Unit 3: The Mathematics of Division

Content Area: **Math**
Course(s): **Discrete Mathematics**
Time Period:
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Status: **Published**

Title Section

Department of Curriculum and Instruction

Belleville Public Schools

Curriculum Guide

DISCRETE MATHEMATICS & STATISTICS, GRADES 11/12

THE MATHEMATICS OF DIVISION

Belleville Board of Education

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Unit Overview

In this unit, students will learn mathematical applications related to social science that can be linked to the mathematics of growth.

These applications include:

- How assets that are commonly owned can be divided in a fair and equitable manner (including the division of assets in an estate that was grown through interest and investment to pass down to the next generation after death)

Enduring Understanding

IN THIS UNIT, STUDENTS WILL UNDERSTAND:

- A fair division problem may be discrete or continuous.
- There are several methods by which an estate or collection of goods can be divided.
- The success of an estate division requires that each player places a value on each object in the estate.

Essential Questions

IN THIS UNIT, WE WILL ASK:

- How do we define fairness?
- How do we determine the value of an appraisable or definitively priced object?
- Are the methods used to divide appraised or definitively priced objects fair?
- How can we divide an estate when the individuals have different views of the values of its parts?

Exit Skills

BY THE END OF THIS UNIT, THE STUDENT SHOULD BE ABLE TO:

- Distinguish between discrete and continuous fair division problems
- Apply appropriate "cake" division algorithms to divide goods fairly between two or more parties
- Divide an estate's goods fairly between multiply parties using the method of sealed bids
- Divide goods fairly between multiple parties using the method of markers

New Jersey Student Learning Standards (NJSLS-S)

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems.

Interdisciplinary Connections

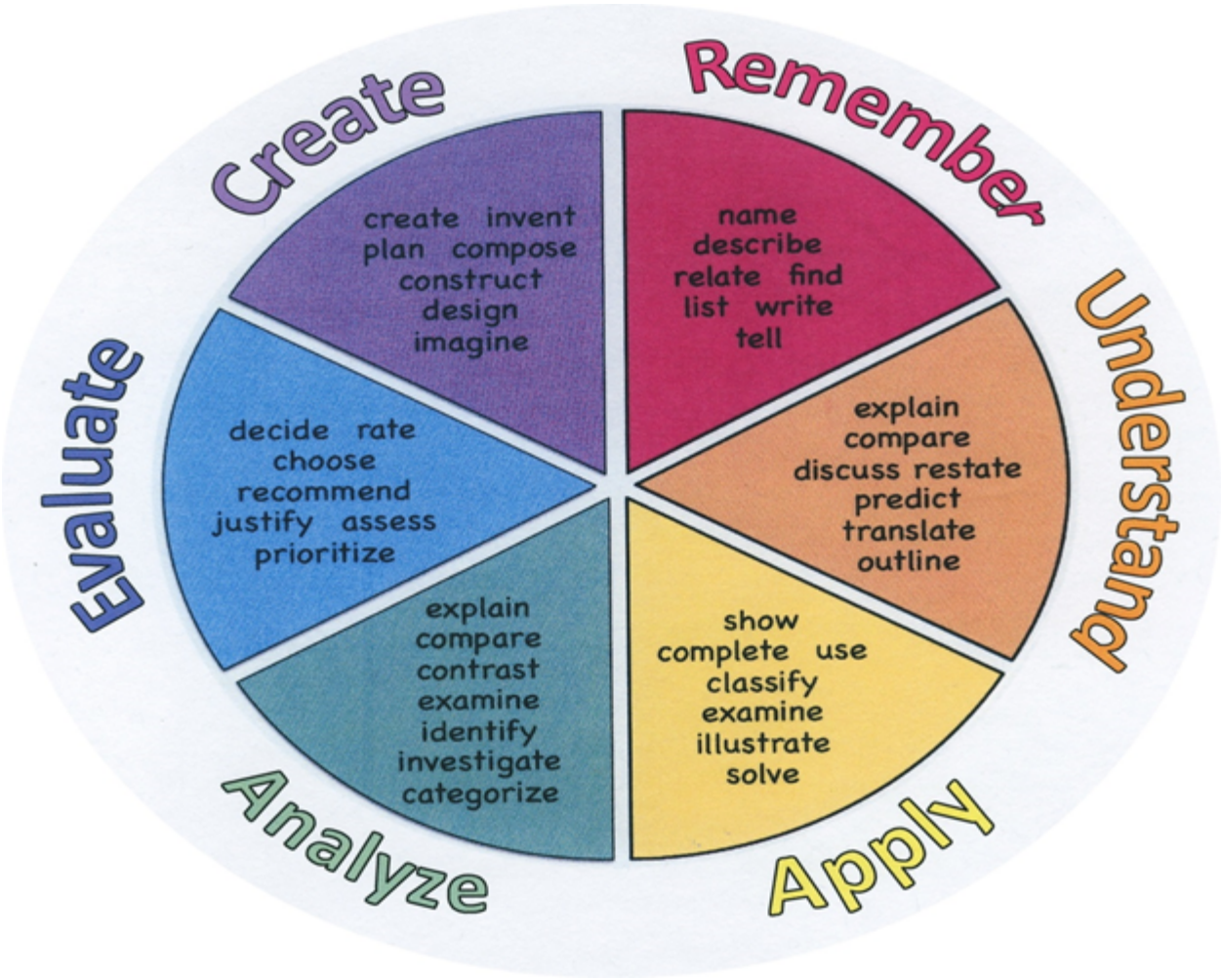
LA.RST.11-12.3	Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.
LA.RST.9-10.5	Analyze the relationships among concepts in a text, including relationships among key terms (e.g., force, friction, reaction force, energy).
SOC.6.3.12.CS3	Collaboratively evaluate possible solutions to problems and conflicts that arise in an interconnected world.
SOC.6.3.12.CS4	Critically analyze information, make ethical judgments, and responsibly address controversial issues.

Learning Objectives

- Interpret key terms relating to fair division (assets, players, value systems, fair share)
- Compare and contrast division games that are discrete and continuous
- Justify fair divisions between two people with the Divider-Chooser method
- Justify fair divisions between three or more people with the Lone-Divider method
- Justify fair divisions between three or more people with the Lone-Chooser method
- Justify fair divisions between multiple people with the method of sealed bids
- Justify fair divisions between multiple people with the method of markers

Action Verbs: Below are examples of action verbs associated with each level of the Revised Bloom's Taxonomy.

Remember	Understand	Apply	Analyze	Evaluate	Create
Choose	Classify	Choose	Categorize	Appraise	Combine
Describe	Defend	Dramatize	Classify	Judge	Compose
Define	Demonstrate	Explain	Compare	Criticize	Construct
Label	Distinguish	Generalize	Differentiate	Defend	Design
List	Explain	Judge	Distinguish	Compare	Develop
Locate	Express	Organize	Identify	Assess	Formulate
Match	Extend	Paint	Infer	Conclude	Hypothesize
Memorize	Give Examples	Prepare	Point out	Contrast	Invent
Name	Illustrate	Produce	Select	Critique	Make
Omit	Indicate	Select	Subdivide	Determine	Originate
Recite	Interrelate	Show	Survey	Grade	Organize
Select	Interpret	Sketch	Arrange	Justify	Plan
State	Infer	Solve	Breakdown	Measure	Produce
Count	Match	Use	Combine	Rank	Role Play
Draw	Paraphrase	Add	Detect	Rate	Drive
Outline	Represent	Calculate	Diagram	Support	Devise
Point	Restate	Change	Discriminate	Test	Generate
Quote	Rewrite	Classify	Illustrate		Integrate
Recall	Select	Complete	Outline		Prescribe
Recognize	Show	Compute	Point out		Propose
Repeat	Summarize	Discover	Separate		Reconstruct
Reproduce	Tell	Divide			Revise
	Translate	Examine			Rewrite
	Associate	Graph			Transform
	Compute	Interpolate			
	Convert	Manipulate			
	Discuss	Modify			
	Estimate	Operate			
	Extrapolate	Subtract			
	Generalize				
	Predict				



Suggested Activities & Best Practices

STUDENTS WILL REACH OBJECTIVES AND ACQUIRE SKILLS & UNDERSTANDING THROUGH:

- Examination and performance on problems selected from the texts
- Student groups with assigned specific roles that can assist each other in overall understanding
- Exit tickets to offer additional summary of key concepts, level of understanding and additional questions

Assessment Evidence - Checking for Understanding (CFU)

- Exit tickets at the close of each lesson will address definitions, concepts and formulas (EX: Recognize why certain apportionments break specific rules at a given moment) (Formative)
- Chapter Test/Quiz (Summative)

- Common Quarterly/Benchmark Exams - Quarter2 Exam for this unit (Benchmark)
- Web-Based Assessments (using Google Forms, ALEKS, Edulastic, Khan Academy, etc.) (Formative/Summative)

- Admit Tickets
- Common Benchmarks
- Compare & Contrast
- Define
- Describe
- Evaluate
- Evaluation rubrics
- Exit Tickets
- Explaining
- Quizzes
- Self- assessments
- Study Guide
- Teacher Observation Checklist
- Think, Pair, Share
- Think, Write, Pair, Share
- Unit review/Test prep
- Unit tests
- Web-Based Assessments

Primary Resources & Materials

- Excursions in Modern Mathematics 9th edition textbook (Frank Tannenbaum)
- Excursions in Modern Mathematics 6th edition textbook (Frank Tannenbaum)

Ancillary Resources

Sample web pages based on material are included here. This list will be edited as more reference material is found.

- http://www.mscf.uky.edu/~lee/ma111fa11/Sharing_Basics.pdf
- http://www.mscf.uky.edu/~lee/ma111fa11/DividerChooser2Player.pdf
- http://www.mscf.uky.edu/~lee/ma111fa11/LoneDivider.pdf
- http://www.mscf.uky.edu/~lee/ma111fa11/LectureNotes12-02.pdf
- http://www.mscf.uky.edu/~lee/ma111fa11/LectureNotes12-05.pdf
- http://www.mscf.uky.edu/~lee/ma111fa11/LectureNotes12-07.pdf

Technology Infusion

GOOGLE SHEETS: Students will use Google Sheets within their Chromebooks for the tasks described:

- CAKE-DIVISIONS: Construction of formulas and tables for finding equal fair shares as a divider, and overall fair values as a chooser
- MARKERS: Construction of formulas and tables to determine running totals of arrays of goods, and determine positioning of markers for fair value

SMART TV: Real-time displays of sealed bid and markers experiments for the purposes of conducting simulated fair divisions



Alignment to 21st Century Skills & Technology

Mastery and infusion of **21st Century Skills & Technology** and their Alignment to the core content areas is essential to student learning. The core content areas include:

- English Language Arts;
- Mathematics;
- Social Studies, including American History, World History, Geography, Government and Civics, and Economics

CRP.K-12.CRP1	Act as a responsible and contributing citizen and employee.
CRP.K-12.CRP3	Attend to personal health and financial well-being.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP5	Consider the environmental, social and economic impacts of decisions.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP9	Model integrity, ethical leadership and effective management.
CRP.K-12.CRP11	Use technology to enhance productivity.
CAEP.9.2.12.C.4	Analyze how economic conditions and societal changes influence employment trends and future education.
CAEP.9.2.12.C.7	Examine the professional, legal, and ethical responsibilities for both employers and employees in the global workplace.
CAEP.9.2.12.C.9	Analyze the correlation between personal and financial behavior and employability.
TECH.8.1.12.E.CS4	Process data and report results.
TECH.8.1.12.F.CS3	Collect and analyze data to identify solutions and/or make informed decisions.
TECH.8.1.12.F.CS4	Use multiple processes and diverse perspectives to explore alternative solutions.

21st Century Skills/Interdisciplinary Themes

- Communication and Collaboration
- Creativity and Innovation
- Critical thinking and Problem Solving
- Life and Career Skills

21st Century Skills

- Civic Literacy
- Financial, Economic, Business and Entrepreneurial Literacy

Differentiation

SPECIFIC EXAMPLES INCLUDE:

- Maniluatives/Group assignments: Students are grouped into 2 or 3 for instant examples of various division methods
- Study guides provided prior to quizzes and tests

Differentiations:

- Small group instruction
- Small group assignments
- Extra time to complete assignments
- Pairing oral instruction with visuals
- Repeat directions
- Use manipulatives
- Study guides
- Rephrase written directions
- Additional time
- Preview vocabulary
- Preview content & concepts
- Highlight text
- Student(s) work with assigned partner
- Visual presentation
- Assistive technology
- Small group setting

Hi-Prep Differentiations:

- Alternative formative and summative assessments
- Games and tournaments
- Group investigations
- Independent research and projects
- Interest groups
- Project-based learning
- Problem-based learning
- Tiered activities/assignments
- Varying organizers for instructions

Lo-Prep Differentiations:

- Exploration by interest
- Flexible grouping
- Goal setting with students
- Mini workshops to re-teach or extend skills
- Open-ended activities
- Think-Pair-Share
- Varied supplemental materials

Special Education Learning (IEP's & 504's)

SPECIFIC EXAMPLES INCLUDE:

- Note cards for assembling Google Sheet formulas for various forms of division share values
- One-on-one oral questioning during testing to elicit responses

- printed copy of board work/notes provided
- additional time for skill mastery
- assistive technology
- behavior management plan
- Center-Based Instruction
- check work frequently for understanding
- computer or electronic device utilizes
- extended time on tests/ quizzes
- have student repeat directions to check for understanding
- highlighted text visual presentation
- modified assignment format
- modified test content
- modified test format
- modified test length
- multi-sensory presentation
- multiple test sessions
- preferential seating
- preview of content, concepts, and vocabulary
- Provide modifications as dictated in the student's IEP/504 plan
- reduced/shortened reading assignments
- Reduced/shortened written assignments
- secure attention before giving instruction/directions
- shortened assignments
- student working with an assigned partner
- teacher initiated weekly assignment sheet
- Use open book, study guides, test prototypes

English Language Learning (ELL)

SPECIFIC EXAMPLES INCLUDE:

- Translated material
- Peer partners for assignments and tests with students that can translate material and meanings of concepts verbally

- teaching key aspects of a topic. Eliminate nonessential information
- using videos, illustrations, pictures, and drawings to explain or clarif
- allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards, charts, graphs, slide shows, videos, etc.) to demonstrate student’s learning;
- allowing students to correct errors (looking for understanding)
- allowing the use of note cards or open-book during testing
- decreasing the amount of workpresented or required

- having peers take notes or providing a copy of the teacher's notes
- modifying tests to reflect selected objectives
- providing study guides
- reducing or omitting lengthy outside reading assignments
- reducing the number of answer choices on a multiple choice test
- tutoring by peers
- using computer word processing spell check and grammar check features
- using true/false, matching, or fill in the blank tests in lieu of essay tests

At Risk

SPECIFIC EXAMPLES INCLUDE:

- Printed or video copy of material missed during excessive absences
- Corrections of incorrect work from tests
- Rewriting of test questions to include options for formuals (lone-divider bidding) for student to execute within the work on free-response test questions

- allowing students to correct errors (looking for understanding)
- teaching key aspects of a topic. Eliminate nonessential information
- allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards, charts, graphs, slide shows, videos, etc.) to demonstrate student's learning
- allowing students to select from given choices
- allowing the use of note cards or open-book during testing
- collaborating (general education teacher and specialist) to modify vocabulary, omit or modify items to reflect objectives for the student, eliminate sections of the test, and determine how the grade will be determined prior to giving the test.
- decreasing the amount of workpresented or required
- having peers take notes or providing a copy of the teacher's notes
- marking students' correct and acceptable work, not the mistakes
- modifying tests to reflect selected objectives
- providing study guides
- reducing or omitting lengthy outside reading assignments
- reducing the number of answer choices on a multiple choice test
- tutoring by peers
- using authentic assessments with real-life problem-solving
- using true/false, matching, or fill in the blank tests in lieu of essay tests
- using videos, illustrations, pictures, and drawings to explain or clarify

Talented and Gifted Learning (T&G)

SPECIFIC EXAMPLES INCLUDE:

- Complete "Running"-level problems in textbook containing higher-level thinking

- Student can construct original examples that can demonstrate full mastery of specific concepts and objectives
- Provide students with resources to allow them to move forward at a faster pace when they display faster mastery of learning objectives

- Above grade level placement option for qualified students
- Advanced problem-solving
- Allow students to work at a faster pace
- Complete activities aligned with above grade level text using Benchmark results
- Create a plan to solve an issue presented in the class or in a text
- Higher order, critical & creative thinking skills, and discovery
- Multi-disciplinary unit and/or project
- Teacher-selected instructional strategies that are focused to provide challenge, engagement, and growth opportunities
- Utilize exploratory connections to higher-grade concepts
- Utilize project-based learning for greater depth of knowledge

Sample Lesson

Using the template below, please develop a **Sample Lesson** for the first unit only.

Unit Name:

NJSLS:

Interdisciplinary Connection:

Statement of Objective:

Anticipatory Set/Do Now:

Learning Activity:

Student Assessment/CFU's:

Materials:

21st Century Themes and Skills:

Differentiation/Modifications:

Integration of Technology: