Unit 6: The Statistics of Chance & Risk

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Title Section

Department of Curriculum and Instruction

Belleville Public Schools

Curriculum Guide

DISCRETE MATHEMATICS & STATISTICS, GRADES 11/12

THE STATISTICS OF CHANCE & RISK

Belleville Board of Education

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Unit Overview

In this unit, students will learn mathematical applications related to the collection, organization and interpretation of statistical data.

These applications include:

- How mathematics are used to measure uncertainty and risk in a real-life situation of chance

Enduring Understanding

IN THIS UNIT, STUDENTS WILL UNDERSTAND:

- There are mathematical ways to determine how many ways an event can occur.
- There are mathematical ways to determine the uncertainty of a specific outcome(s) of an event.
- There are different ways in which the uncertainty of an outcome can be quantified and expressed.
- Knowing the uncertainty of an outcome can assist in decision-making.

Essential Questions

IN THIS UNIT, WE WILL ASK:

- How can all of the possible outcomes of an event be determined?
- How can the level of uncertainty of a specific outcome(s) of an event be determined?
- How can a level of uncertainty be expressed in a quantitative manner?
- How can the level of uncertainty be used in decision-making?

Exit Skills

By the end of this unit, the student should be able to:

- Understand the concepts and terms of a basic probability framework, including random experiment, sample space and probability spaces
- Calculate, interpret, and understand the differences between probability, chances and odds
- Calculate and interpret permutations and combinations, and apply them to calculate probability, chances and odds
- Apply probabilities to calculate the expected value of a random experiment, and use the results to assist in decision-making

New Jersey Student Learning Standards (NJSLS)

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.S-CP.A.1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).
MA.S-CP.A.2	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
MA.S-CP.A.3	Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
MA.S-CP.A.5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
MA.S-CP.B	Use the rules of probability to compute probabilities of compound events in a uniform

	probability model
MA.S-CP.B.6	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
MA.S-CP.B.7	Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.
MA.S-CP.B.8	Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = [P(A)] \times [P(B \mid A)] = [P(B)] \times [P(A \mid B)]$, and interpret the answer in terms of the model.
MA.S-CP.B.9	Use permutations and combinations to compute probabilities of compound events and solve problems.
MA.S-MD	Using Probability to Make Decisions
MA.S-MD.A	Calculate expected values and use them to solve problems
MA.S-MD.A.1	Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
MA.S-MD.A.2	Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
MA.S-MD.A.3	Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.
MA.S-MD.A.4	Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.
MA.S-MD.B	Use probability to evaluate outcomes of decisions
MA.S-MD.B.5	Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
MA.S-MD.B.5a	Find the expected payoff for a game of chance.
MA.S-MD.B.5b	Evaluate and compare strategies on the basis of expected values.
MA.S-MD.B.6	Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
MA.S-MD.B.7	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Interdisciplinary Connections

LA.RST.11-12.3	Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.
LA.RST.9-10.5	Analyze the relationships among concepts in a text, including relationships among key terms (e.g., force, friction, reaction force, energy).
9-12.HS-ETS1-4	Use a computer simulation to model the impact of proposed solutions to a complex real-world problem with numerous criteria and constraints on interactions within and between systems relevant to the problem.

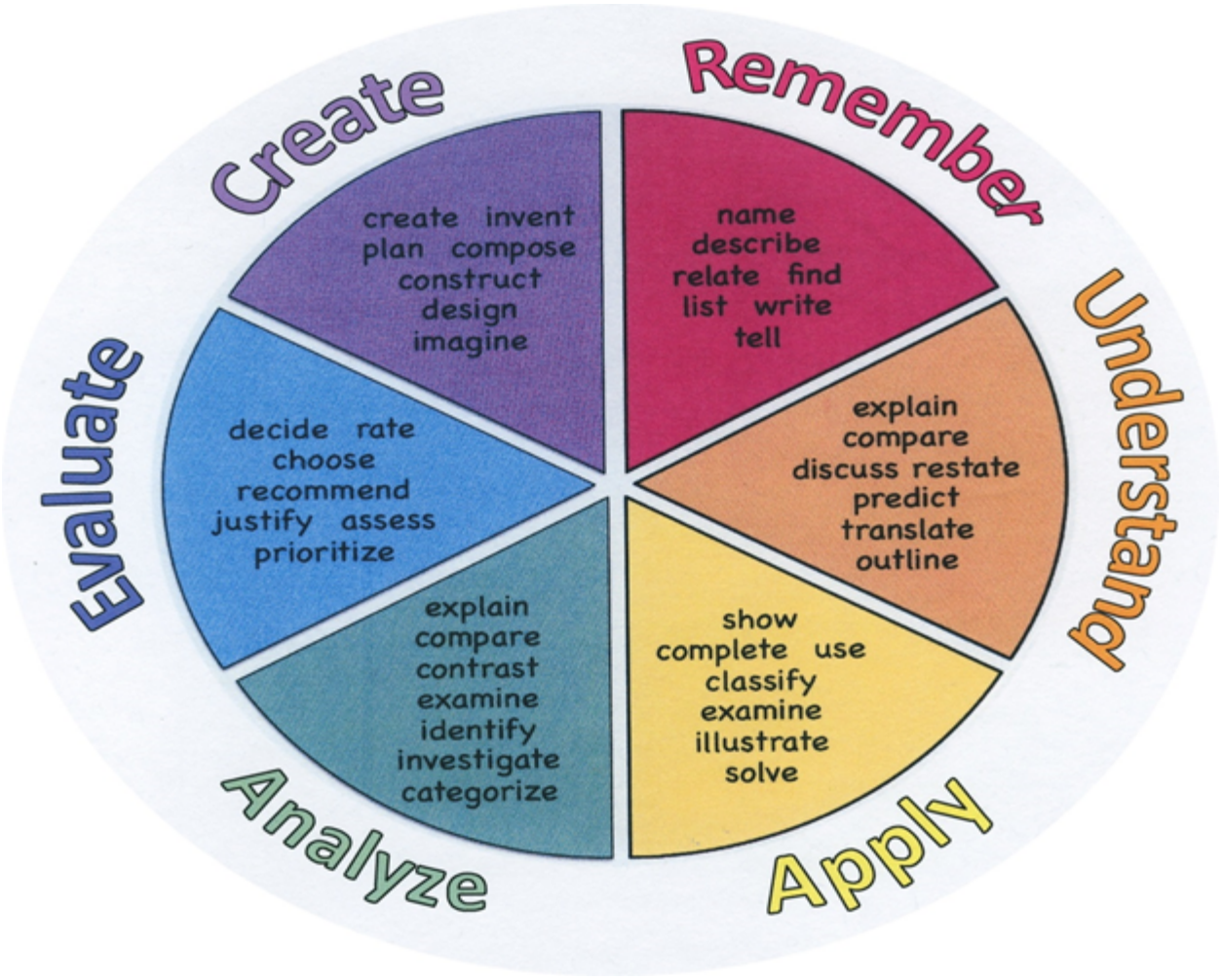
Learning Objectives

- Interpret the key terms relating to probability (random experiment, sample space, event, probability space)
- Calculate permutations and combinations to determine sample spaces for an event
- Calculate the probability, chances and odds for an event, and compare and contrast the results

- Combine probabilities with weights to calculate the expected value of a random experiment
- Judge the results of an expected value calculation to hypothesize the risk of a random event

Action Verbs: Below are examples of action verbs associated with each level of the Revised Bloom's Taxonomy.

Remember	Understand	Apply	Analyze	Evaluate	Create
Choose	Classify	Choose	Categorize	Appraise	Combine
Describe	Defend	Dramatize	Classify	Judge	Compose
Define	Demonstrate	Explain	Compare	Criticize	Construct
Label	Distinguish	Generalize	Differentiate	Defend	Design
List	Explain	Judge	Distinguish	Compare	Develop
Locate	Express	Organize	Identify	Assess	Formulate
Match	Extend	Paint	Infer	Conclude	Hypothesize
Memorize	Give Examples	Prepare	Point out	Contrast	Invent
Name	Illustrate	Produce	Select	Critique	Make
Omit	Indicate	Select	Subdivide	Determine	Originate
Recite	Interrelate	Show	Survey	Grade	Organize
Select	Interpret	Sketch	Arrange	Justify	Plan
State	Infer	Solve	Breakdown	Measure	Produce
Count	Match	Use	Combine	Rank	Role Play
Draw	Paraphrase	Add	Detect	Rate	Drive
Outline	Represent	Calculate	Diagram	Support	Devise
Point	Restate	Change	Discriminate	Test	Generate
Quote	Rewrite	Classify	Illustrate		Integrate
Recall	Select	Complete	Outline		Prescribe
Recognize	Show	Compute	Point out		Propose
Repeat	Summarize	Discover	Separate		Reconstruct
Reproduce	Tell	Divide			Revise
	Translate	Examine			Rewrite
	Associate	Graph			Transform
	Compute	Interpolate			
	Convert	Manipulate			
	Discuss	Modify			
	Estimate	Operate			
	Extrapolate	Subtract			
	Generalize				
	Predict				



Suggested Activities & Best Practices

STUDENTS WILL REACH OBJECTIVES AND ACQUIRE SKILLS & UNDERSTANDING THROUGH:

- Examination and performance on problems selected from the texts
- Student groups with assigned specific roles that can assist each other in overall understanding
- Exit tickets to offer additional summary of key concepts, level of understanding and additional questions
- Project-based learning for students to play probability and expected value games

Assessment Evidence - Checking for Understanding (CFU)

SPECIFIC SAMPLES INCLUDE:

- Exit tickets at the close of each lesson will address definitions, concepts and formulas (EX: Recognize where events are

permutations or combinations) (Formative)
- Define lists of combinations after calculating them (Formative)
- Chapter Test/Quiz (Summative)
- Common Quarterly/Benchmark Exams - Quarter 4 Exam for this unit (Benchmark)
- Web-Based Assessments (using Google Forms, ALEKS, Edulastic, Khan Academy, etc.) (Formative/Summative)

- Admit Tickets
- Anticipation Guide
- Common Benchmarks
- Compare & Contrast
- Define
- Describe
- Evaluate
- Evaluation rubrics
- Exit Tickets
- Explaining
- Quizzes
- Self- assessments
- Study Guide
- Surveys
- Teacher Observation Checklist
- Think, Pair, Share
- Think, Write, Pair, Share
- Unit review/Test prep
- Unit tests
- Web-Based Assessments

Primary Resources & Materials

- Excursions in Modern Mathematics 9th edition textbook (Frank Tannenbaum)
- Excursions in Modern Mathematics 6th edition textbook (Frank Tannenbaum)

Ancillary Resources

Sample web pages based on material are included here. This list will be edited as more reference material is found.

- https://slideplayer.com/slide/4547262/

Technology Infusion

GOOGLE SHEETS: Students will use Google Sheets within their Chromebooks for the tasks described:

- Construction of permutation and combination formulas
- Lists of perumation and combination layouts
- Construction of exclusive and inclusive probability tables
- Create formulas to calculate expected values

SMART TV: Real-time demonstrations of changes in expected value scenarios, and displays of interactive probability games



Alignment to 21st Century Skills & Technology

Mastery and infusion of **21st Century Skills & Technology** and their Alignment to the core content areas is essential to student learning. The core content areas include:

- English Language Arts;
- Mathematics;
- Technology

CRP.K-12.CRP1	Act as a responsible and contributing citizen and employee.
CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP3	Attend to personal health and financial well-being.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP5	Consider the environmental, social and economic impacts of decisions.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP10	Plan education and career paths aligned to personal goals.
CRP.K-12.CRP11	Use technology to enhance productivity.
PFL.9.1.12.A.6	Summarize the financial risks and benefits of entrepreneurship as a career choice.
PFL.9.1.12.D.13	Determine the impact of various market events on stock market prices and on other savings and investments.
CAEP.9.2.12.C.1	Review career goals and determine steps necessary for attainment.
CAEP.9.2.12.C.9	Analyze the correlation between personal and financial behavior and employability.
TECH.8.1.12.A.2	Produce and edit a multi-page digital document for a commercial or professional audience and present it to peers and/or professionals in that related area for review.
TECH.8.1.12.A.4	Construct a spreadsheet workbook with multiple worksheets, rename tabs to reflect the data on the worksheet, and use mathematical or logical functions, charts and data from all worksheets to convey the results.
TECH.8.1.12.B.CS2	Create original works as a means of personal or group expression.
TECH.8.1.12.C	Communication and Collaboration: Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
TECH.8.1.12.E.CS1	Plan strategies to guide inquiry.
TECH.8.1.12.E.CS3	Evaluate and select information sources and digital tools based on the appropriateness for specific tasks.
TECH.8.1.12.E.CS4	Process data and report results.
TECH.8.1.12.F.CS3	Collect and analyze data to identify solutions and/or make informed decisions.

21st Century Skills/Interdisciplinary Themes

- Communication and Collaboration
- Creativity and Innovation
- Critical thinking and Problem Solving
- ICT (Information, Communications and Technology) Literacy
- Life and Career Skills

21st Century Skills

- Civic Literacy
- Financial, Economic, Business and Entrepreneurial Literacy

Differentiation

SPECIFIC EXAMPLES INCLUDE:

- Small task-oriented groups where where each member is responsible for tabulating, solution layouts, etc. for probability or combination problems
- Games/Maniluatives: Live games using dice, cards, etc. for instant examples of probability, expected value, etc.
- Study guides provided prior to quizzes and tests

Differentiations:

- Small group instruction
- Small group assignments
- Extra time to complete assignments
- Pairing oral instruction with visuals
- Repeat directions
- Use manipulatives
- Study guides
- Rephrase written directions
- Additional time
- Preview vocabulary
- Preview content & concepts
- Highlight text
- Student(s) work with assigned partner
- Visual presentation
- Assistive technology
- Small group setting

Hi-Prep Differentiations:

- Alternative formative and summative assessments
- Games and tournaments
- Group investigations
- Independent research and projects
- Interest groups
- Project-based learning
- Problem-based learning
- Tiered activities/assignments
- Varying organizers for instructions

Lo-Prep Differentiations:

- Exploration by interest
- Flexible grouping

- Goal setting with students
- Mini workshops to re-teach or extend skills
- Open-ended activities
- Think-Pair-Share
- Varied supplemental materials

Special Education Learning (IEP's & 504's)

SPECIFIC EXAMPLES INCLUDE:

- Note cards for assembling Google Sheet formulas for probability, permutation and combination
- One-on-one oral questioning during testing to elicit responses

- printed copy of board work/notes provided
- additional time for skill mastery
- assistive technology
- behavior management plan
- Center-Based Instruction
- check work frequently for understanding
- computer or electronic device utilizes
- extended time on tests/ quizzes
- have student repeat directions to check for understanding
- highlighted text visual presentation
- modified assignment format
- modified test content
- modified test format
- modified test length
- multi-sensory presentation
- multiple test sessions
- preferential seating
- preview of content, concepts, and vocabulary
- Provide modifications as dictated in the student's IEP/504 plan
- reduced/shortened reading assignments
- Reduced/shortened written assignments
- secure attention before giving instruction/directions
- shortened assignments
- student working with an assigned partner
- teacher initiated weekly assignment sheet
- Use open book, study guides, test prototypes

English Language Learning (ELL)

SPECIFIC EXAMPLES INCLUDE:

- Translated material
- Peer partners for assignments and tests with students that can translate material and meanings of concepts verbally

- teaching key aspects of a topic. Eliminate nonessential information
- using videos, illustrations, pictures, and drawings to explain or clarif
- allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards, charts, graphs, slide shows, videos, etc.) to demonstrate student's learning;
- allowing students to correct errors (looking for understanding)
- allowing the use of note cards or open-book during testing
- decreasing the amount of workpresented or required
- having peers take notes or providing a copy of the teacher's notes
- modifying tests to reflect selected objectives
- providing study guides
- reducing or omitting lengthy outside reading assignments
- reducing the number of answer choices on a multiple choice test
- tutoring by peers
- using computer word processing spell check and grammar check features
- using true/false, matching, or fill in the blank tests in lieu of essay tests

At Risk

SPECIFIC EXAMPLES INCLUDE:

- Printed or video copy of material missed during excessive absences
- Corrections of incorrect work from tests
- Rewriting of test questions to include options for formulas (probability, permutation/combination) for student to execute within the work on free-response test questions

- allowing students to correct errors (looking for understanding)
- teaching key aspects of a topic. Eliminate nonessential information
- allowing products (projects, timelines, demonstrations, models, drawings, dioramas, poster boards, charts, graphs, slide shows, videos, etc.) to demonstrate student's learning
- allowing students to select from given choices
- allowing the use of note cards or open-book during testing
- collaborating (general education teacher and specialist) to modify vocabulary, omit or modify items to reflect objectives for the student, eliminate sections of the test, and determine how the grade will be determined prior to giving the test.
- decreasing the amount of workpresented or required
- having peers take notes or providing a copy of the teacher's notes
- marking students' correct and acceptable work, not the mistakes

- modifying tests to reflect selected objectives
- providing study guides
- reducing or omitting lengthy outside reading assignments
- reducing the number of answer choices on a multiple choice test
- tutoring by peers
- using authentic assessments with real-life problem-solving
- using true/false, matching, or fill in the blank tests in lieu of essay tests
- using videos, illustrations, pictures, and drawings to explain or clarify

Talented and Gifted Learning (T&G)

SPECIFIC EXAMPLES INCLUDE:

- Complete "Running"-level problems in textbook containing higher-level thinking
- Student can construct original examples that can demonstrate full mastery of specific concepts and objectives
- Provide students with resources to allow them to move forward at a faster pace when they display faster mastery of learning objectives

- Above grade level placement option for qualified students
- Advanced problem-solving
- Allow students to work at a faster pace
- Complete activities aligned with above grade level text using Benchmark results
- Create a plan to solve an issue presented in the class or in a text
- Higher order, critical & creative thinking skills, and discovery
- Multi-disciplinary unit and/or project
- Teacher-selected instructional strategies that are focused to provide challenge, engagement, and growth opportunities
- Utilize exploratory connections to higher-grade concepts
- Utilize project-based learning for greater depth of knowledge

Sample Lesson

Using the template below, please develop a **Sample Lesson** for the first unit only.

Unit Name:

NJSLS:

Interdisciplinary Connection:

Statement of Objective:

Anticipatory Set/Do Now:

Learning Activity:

Student Assessment/CFU's:

Materials:

21st Century Themes and Skills:

Differentiation/Modifications:

Integration of Technology: