Unit 3 - Geometry: Pythagorean Theorem, Congruence and Similarity Transformations

Content Area: **Mathematics**
Course(s):
Time Period: **Marking Period 3**
Length: **9-10 weeks**
Status: **Published**

Brief Summary of Unit

The unit covers a comprehensive exploration of real numbers, the Pythagorean Theorem, geometric transformations, angles and triangles, and volumes of solids. Students begin by understanding real numbers, including square and cube roots, before delving into the Pythagorean Theorem's applications and the converse. Geometry topics include translations, reflections, rotations, congruence, similarity, and angles within polygons and triangles, culminating in an exploration of volumes for cylinders, cones, and spheres, as well as surface areas of similar solids. The unit integrates theoretical concepts with practical applications, fostering a deep understanding of foundational mathematical principles and geometric properties.

Revision Date: 5/30/24

Standards

1	Make sense of problems and persevere in solving them
8.NS.A.1	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually and convert a decimal expansion which repeats eventually into a rational number.
8.NS.A.2	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., $\square^2$).
8.NS.A.3	Understand that the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
4	Model with mathematics
5	Use appropriate tools strategically
L.KL.8.2.B	Gather vocabulary knowledge when selecting a word or phrase important to comprehension or expression.
7	Look for and make use of structure
8	Look for and express regularity in repeated reasoning
L.VI.8.4.B	Use the relationship between particular words to better understand each of the words.
8.F.A.1	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

8.G.A.1	Verify experimentally the properties of rotations, reflections, and translations:
8.G.A.2	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
8.G.A.3	Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
8.G.A.4	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
8.G.A.5	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.
8.G.B.6	Explain a proof of the Pythagorean Theorem and its converse.
8.G.B.7	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
8.G.B.8	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
8.G.C.9	Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.
WORK.K-12.9.1	All students will develop career awareness and planning, employability skills and foundational knowledge necessary for success in the workplace.

Essential Questions

- How can models be used to compare three-dimensional objects?
- How can relationships between shapes and angles be used to identify properties and missing values?
- How can the Pythagorean Theorem be applied to real life scenarios?
- How can the Pythagorean Theorem calculate missing sides of right triangles?
- How can we use our knowledge of geometric properties to visualize transformations between figures?
- How can you determine if a two-dimensional figure is similar or congruent to another transformation?
- How can you justify facts about angles in triangles and transversals?
- How do you find and simplify square roots? What about cube roots?
- What are real numbers, and how are they classified into rational and irrational numbers?
- What does it mean for figures and angles to be congruent or similar?
- What is the volume formula for cylinders, cones, and spheres, and how do you calculate it?

Enduring Understandings

- Properties of the angles of a triangle are able to be used to state facts and find missing values of similar triangles and interior/exterior angles.
- Real numbers encompass both rational and irrational numbers, with square roots and cube roots being essential components for solving equations and understanding number properties.
- The meaning of volume and how it can be applied to three-dimensional figures.

- Transversals create sets of congruent and supplementary angles.
- Two-dimensional figures are congruent if obtained by a series of translations, reflections and/or rotations.
- Two-dimensional figures are similar if obtained by dilations.
- Understanding geometric concepts and their applications enhances problem-solving skills and fosters critical thinking in various real-world scenarios.
- What it means for figures to be congruent or similar.
- When the Pythagorean Theorem can be applied.

Students Will Know

- How to find and identify angle measures in parallel lines when cut by transversals.
- How to find and identify interior and exterior angles in similar triangles.
- How to graph figures on the coordinate plane.
- How to identify similar and congruent figures.
- How to prove the Pythagorean Theorem and its converse.
- How to translate, rotate, reflect, and dilate figures on the coordinate plane to obtain their resulting images.
- How to use the Pythagorean Theorem to find missing side lengths.
- Relationship between the area of square and the length of its side.
- Students will understand the volume formulas for cylinders, cones, and spheres, and they will be able to calculate the volumes of these solids.
- Various volume formulas and their applications.

Students Will Be Skilled At

- Applying the Pythagorean Theorem to real life situations.
- Calculating the volumes of cylinders, cones, and spheres using the appropriate volume formulas.
- Classifying real numbers into rational and irrational categories and performing operations with them, including finding square roots and cube roots.
- Finding missing angle measures when parallel lines are cut by a transversal.
- Finding missing sides of right triangles using the Pythagorean Theorem.
- Graphing figures.
- Identifying alternate interior, alternate exterior, corresponding, vertical, and similar angles.
- Identifying right triangles using the Pythagorean Theorem.
- Identifying the series of transformations that took place between a pre-image and an image.
- Identifying what makes figures congruent and similar.
- Translating, rotating, reflecting and dilating figures with and without the use of a coordinate plane.
- Using angle properties of triangles to find, identify, and prove measures of interior and exterior angles of a triangle.

Evidence/Performance Tasks

Assessments

- Formative: Daily assessments using examples from class notes, iReady MyPath, Big Ideas Math online platform problems, and NJSLA test bank problems
- Summative: Teacher-created assessments, NJSLA test bank problems, Big Ideas Math online platform problems, Big Ideas Math unit assessments
- Benchmark: iReady diagnostic assessments and district placement assessments in addition to unit assessments from Big Ideas Math
- Alternative Assessments: Student-centered activities such as scavenger hunts, various projects involving real world applications, and adaptive learning tasks in iReady, Khan Academy, and Big Ideas Math

- Answer essential questions
- Class discussion of daily topic
- Classwork and homework that assess the essential questions
- Entrance/exit slips
- Provide alternative means of assessments for certain students
- Teacher Observation
- Tests and quizzes that assess the essential questions
- Written assignments that assess the essential questions that involves providing explanations

Learning Plan

Day 1: Real Numbers and Classification

- Objective: Introduce real numbers, their classification (rational, irrational), and basic operations.
- Activity:
    1. Lecture on real numbers and classification.
    2. Teach basic operations (addition, subtraction, multiplication, division) with real numbers.
    3. Practice problems on identifying rational and irrational numbers.

Day 2: More on Real Numbers and Operations

- Objective: Reinforce understanding of real numbers and operations.
- Activity:

1. Review classification of real numbers.
2. Practice more complex operations with real numbers.
3. Introduce the concept of square roots.

Day 3: Square Roots

- Objective: Teach how to find square roots and simplify square roots.
- Activity:
    1. Define square roots and the square root symbol.
    2. Teach methods to find square roots manually.
    3. Practice simplifying square roots.

Day 4: Pythagorean Theorem Introduction

- Objective: Introduce the Pythagorean Theorem and its application in right triangles.
- Activity:
    1. Lecture on the Pythagorean Theorem and its history.
    2. Provide examples of right triangles and demonstrate the theorem.

Day 5: Pythagorean Theorem Practice

- Objective: Practice problems using the Pythagorean Theorem.
- Activity:
    1. Practice problems finding missing sides of right triangles.
    2. Explore real-life applications of the Pythagorean Theorem.

Day 6: Cube Roots

- Objective: Introduce cube roots and teach how to find them.
- Activity:
    1. Define cube roots and the cube root symbol.
    2. Teach methods to find cube roots manually.
    3. Practice finding cube roots.

Day 7: Equations with Cube Roots

- Objective: Solve equations involving cube roots.

- Activity:
    1. Solve equations containing cube roots.
    2. Discuss properties of rational and irrational numbers.

Day 8: Review and Quiz

- Objective: Review concepts covered in the past week and assess understanding.
- Activity:
    1. Review real numbers, square roots, Pythagorean Theorem, and cube roots.
    2. Quiz or assignment to assess understanding.

Day 9: Converse of Pythagorean Theorem

- Objective: Introduce the converse of the Pythagorean Theorem.
- Activity:
    1. Lecture on the converse of the Pythagorean Theorem.
    2. Provide examples and practice problems related to the converse.

Day 10: Proofs and Converse Practice

- Objective: Explore proofs and reasoning behind the converse of the Pythagorean Theorem.
- Activity:
    1. Explore proofs of the converse of the Pythagorean Theorem.
    2. Practice problems applying the converse.

Day 11: Translations in Geometry

- Objective: Define translations and how they affect geometric figures.
- Activity:
    1. Define translations and their properties.
    2. Provide examples and practice problems on performing translations.

Day 12: Reflections in Geometry

- Objective: Define reflections and their effects on geometric figures.
- Activity:
    1. Define reflections and their properties.

2. Teach how to perform reflections and identify reflection properties.

Day 13: Rotations in Geometry

- Objective: Define rotations and their impact on geometric figures.
- Activity:
    1. Define rotations and their properties, including angle of rotation and direction.
    2. Provide examples and practice problems on performing rotations.

Day 14: Congruent Figures and Dilations

- Objective: Discuss congruence and how to identify congruent figures. Introduce dilations and their effects on geometric figures.
- Activity:
    1. Discuss congruence and teach how to identify congruent figures.
    2. Introduce dilations and discuss their effects on figures.

Day 15: Similar Figures

- Objective: Define similarity and how to identify similar figures.
- Activity:
    1. Define similarity and discuss its properties.
    2. Teach methods for determining similarity between figures.

Day 16: Perimeters and Areas of Similar Figures

- Objective: Discuss how perimeters and areas change in similar figures.
- Activity:
    1. Lecture on perimeters and areas of similar figures.
    2. Teach methods for calculating perimeters and areas of similar figures.

Day 17: Recap and Review

- Objective: Review concepts covered in geometry and assess understanding.
- Activity:
    1. Review translations, reflections, rotations, congruence, dilations, similarity, perimeters, and areas.
    2. Quiz or assignment to assess understanding.

Day 18: Angles of Triangles

- Objective: Discuss angles within triangles, including the sum of interior angles.
- Activity:
    1. Explore different types of triangles based on angles and sides.
    2. Discuss the sum of interior angles in triangles.
    3. Practice problems on finding missing angles in triangles.

Day 19: Pythagorean Theorem and Triangle Angles

- Objective: Introduce the Pythagorean Theorem's relation to triangle angles.
- Activity:
    1. Review the Pythagorean Theorem and its applications.
    2. Discuss how the Pythagorean Theorem can help find triangle angles.
    3. Practice problems applying the Pythagorean Theorem to triangle angles.

Day 20: Angles of Polygons

- Objective: Teach how to calculate angles in polygons using exterior and interior angles.
- Activity:
    1. Discuss the sum of interior angles in polygons.
    2. Introduce exterior angles of polygons and their properties.
    3. Practice problems on calculating angles in polygons.

Day 21: Using Similar Triangles

- Objective: Introduce similar triangles and their properties.
- Activity:
    1. Define similar triangles and discuss their properties.
    2. Teach methods for determining similarity between triangles.
    3. Practice problems using similar triangles.

Day 22: Volumes of Cylinders

- Objective: Introduce cylinders and their volume formula.
- Activity:

1. Define cylinders and their properties.
2. Introduce the formula for calculating the volume of cylinders.
3. Practice problems on finding volumes of cylinders.

Day 23: Volumes of Cones

- Objective: Define cones and their volume formula.
- Activity:
    1. Define cones and their properties.
    2. Introduce the formula for calculating the volume of cones.
    3. Practice problems on finding volumes of cones.

Day 24: Volumes of Spheres

- Objective: Discuss spheres and their volume formula.
- Activity:
    1. Define spheres and their properties.
    2. Introduce the formula for calculating the volume of spheres.
    3. Practice problems on finding volumes of spheres.

Day 25: Surface Areas of Similar Solids

- Objective: Introduce surface area formulas for similar solids.
- Activity:
    1. Discuss surface area formulas for similar solids.
    2. Introduce methods for calculating surface areas of similar solids.
    3. Practice problems on finding surface areas of similar solids.

Day 26: Review and Practice

- Objective: Review concepts covered in the past week and practice problems.
- Activity:
    1. Review angles, triangle properties, similar triangles, and volumes of solids.
    2. Practice problems covering a variety of topics.

Day 27: Assessment

- Objective: Assess understanding of the entire learning plan.
- Activity:
    1. Comprehensive quiz or test covering all topics.
    2. Review any areas where students may need additional practice or clarification.

This breakdown covers the learning plan over approximately 27 days, ensuring each topic is thoroughly covered with objectives and activities aligned accordingly. Adjustments can be made based on the pace of learning and student needs.

Materials

Core instructional materials: Core Book List including Big Ideas Math textbook and online platform for all levels of grade 6, 7, 8, and Algebra 1

Supplemental materials: Khan Academy, Edia, and DeltaMath

- Calculators/Math Tools
- District approved textbook
- Manipulatives
- Teacher created activiites
- Teacher created notes
- Websites, such as Khan Academy

Suggested Strategies for Modifications

Possible accommodations/modification for Grade 8