zeros and at least one

irrational or complex solution.

4.8, 4.1: Graphing Polynomials (2 days, 1 day practice)

- Introduce the concept of end behavior and how it can be predicted by the characteristics of the given polynomial. Review parent function x squared and the reflection across the x-axis. Connect degree of quadratic and sign of lead coefficient with end behavior. Also, introduce the function x cubed and negative x cubed. Use these functions to remember end behaviors given degrees and signs of lead coefficients.

- Review multiplicities of factors from 4.5. When the multiplicity is even, the graph will stay of the same side of the x-axis or "bounce". When the multiplicity is odd, the graph will cross through the x-axis; as the multiplicity increases, the shape of the graph at the zero flattens. Use technology (Delta Math) to illustrate this concept. Students are given equations in factored form and should use Delta Math to graph and observe how the change of multiplicity is reflected in the graph.

- Graph polynomials using end behavior, zeros, and multiplicities. Emphasize to students that they should connect tails and zeros, with correct shapes according to multiplicities, moving from left to right.

- Students should evaluate an x value into the given polynomial to confirm the graph is correct.

- Sketch graph given constraints including increasing and decreasing intervals and values for x when $f(x) > 0$ and $f(x) < 0$. Students should also use interval notation.

- Students will identify odd and even functions and realize that functions could also be neither odd or even. They should see the symmetry visually and be able to decide if a polynomial is odd or even algebraically.

**Test on second half of Chapter 4: 4.6, 4.8, 4.1

Materials

Core instructional materials:Core Book List including Big Ideas Math Algebra 2 2022

Supplemental materials: Khan Academy, Edia, and DeltaMath

- District approved textbook and ancillary materials.
- Online materials such as: Khan Academy, Delta Math, Ed Puzzle, Edia

- Teacher created activiites
- Teacher created notes

Suggested Strategies for Modifications

Suggested Strategies for Modification for Algebra 2/Intro to Trig CP