Unit 2: The Mathematics of Growth

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Title Section

Department of Curriculum and Instruction

Belleville Public Schools

Curriculum Guide

DISCRETE MATHEMATICS & STATISTICS, GRADES 11/12

THE MATHEMATICS OF GROWTH

Belleville Board of Education

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Unit Overview

In this unit, students will learn mathematical applications related to growth, decay, profit and loss.

These applications include:

- Modeling of population growth of people and other quantifiable objects
- Modeling of various forms of financial mathematics, including interest, investments, retirement, loans and debt

Enduring Understanding

IN THIS CHAPTER, STUDENTS WILL UNDERSTAND:

- Populations can grow or decay by a specific pattern or rate.
- Based on its growth rate, a population may approach a specific amount over an infinite period of time, or may forever oscillate.
- There are several types of interest that each affect the growth and future value of an investment or debt over a period of time.

Essential Questions

IN THIS CHAPTER, WE WILL ASK:

- How can we measure and predict the way that a population will increase or decrease over time?
- When, if ever, will a population reach a certain value?
- What will be the total amount of a periodically collected inventory that grows at a specific rate?
- How does interest cause an investment or debt to grow?
- What is the future value of an investment?
- How long will it take to pay off a loan?

Exit Skills

BY THE END OF THIS UNIT, THE STUDENT SHOULD BE ABLE TO:

- Apply recursions to calculate and create population sequences
- Apply recursions to and create explicit formulas for growth models of a population
- Calculate simple percentages
- Apply percentages to calculate simple and compound interest
- Apply percentages to calculate retirement savings
- Apply percentages to determine how to finance a debt

New Jersey Student Learning Standards (NJSLS)

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.F-IF.A.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.7	Look for and make use of structure.

MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.F-BF.A.1a	Determine an explicit expression, a recursive process, or steps for calculation from a context.
MA.F-BF.A.1b	Combine standard function types using arithmetic operations.
MA.F-BF.A.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
MA.F-LE.A.1	Distinguish between situations that can be modeled with linear functions and with exponential functions.
MA.F-LE.A.1a	Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
MA.F-LE.A.1b	Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
MA.F-LE.A.1c	Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
MA.F-LE.B.5	Interpret the parameters in a linear or exponential function in terms of a context.

Interdisciplinary Connections

PFL.9.1.12.A.4	Identify a career goal and develop a plan and timetable for achieving it, including educational/training requirements, costs, and possible debt.
PFL.9.1.12.A.6	Summarize the financial risks and benefits of entrepreneurship as a career choice.
PFL.9.1.12.B.1	Prioritize financial decisions by systematically considering alternatives and possible consequences.
PFL.9.1.12.B.2	Compare strategies for saving and investing and the factors that influence how much should be saved or invested to meet financial goals.
PFL.9.1.12.B.3	Construct a plan to accumulate emergency “rainy day” funds.
PFL.9.1.12.B.8	Describe and calculate interest and fees that are applied to various forms of spending, debt, and saving.
PFL.9.1.12.C.2	Compare and compute interest and compound interest and develop an amortization table using business tools.
PFL.9.1.12.C.3	Compute and assess the accumulating effect of interest paid over time when using a variety of sources of credit.
PFL.9.1.12.C.4	Compare and contrast the advantages and disadvantages of various types of mortgages.
PFL.9.1.12.D.1	Calculate short- and long-term returns on various investments (e.g., stocks, bonds, mutual funds, IRAs, deferred pension plans, and so on).
PFL.9.1.12.D.4	Assess factors that influence financial planning.
PFL.9.1.12.D.5	Justify the use of savings and investment options to meet targeted goals.
PFL.9.1.12.D.9	Relate savings and investment results to achievement of financial goals.

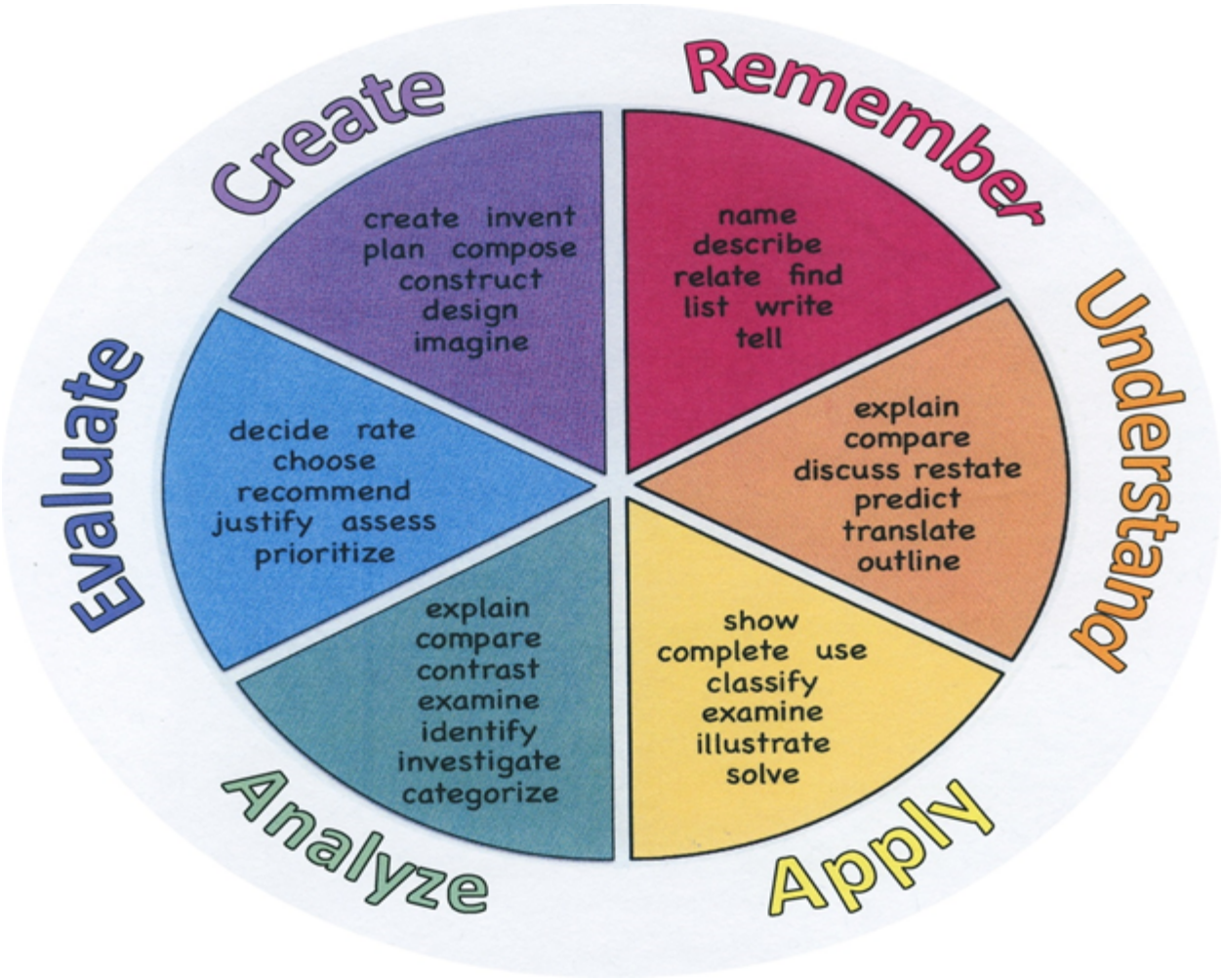
Learning Objectives

- Generate a population sequence by calculating with a recursive formula
- Construct an explicit formula for a recursive growth formula based on its type (linear, exponential, logistic)
- Compare and contrast each type of growth formula
- Conclude if and what the long-term value that a logistic growth model is

- Calculate simple percentages from a model situation
- Calculate the final value of a simple or compound interest investment
- Compare and contrast a series of simple and compound interest investments
- Calculate the final value of a retirement savings account
- Generate a payment plan for a loan debt

Action Verbs: Below are examples of action verbs associated with each level of the Revised Bloom's Taxonomy.

Remember	Understand	Apply	Analyze	Evaluate	Create
Choose	Classify	Choose	Categorize	Appraise	Combine
Describe	Defend	Dramatize	Classify	Judge	Compose
Define	Demonstrate	Explain	Compare	Criticize	Construct
Label	Distinguish	Generalize	Differentiate	Defend	Design
List	Explain	Judge	Distinguish	Compare	Develop
Locate	Express	Organize	Identify	Assess	Formulate
Match	Extend	Paint	Infer	Conclude	Hypothesize
Memorize	Give Examples	Prepare	Point out	Contrast	Invent
Name	Illustrate	Produce	Select	Critique	Make
Omit	Indicate	Select	Subdivide	Determine	Originate
Recite	Interrelate	Show	Survey	Grade	Organize
Select	Interpret	Sketch	Arrange	Justify	Plan
State	Infer	Solve	Breakdown	Measure	Produce
Count	Match	Use	Combine	Rank	Role Play
Draw	Paraphrase	Add	Detect	Rate	Drive
Outline	Represent	Calculate	Diagram	Support	Devise
Point	Restate	Change	Discriminate	Test	Generate
Quote	Rewrite	Classify	Illustrate		Integrate
Recall	Select	Complete	Outline		Prescribe
Recognize	Show	Compute	Point out		Propose
Repeat	Summarize	Discover	Separate		Reconstruct
Reproduce	Tell	Divide			Revise
	Translate	Examine			Rewrite
	Associate	Graph			Transform
	Compute	Interpolate			
	Convert	Manipulate			
	Discuss	Modify			
	Estimate	Operate			
	Extrapolate	Subtract			
	Generalize				
	Predict				



Suggested Activities & Best Practices

STUDENTS WILL REACH OBJECTIVES AND ACQUIRE SKILLS & UNDERSTANDING THROUGH:

- Examination and performance on problems selected from the texts
- Student groups with assigned specific roles that can assist each other in overall understanding
- Exit tickets to offer additional summary of key concepts, level of understanding and additional student questions
- Project-based learning for students to play investment and interest games

Assessment Evidence - Checking for Understanding (CFU)

- Exit tickets at the close of each lesson will address definitions, concepts and formulas (EX: Recognize why certain apportionments break specific rules at a given moment) (Formative)

- Demonstrating a proper comparison of voting methods for the same set of voting ballots (Formative)
- Chapter Test/Quiz (Summative)
- Common Quarterly/Benchmark Exams - Quarter 1 Exam for this unit (Benchmark)
- Web-Based Assessments (using Google Forms, ALEKS, Edulastic, Khan Academy, etc.) (Formative/Summative)

- Admit Tickets
- Common Benchmarks
- Compare & Contrast
- Define
- Describe
- Evaluate
- Evaluation rubrics
- Exit Tickets
- Explaining
- Quizzes
- Self- assessments
- Study Guide
- Teacher Observation Checklist
- Think, Pair, Share
- Think, Write, Pair, Share
- Unit review/Test prep
- Unit tests
- Web-Based Assessments

Primary Resources & Materials

- Excursions in Modern Mathematics 9th edition textbook (Frank Tannenbaum)
- Excursions in Modern Mathematics 6th edition textbook (Frank Tannenbaum)

Ancillary Resources

Sample web pages based on material are included here. This list will be edited as more reference material is found.

- http://www.ms.uky.edu/~lee/ma111fa11/slides10.pdf
- http://www.mscf.uky.edu/~lee/ma111fa11/CreditCardsKoester.pdf

Technology Infusion

GOOGLE SHEETS: Students will use Google Sheets within Chromebooks for the tasks described:

- GROWTH: Construction of formulas and displays growth sequences and identification of logistic growth limits
- FINANCE: Construction of formulas to calculate compound interest scenarios and sequences to determine final values and end dates for savings totals and financing length options

SMART TV: Real-time comparisons of growth and compound interest formulas based on changes in parameters



Alignment to 21st Century Skills & Technology

Mastery and infusion of **21st Century Skills & Technology** and their Alignment to the core content areas is essential to student

learning. The core content areas include:

- English Langauge Arts
- Mathematics;
- Science and Scientific Inquiry (Next Generation);
- Social Studies, including American History, World History, Geography, Government and Civics, and Economics;
- Technology

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP3	Attend to personal health and financial well-being.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP11	Use technology to enhance productivity.
CAEP.9.2.12.C.1	Review career goals and determine steps necessary for attainment.
CAEP.9.2.12.C.2	Modify Personalized Student Learning Plans to support declared career goals.
CAEP.9.2.12.C.4	Analyze how economic conditions and societal changes influence employment trends and future education.
CAEP.9.2.12.C.9	Analyze the correlation between personal and financial behavior and employability.
TECH.8.1.12.E.CS4	Process data and report results.
TECH.8.1.12.F.CS2	Plan and manage activities to develop a solution or complete a project.
TECH.8.1.12.F.CS3	Collect and analyze data to identify solutions and/or make informed decisions.
TECH.8.1.12.F.CS4	Use multiple processes and diverse perspectives to explore alternative solutions.

21st Century Skills/Interdisciplinary Themes

- Communication and Collaboration
- Creativity and Innovation
- Critical thinking and Problem Solving
- ICT (Information, Communications and Technology) Literacy
- Information Literacy
- Life and Career Skills
- Media Literacy

21st Century Skills

- Civic Literacy
- Environmental Literacy
- Financial, Economic, Business and Entrepreneurial Literacy
- Global Awareness

Differentiation

SPECIFIC EXAMPLES INCLUDE:

- Game projects where students can determine proper compound interest investments for long-term growth
- Study guides provided prior to quizzes and tests

Differentiations:

- Small group instruction
- Small group assignments
- Extra time to complete assignments
- Pairing oral instruction with visuals
- Repeat directions
- Use manipulatives
- Study guides
- Rephrase written directions
- Additional time
- Preview vocabulary
- Preview content & concepts
- Highlight text
- Student(s) work with assigned partner
- Visual presentation
- Assistive technology
- Small group setting

Hi-Prep Differentiations:

- Alternative formative and summative assessments
- Games and tournaments
- Group investigations
- Independent research and projects
- Interest groups
- Project-based learning
- Problem-based learning
- Tiered activities/assignments
- Varying organizers for instructions

Lo-Prep Differentiations:

- Exploration by interest
- Flexible grouping
- Goal setting with students
- Mini workshops to re-teach or extend skills
- Open-ended activities
- Think-Pair-Share
- Varied supplemental materials

Special Education Learning (IEP's & 504's)

SPECIFIC EXAMPLES INCLUDE:

- Note cards for assembling Google Sheet formulas for exponential growth, logistic growth, interest, etc.
- One-on-one oral questioning during testing to elicit responses

- printed copy of board work/notes provided
- additional time for skill mastery
- assistive technology
- behavior management plan
- Center-Based Instruction
- check work frequently for understanding
- computer or electronic device utilizes
- extended time on tests/ quizzes
- have student repeat directions to check for understanding
- highlighted text visual presentation
- modified assignment format
- modified test content
- modified test format
- modified test length
- multi-sensory presentation
- multiple test sessions
- preferential seating
- preview of content, concepts, and vocabulary
- Provide modifications as dictated in the student's IEP/504 plan
- reduced/shortened reading assignments
- Reduced/shortened written assignments
- secure attention before giving instruction/directions
- shortened assignments
- student working with an assigned partner
- teacher initiated weekly assignment sheet
- Use open book, study guides, test prototypes

English Language Learning (ELL)

SPECIFIC EXAMPLES INCLUDE:

- Translated material
- Peer partners for assignments and tests with students that can translate material and meanings of concepts verbally

- teaching key aspects of a topic. Eliminate nonessential information
- using videos, illustrations, pictures, and drawings to explain or clarif

- allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards, charts, graphs, slide shows, videos, etc.) to demonstrate student's learning;
- allowing students to correct errors (looking for understanding)
- allowing the use of note cards or open-book during testing
- decreasing the amount of workpresented or required
- having peers take notes or providing a copy of the teacher's notes
- modifying tests to reflect selected objectives
- providing study guides
- reducing or omitting lengthy outside reading assignments
- reducing the number of answer choices on a multiple choice test
- tutoring by peers
- using computer word processing spell check and grammar check features
- using true/false, matching, or fill in the blank tests in lieu of essay tests

At Risk

SPECIFIC EXAMPLES INCLUDE:

- Printed or video copy of material missed during excessive absences
- Corrections of incorrect work from tests
- Rewriting of test questions to include options for formuals (exponential growth, logistic growth, interest, percentage yield) for student to execute within the work on free-response test questions

- allowing students to correct errors (looking for understanding)
- teaching key aspects of a topic. Eliminate nonessential information
- allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards, charts, graphs, slide shows, videos, etc.) to demonstrate student's learning
- allowing students to select from given choices
- allowing the use of note cards or open-book during testing
- collaborating (general education teacher and specialist) to modify vocabulary, omit or modify items to reflect objectives for the student, eliminate sections of the test, and determine how the grade will be determined prior to giving the test.
- decreasing the amount of workpresented or required
- having peers take notes or providing a copy of the teacher's notes
- marking students' correct and acceptable work, not the mistakes
- modifying tests to reflect selected objectives
- providing study guides
- reducing or omitting lengthy outside reading assignments
- reducing the number of answer choices on a multiple choice test
- tutoring by peers
- using authentic assessments with real-life problem-solving
- using true/false, matching, or fill in the blank tests in lieu of essay tests
- using videos, illustrations, pictures, and drawings to explain or clarify

Talented and Gifted Learning (T&G)

SPECIFIC EXAMPLES INCLUDE:

- Complete "Running"-level problems in textbook containing higher-level thinking
- Student can construct original examples that can demonstrate full mastery of specific concepts and objectives
- Provide students with resources to allow them to move forward at a faster pace when they display faster mastery of learning objectives

- Above grade level placement option for qualified students
- Advanced problem-solving
- Allow students to work at a faster pace
- Complete activities aligned with above grade level text using Benchmark results
- Create a plan to solve an issue presented in the class or in a text
- Higher order, critical & creative thinking skills, and discovery
- Multi-disciplinary unit and/or project
- Teacher-selected instructional strategies that are focused to provide challenge, engagement, and growth opportunities
- Utilize exploratory connections to higher-grade concepts
- Utilize project-based learning for greater depth of knowledge

Sample Lesson

Using the template below, please develop a **Sample Lesson** for the first unit only.

Unit Name:

NJSLS:

Interdisciplinary Connection:

Statement of Objective:

Anticipatory Set/Do Now:

Learning Activity:

Student Assessment/CFU's:

Materials:

21st Century Themes and Skills:

Differentiation/Modifications:

Integration of Technology: