Unit 03 Quadratic Equations and Complex Numbers

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 2**
Length: **3 weeks**
Status: **Published**

Brief Summary of Unit

The major work of this chapter is solving quadratic equations and inequalities, which may include imaginary solutions. Complex numbers are introduced after students review several methods they used in Algebra 1 to solve quadratic equations with real solutions. Students will also refresh and extend their understanding of solving nonlinear systems of equations. The methods for solving quadratic equations used in the first four sections were introduced in Algebra 1. In this chapter, solutions are not restricted to real numbers. In the second section, complex numbers are defined and operations with complex numbers are presented. This is followed by the method of completing the square so that the Quadratic Formula can be derived. In total, students will use five methods for solving quadratic equations: graphing, using square roots, factoring, completing the square, and using the Quadratic Formula. As the number of methods increases in the chapter, students should be making informed decisions as to which method to use for a given equation. The last two sections extend work with solving quadratic equations to solving nonlinear systems of equations and solving quadratic inequalities. Each of these topics requires recall of connected skills from work with linear equations and inequalities and transfer of these skills to quadratic equations and inequalities. For example, students use the methods of graphing, substitution, and elimination to solve nonlinear systems.

Revised Date: June 2024

Standards

1	Developing Responsibility for Learning: Cultivating independence, self-reflection, and responsibility for one's own learning.
4	Building Knowledge: Building strong content knowledge and connecting ideas across disciplines using a variety of text resources and media.
N.CN.A.1	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
N.CN.A.2	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
A.CED.A.1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
A.CED.A.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
N.CN.C.7	Solve quadratic equations with real coefficients that have complex solutions.

A.REI.B.4.a	Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
A.REI.B.4.b	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
A.REI.C.7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
F.IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
A.REI.D.11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
F.IF.C.8.a	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Essential Questions

- How does knowledge of real numbers help when working with complex numbers?
- Is there a most efficient method to solve a quadratic equation? Explain why.
- What real world situation results in a quantity whose rate of change with respect to another quantity is not constant?

Enduring Understandings

- Operations and properties of the real number system can be extended to situations involving complex numbers, which have many useful applications when working with quadratic functions.
- There are multiple methods used to solve quadratic equations; each equation should be analyzed and the solver should make a strategic decision to choose the most efficient method to solve the equation. Often, quadratic equations have solutions that are complex numbers.

Students Will Know

- Nonlinear systems can be solved using multiple methods.
- Operations can be performed with complex numbers.
- Quadratic equations can be solved by completing the square.
- Quadratic inequalities can be solved algebraically and graphically.
- Quadratics may be used to model real-world situations.
- The Quadratic Formula can be used to solve quadratic equations.

Students Will Be Skilled At

- Add, subtract, and multiply complex numbers.
- Define imaginary unit I and use it to rewrite the square root of a negative number.
- Describe the graph of the Quadratic inequality.
- Describe what a nonlinear systems of equations is.
- Find and interpret the discriminant of an equation.
- Find complex solutions of quadratic equations and Complex zeros of quadratic equations.
- Graph quadratic inequalities.
- Graph system of quadratic inequalities.
- Solve nonlinear systems using graphing, substitution, or elimination.
- Solve quadratic equations algebraically.
- Solve quadratic equations by completing the square.
- Solve quadratic equations by graphing each side of the equation.
- Solve quadratic equations by graphing.
- Solve quadratic equations using square roots.
- Solve quadratic equations using the Quadratic Formula.
- Solve quadratic inequalities algebraically and graphically.
- Use quadratic equations to solve real-life problems.

Evidence/Performance Tasks

Assessments

- Formative: Daily assessments using examples from class notes, NJSLA test bank problems, and/or Albert/AP Classroom assessments
- Summative: Teacher-created assessments, NJSLA test bank problems, Big Ideas Math online platform problems, Albert/AP Classroom and/or Big Ideas Math unit assessments
- Benchmark: IXL or teacher created diagnostic assessments in addition to unit assessments from Big Ideas Math
- Alternative Assessments: Student-centered activities such as scavenger hunts, various projects involving real world applications, and differentiated learning tasks in Khan Academy, DeltaMath, and IXL

- Answer essential questions
- Class discussion of daily topic
- Classwork and homework that assess the essential questions
- Provide alternative means of assessments for certain students
- Teacher Observation

- Tests and quizzes that assess the essential questions
- Written assignments that assess the essential questions that involves providing explanations

Learning Plan

Unit 3 should begin during the first marking period, but most likely will not be completed until the second marking period has begun.

Unit 3 Quadratic Equations and Complex Numbers (3 weeks) (Chapter 3)

*Before starting with the sections in the textbook, students should review factoring: AC (include Grouping) or Trial and Error, Difference of Squares, GCF (2 days) This need to spend time reviewing factoring from Algebra 1 should lessen over time as foundational skills will improve.

3.1 Solving Quadratic Equations (2 days new material, 1 day practice)

- Make the connection, using graphing from the previous chapter, that the solution to a quadratic equation is the x-intercept. Also, use the term root, which can be interchanged with solution.
- Discuss three methods for solving quadratic equations: factoring, taking the square root, and graphing. Discuss characteristics of equations that would suggest the most efficient method for solving.
- Discuss the Zero Product Property and why it works when factoring to solve.
- Review simplifying radicals when solving by taking the square root.
- This section may take three days, if you include a pure work day, where students solve equations. These methods are important to the class and students may need more practice, other than homework.

**Quiz on factoring and solving quadratics using the first three methods. Students should recognize the difference between the instructions to “solve” and to “factor”.

3.2 Complex Numbers (2 days)

- Introduce Complex numbers by asking the solutions to a quadratic that doesn’t intersect the x-axis.
- Discuss the classification of numbers, now including Complex Numbers and Pure Imaginary numbers.
- Define i=-1
- Simplify square roots of negative numbers.
- Students should understand that an imaginary number multiplied by another imaginary number gives a real number: i2 = -1
- Operations on complex numbers: adding, subtracting, multiplying and combine operations (distributive property).

- Recognize complex conjugates: (a+bi)(a-bi) = a2- (-1)(b2).
- Omit division of complex numbers, including by a single term and binomial. Omit higher powers of i
- Solve quadratic equations by taking the square root and simplifying the negative square root into two imaginary solutions.

3.3 Solving Quadratic Equations by Completing the Square (1 day)

- Review re-writing a perfect square trinomial as a binomial squared.
- Solve a quadratic where a perfect square trinomial is given and students rewrite as a binomial squared. Take the square root of both sides and continue to solve, where the solution could be any form of a complex number.
- Decide the process for creating a perfect square trinomial by adding (b/2)^2 to both sides of the equation.
- Solve, when a is not equal to 1, by completing the square.

3.4 Solve Quadratic Equations by using the Quadratic Formula (1 day plus 1 practice day for 3.3 and 3.4)

- Use a mnemonic device (video, song, etc) so students memorize the Quadratic Formula.
- Review solving Quadratics using the Quadratic Formula by setting the equation equal and combing like terms.
- Discuss, in detail, how to simplify solutions. Students tend to cancel common factors in only two terms, but must cancel from all three terms.
- Find and interpret the discriminant of a quadratic equation. Students should find the discriminant to decide which method of solving the equation to use. Students should understand that the discriminant is not the solution to the equation.

**Quiz on 3.2, 3.3 and 3.4

Modeling with Quadratic Equations

Use real world problems throughout the chapter to review techniques in preparation for a test on Factoring and 3.1-3.4. Also, projectile problems can be used. 11th grade students will make a connection with problems from Physics class.

** Test on Factoring and 3.1-3.4

The next two sections can be completed as independent units, depending on timing within the school building and calendar.

3.5: Solving Nonlinear systems. (2 days)

- Review graphing circles in standard form, covered in Geometry. (If time permits: Convert from Standard Form to (h,k) form by completing the square for both x and y.)
- Solve nonlinear systems (linear, quadratic, circles) by graphing. Review graphing and transformation techniques as well as technology to find intersection points.
- Solve nonlinear systems by substitution. Students need to be reminded that the solution is an ordered pair, and that they need to substitute their value for x back into either equation.
- Solve nonlinear systems by elimination.
- Students should be aware of limitations of each method and decide which method to use for any given system.

3.6 Quadratic Inequalities (2 days)

- Use quadratic graphing techniques to graph a quadratic inequality in two variables. Use solid or dotted lines to include or exclude border points.
- Graph a system of quadratic inequalities in two variables, shading the final solution set.
- Solve quadratic inequalities in one variable algebraically and graphically.
- Introduce interval notation to represent the solution set as opposed to inequality notation.

No sums/differences of cubes and no focus of parabola as per evidence statements need to be included in this unit.

Materials

Core instructional materials:Core Book List including Big Ideas Math Algebra 2 2022

Supplemental materials: Khan Academy, Edia, and DeltaMath

- District approved textbook and ancillary materials.
- Online resources: Delta Math, Ed Puzzle, Edia, Khan Academy
- Teacher created activiites
- Teacher created notes

Suggested Strategies for Modification

Suggested Strategies for Modification for Algebra 2/Intro to Trig CP