Unit 1: The Mathematics of Social Choice & Government

Content Area: **Math**
Course(s): **Discrete Mathematics**
Time Period:
Length: **35-40 Days/Grades 11-12**
Status: **Published**

Title Section

Department of Curriculum and Instruction

Belleville Public Schools

Curriculum Guide

DISCRETE MATHEMATICS & STATISTICS, GRADES 11/12

THE MATHEMATICS OF SOCIAL CHOICE & GOVERNMENT

Belleville Board of Education

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Unit Overview

In this unit, students will learn mathematical applications related to political science, social science and government.

These applications include:

- How elections are decided
- How the power of individuals, groups or voting blocs are measured
- How the seats within a governmental body are apportioned to its states

Enduring Understanding

IN THIS CHAPTER, STUDENTS WILL UNDERSTAND:

- A winner of a voting process can vary depending on the method that is used to determine it.
- There is no optimal voting method for all situations. Each method has its own inherent flaws.
- In a decision-making body, each member may contribute a different number of votes to the process.
- The number of votes a decision maker has can possibly grant them all the power to control a decision, or none at all.

- Congressional/Governmental representation that is determined by an apportionment method is based on the results of a census.
- There is no apportionment method that can distribute representation to each state of a nation in every possible circumstance.

Essential Questions

IN THIS CHAPTER, WE WILL ASK:

- How are the desires of many individuals combined to produce a single result?
- What are the advantages and disadvantages of using different voting methods?
- How do we define fairness in a voting procedure?
- How can the power of the members of a decision-making body be measured?
- How do populations determine the number of representatives of each state of a nation's government?
- In what ways can each apportionment method produce a unfair distribution of representatives?

Exit Skills

BY THE END OF THIS UNIT, THE STUDENT SHOULD BE ABLE TO:

- Create, read and organize preference ballots and schedules
- Determine winners and rankings of a voting process by using several procedures, and compare & contrast their outcomes
- Applying several criteria to determine the fairness and/or flaws of a voting process
- Analyze voter's power in a weighted voting process
- Apportion votes based on population using several methods, and compare & contrast their outcomes
- Applying several criteria to determine the flaws of an apportion method

New Jersey Student Learning Standards (NJSLS)

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems.
MA.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Interdisciplinary Connections

LA.RH.11-12.6	Evaluate authors' differing perspectives on the same historical event or issue by assessing the authors' claims, reasoning, and evidence.
LA.RH.11-12.8	Evaluate an author's claims, reasoning, and evidence by corroborating or challenging them with other sources.
LA.RST.11-12.3	Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.
LA.RST.9-10.5	Analyze the relationships among concepts in a text, including relationships among key terms (e.g., force, friction, reaction force, energy).
9.3.12.BM.1	Utilize mathematical concepts, skills and problem solving to obtain necessary information for decision-making in business.
SOC.6.1.12.A.14.e	Evaluate the effectiveness and fairness of the process by which national, state, and local officials are elected and vote on issues of public concern.

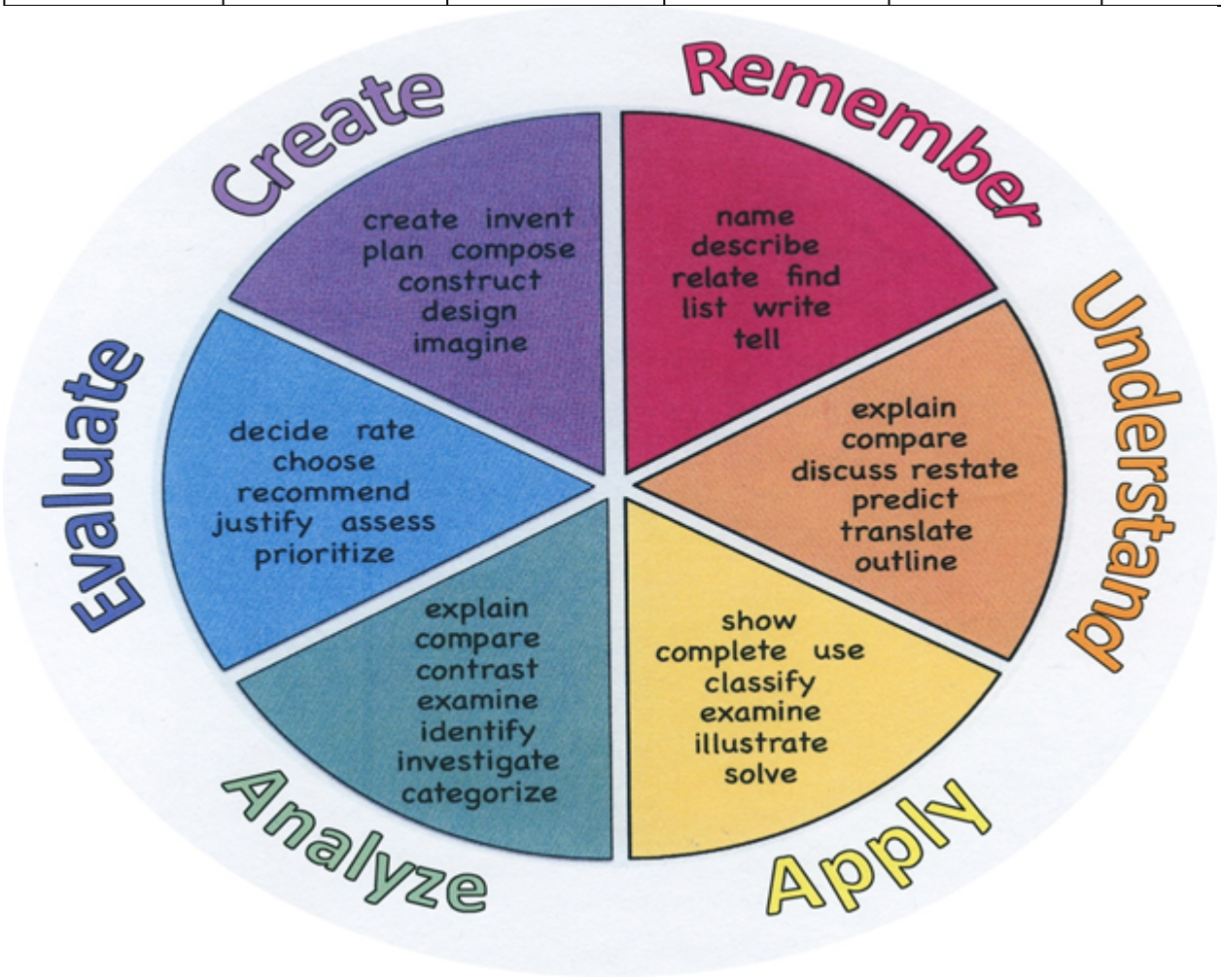
Learning Objectives

- Construct and distinguish between different types of voting ballots (single-choice, preference, truncated preference)
- Organize a set of preference ballot votes into a preference schedule
- Determine winners and rankings of a voting process by using a specific procedure (plurality, Borda Count, elimination, pairwise comparisons)
- Compare & contrast the outcomes of each voting method
- Judge the fairness and/or flaws of a voting process by applying a criterion to it (majority, Condorcet, monotonicity)
- Define the elements of a weighted voting system, and detect players who are dictators or have veto power
- Calculate a voter's power in a weighted voting system by developing a list of coalitions under a specific index rule (Banzhaf, Shapley-Shubik)
- Determine the number of subsets and coalitions in a weighted voting system by calculating exponential functions and permutations
- Determine the apportionment of votes based on population using a specific method (Hamilton, Jefferson, Adams, Webster-Wilcox, Huntington-Hill)
- Compare & contrast the outcomes of each apportionment method
- Reconstruct the flaws of an apportion method according to the Quota Rule, a violation (upper, lower) or a paradox (Alabama, Population, New States)

Action Verbs:

Remember	Understand	Apply	Analyze	Evaluate	Create
Choose	Classify	Choose	Categorize	Appraise	Combine
Describe	Defend	Dramatize	Classify	Judge	Compose
Define	Demonstrate	Explain	Compare	Criticize	Construct
Label	Distinguish	Generalize	Differentiate	Defend	Design
List	Explain	Judge	Distinguish	Compare	Develop
Locate	Express	Organize	Identify	Assess	Formulate
Match	Extend	Paint	Infer	Conclude	Hypothesize
Memorize	Give Examples	Prepare	Point out	Contrast	Invent
Name	Illustrate	Produce	Select	Critique	Make
Omit	Indicate	Select	Subdivide	Determine	Originate
Recite	Interrelate	Show	Survey	Grade	Organize
Select	Interpret	Sketch	Arrange	Justify	Plan
State	Infer	Solve	Breakdown	Measure	Produce
Count	Match	Use	Combine	Rank	Role Play
Draw	Paraphrase	Add	Detect	Rate	Drive
Outline	Represent	Calculate	Diagram	Support	Devise

Point Quote Recall Recognize Repeat Reproduce	Restate Rewrite Select Show Summarize Tell Translate Associate Compute Convert Discuss Estimate Extrapolate Generalize Predict	Change Classify Complete Compute Discover Divide Examine Graph Interpolate Manipulate Modify Operate Subtract	Discriminate Illustrate Outline Point out Separate	Test	Generate Integrate Prescribe Propose Reconstruct Revise Rewrite Transform



Suggested Activities & Best Practices

STUDENTS WILL REACH OBJECTIVES AND ACQUIRE SKILLS & UNDERSTANDING THROUGH:

- Examination and performance on problems selected from the texts
- Student groups with assigned specific roles that can assist each other in overall understanding
- Exit tickets to offer additional summary of key concepts, level of understanding and additional student questions
- Project-based learning for students to conduct voting experiments

Assessment Evidence - Checking for Understanding (CFU)

SPECIFIC SAMPLES INCLUDE:

- Exit tickets at the close of each lesson will address definitions, concepts and formulas (EX: Recognize why certain apportionments break specific rules at a given moment) (Formative)
- Demonstrating a proper comparison of voting methods for the same set of voting ballots (Formative)
- Chapter Test/Quiz (Summative)
- Common Quarterly/Benchmark Exams - Quarter 1 Exam for this unit (Benchmark)
- Web-Based Assessments (using Google Forms, ALEKS, Edulastic, Khan Academy, etc.) (Formative/Summative)
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- Admit Tickets
- Common Benchmarks
- Compare & Contrast
- Define
- Describe
- Evaluate
- Evaluation rubrics
- Exit Tickets
- Explaining
- Quizzes
- Self- assessments
- Study Guide
- Surveys
- Teacher Observation Checklist
- Think, Pair, Share
- Think, Write, Pair, Share
- Unit review/Test prep
- Unit tests
- Web-Based Assessments

Primary Resources & Materials

- Excursions in Modern Mathematics 9th edition textbook (Frank Tannenbaum)

- Excursions in Modern Mathematics 6th edition textbook (Frank Tannenbaum)

Ancillary Resources

Sample web pages based on material are included here. This list will be edited as more reference material is found.

- http://www.mscf.uky.edu/~lee/ma111fa11/slides01.pdf
- https://marinmathcircledotorg.files.wordpress.com/2015/12/mmcadv-20120111-votinglecture-ernestodiaz.pdf

Technology Infusion

CHROMEBOOK: Students will use the following functions within their Chromebooks for the tasks described:

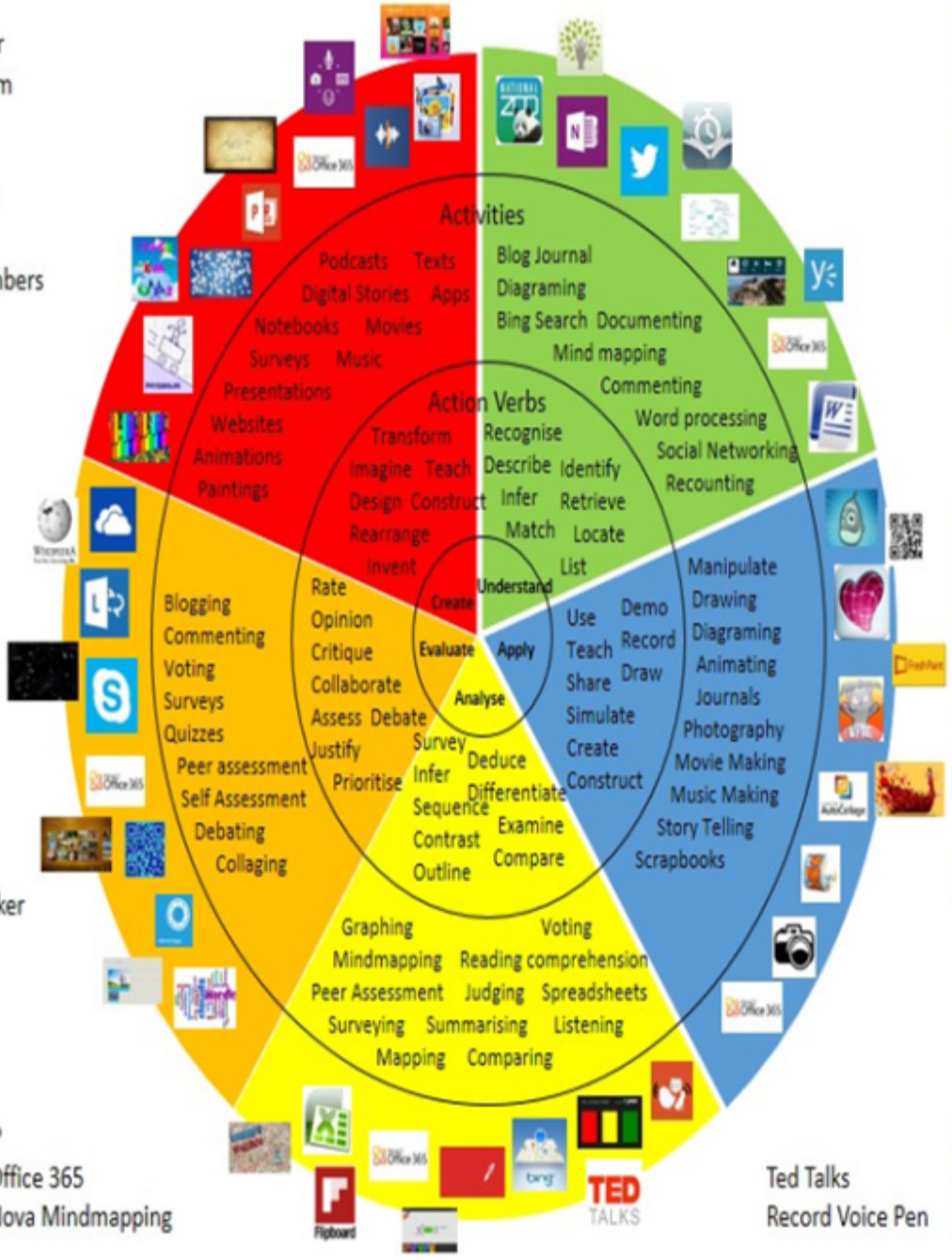
- Google Forms: Creation of survey questions for fellow students to answer for voting projects
- Google Sheets: Construction of lists for coalitions and critical players for weighted voting power indexes, and formulas and tables for simplifying apportionment experiment scenarios

SMART TV: Real-time tests of apportionment experiment scenarios and demonstrations of paradoxes and rule violations

Win 8.1 Apps/Tools Pedagogy Wheel

Podcasts
Photostory 3
Kid Story Builder
Music Maker Jam
Paint A Story
Office 365
MS PowerPoint
Stack 'Em Up
NqSquared Numbers
Physamajig
Xylophone 8

Wikipedia
Skydrive
Lync
SkyMap
Skype
Office 365
Puzzle Touch
Easy QR
Memorylage
Life Moments
Word Cloud Maker



Where's Waldo?
MS Excel
Flipboard
Office 365
Nova Mindmapping

Ted Talks
Record Voice Pen

Originally taken from http://www.coetail.com/vzimmer/files/2013/02/iPadagogy-Wheel.001.jpg
And adapted for Windows 8.1 devices by Charlotte Beckhurst @CharBeckhurst

Alignment to 21st Century Skills & Technology

Mastery and infusion of **21st Century Skills & Technology** and their Alignment to the core content areas is essential to student learning. The core content areas include:

- English Language Arts;
- Mathematics;
- Social Studies, including American History, World History, Geography, Government and Civics, and Economics;
- Technology

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP5	Consider the environmental, social and economic impacts of decisions.
CRP.K-12.CRP5.1	Career-ready individuals understand the interrelated nature of their actions and regularly make decisions that positively impact and/or mitigate negative impact on other people, organization, and the environment. They are aware of and utilize new technologies, understandings, procedures, materials, and regulations affecting the nature of their work as it relates to the impact on the social condition, the environment and the profitability of the organization.
CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP11	Use technology to enhance productivity.
CAEP.9.2.12.C.2	Modify Personalized Student Learning Plans to support declared career goals.
TECH.8.1.12.E.CS4	Process data and report results.
TECH.8.1.12.F.CS3	Collect and analyze data to identify solutions and/or make informed decisions.
TECH.8.1.12.F.CS4	Use multiple processes and diverse perspectives to explore alternative solutions.

21st Century Skills/Interdisciplinary Themes

- Communication and Collaboration
- Creativity and Innovation
- Critical thinking and Problem Solving
- ICT (Information, Communications and Technology) Literacy
- Information Literacy
- Life and Career Skills
- Media Literacy

21st Century Skills

- Civic Literacy
- Financial, Economic, Business and Entrepreneurial Literacy
- Global Awareness

Differentiation

SPECIFIC EXAMPLES INCLUDE:

- Small task-oriented groups where each member is responsible for tallying, solution layouts, etc.
- Maniluatives: Sample ballots for students to fill out for instant examples of voting methods
- Study guides provided prior to quizzes and tests

Differentiations:

- Small group instruction
- Small group assignments
- Extra time to complete assignments
- Pairing oral instruction with visuals
- Repeat directions
- Use manipulatives
- Study guides
- Rephrase written directions
- Additional time
- Preview vocabulary
- Preview content & concepts
- Highlight text
- Student(s) work with assigned partner
- Visual presentation
- Assistive technology
- Small group setting

Hi-Prep Differentiations:

- Alternative formative and summative assessments
- Games and tournaments
- Group investigations
- Independent research and projects
- Interest groups
- Project-based learning
- Problem-based learning
- Tiered activities/assignments
- Varying organizers for instructions

Lo-Prep Differentiations

- Exploration by interest
- Flexible grouping
- Goal setting with students
- Mini workshops to re-teach or extend skills
- Open-ended activities
- Think-Pair-Share
- Varied supplemental materials

Special Education Learning (IEP's & 504's)

SPECIFIC EXAMPLES INCLUDE:

- Note cards for assembling Google Sheet formulas for apportionment
- One-on-one oral questioning during testing to elicit responses

- printed copy of board work/notes provided
- additional time for skill mastery
- assistive technology
- behavior management plan
- Center-Based Instruction
- check work frequently for understanding
- computer or electronic device utilizes
- extended time on tests/ quizzes
- have student repeat directions to check for understanding
- highlighted text visual presentation
- modified assignment format
- modified test content
- modified test format
- modified test length
- multi-sensory presentation
- multiple test sessions
- preferential seating
- preview of content, concepts, and vocabulary
- Provide modifications as dictated in the student's IEP/504 plan
- reduced/shortened reading assignments
- Reduced/shortened written assignments
- secure attention before giving instruction/directions
- shortened assignments
- student working with an assigned partner
- teacher initiated weekly assignment sheet
- Use open book, study guides, test prototypes

English Language Learning (ELL)

SPECIFIC EXAMPLES INCLUDE:

- Translated material
- Peer partners for assignments and tests with students that can translate material and meanings of concepts verbally

- teaching key aspects of a topic. Eliminate nonessential information
- using videos, illustrations, pictures, and drawings to explain or clarif
- allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards, charts, graphs, slide shows, videos, etc.) to demonstrate student's learning;
- allowing students to correct errors (looking for understanding)
- allowing the use of note cards or open-book during testing
- decreasing the amount of workpresented or required
- having peers take notes or providing a copy of the teacher's notes
- modifying tests to reflect selected objectives
- providing study guides
- reducing or omitting lengthy outside reading assignments
- reducing the number of answer choices on a multiple choice test
- tutoring by peers
- using computer word processing spell check and grammar check features
- using true/false, matching, or fill in the blank tests in lieu of essay tests

At Risk

SPECIFIC EXAMPLES INCLUDE:

- Printed or video copy of material missed during excessive absences
- Corrections of incorrect work from tests
- Rewriting of test questions to include limited range of apportionment divisors for student to execute within the work on free-response test questions

- allowing students to correct errors (looking for understanding)
- teaching key aspects of a topic. Eliminate nonessential information
- allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards, charts, graphs, slide shows, videos, etc.) to demonstrate student's learning
- allowing students to select from given choices
- allowing the use of note cards or open-book during testing
- collaborating (general education teacher and specialist) to modify vocabulary, omit or modify items to reflect objectives for the student, eliminate sections of the test, and determine how the grade will be determined prior to giving the test.
- decreasing the amount of workpresented or required
- having peers take notes or providing a copy of the teacher's notes
- marking students' correct and acceptable work, not the mistakes
- modifying tests to reflect selected objectives
- providing study guides
- reducing or omitting lengthy outside reading assignments
- reducing the number of answer choices on a multiple choice test
- tutoring by peers
- using authentic assessments with real-life problem-solving

- using true/false, matching, or fill in the blank tests in lieu of essay tests
- using videos, illustrations, pictures, and drawings to explain or clarify

Talented and Gifted Learning (T&G)

SPECIFIC EXAMPLES INCLUDE:

- Complete "Running"-level problems in textbook containing higher-level thinking
- Student can construct original examples that can demonstrate full mastery of specific concepts and objectives
- Provide students with resources to allow them to move forward at a faster pace when they display faster mastery of learning objectives

- Above grade level placement option for qualified students
- Advanced problem-solving
- Allow students to work at a faster pace
- Complete activities aligned with above grade level text using Benchmark results
- Create a plan to solve an issue presented in the class or in a text
- Higher order, critical & creative thinking skills, and discovery
- Multi-disciplinary unit and/or project
- Teacher-selected instructional strategies that are focused to provide challenge, engagement, and growth opportunities
- Utilize exploratory connections to higher-grade concepts
- Utilize project-based learning for greater depth of knowledge

Sample Lesson

Unit Name: The Mathematics of Social Choice & Government

NJSLS:

Interdisciplinary Connection:

Statement of Objective: SWBAT calculate votes and determine the results of an election by the Borda Count method.

Anticipatory Set/Do Now: Determine the winner of a Plurality vote in two sample election preference ballots

Learning Activity:

DEFIINTIONS OF THE DAY: Borda Count

LECTURE NOTES: Assigning scores to ranked votes in a preference ballot, Calculating the total scores to find a winner, Comparing the results to the winner of a Plurality vote, Critiquing fairness of the process through identifying possible Majority & Condorcet Rule violations

TRY IT: Students work independently or in assigned groups on practice questions in notebooks as notes are checked for complete verbal information

Student Assessment/CFU's: Define, Surveys, Evaluate, Compare & Contrast, Self-Assessment, Teacher Observation Checklist, Exit Tickets

Materials: Notebook, Chromebook (Google Sheets)

21st Century Themes and Skills: Criticial Thinking & Problem Solving, Media Literacy, Communication & Collaboration

Differentiation/Modifications: Students may be paired to guide others in the setup of scoring votes/creating Google Sheet formulas, etc.

Integration of Technology: Chromebook (Google Sheets) to create counting formulas to more quickly calculate total scores

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